# Hot axions and $N_{eff}$: towards an understanding of hot QCD effects

NUCLÉAIRE
& PARTICULES

Jacopo Ghiglieri, SUBATECH, Nantes

CosmoFONDUE, Genève, June 11th 2025

# My CV

2002-08
Bachelor&Master
Milan U.

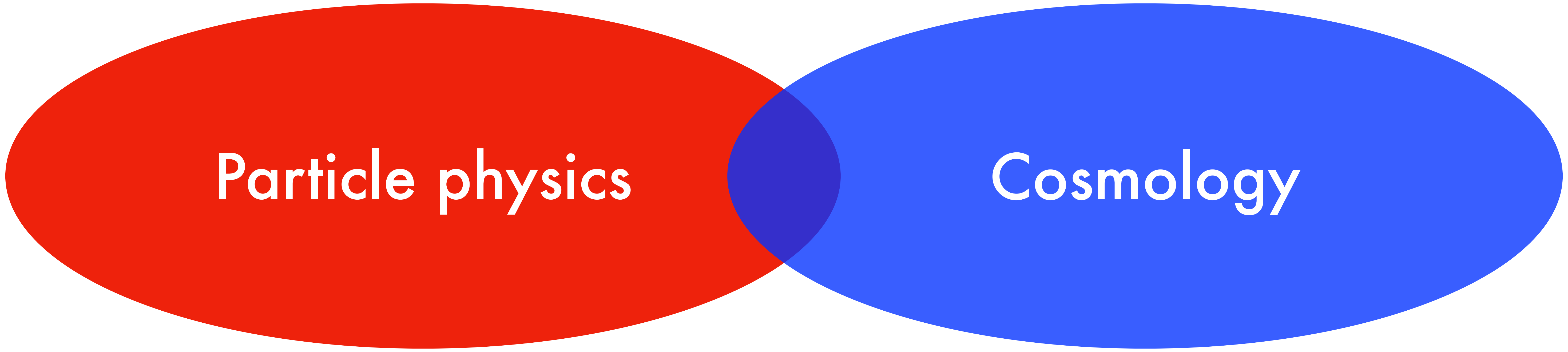
2008-11
PhD
Technical U. Munich

2011-14
Postdoc
McGill U., Montréal

2014-16
Postdoc
Bern U.

2016-19
CERN Fellow

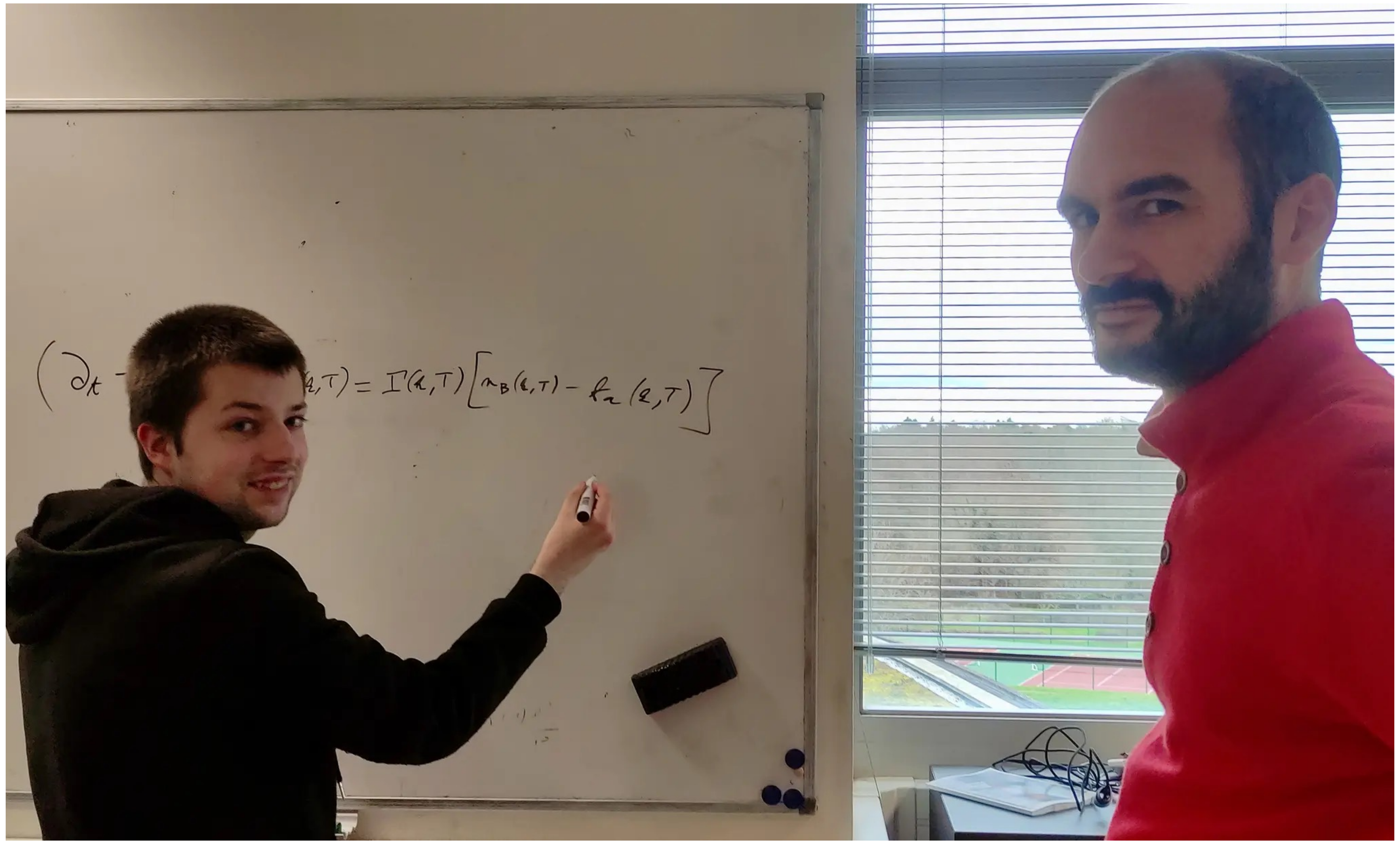
2019-
CNRS staff scientist
SUBATECH, Nantes




Application of *Thermal Field Theory* and *Effective Field Theories* to **heavy-ion collisions** and the **early universe**

# In this talk

- Meet the axion
- Hot axion freezeout
- Thermal axion production at LO: dos and don'ts
- Results and conclusions
- Based on work with Ph.D. candidate Killian Bouzoud, see Bouzoud JG **2404.06113** published on **JHEP**



# In this talk: the axion

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^{\mu\nu\, a} F^a_{\mu\nu} + \sum_{i=1}^{6} \bar{q}_i (i \not{D} - m_i) q_i + \,?$$

- **Strong CP problem:** a term $\propto \theta\, \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$ in the QCD Lagrangian is allowed by gauge symmetry and renormalizability. It would violate CP.
- Unobserved neutron electric dipole moment implies $|\theta| < 10^{-10}$. Why so small?

# In this talk: the axion

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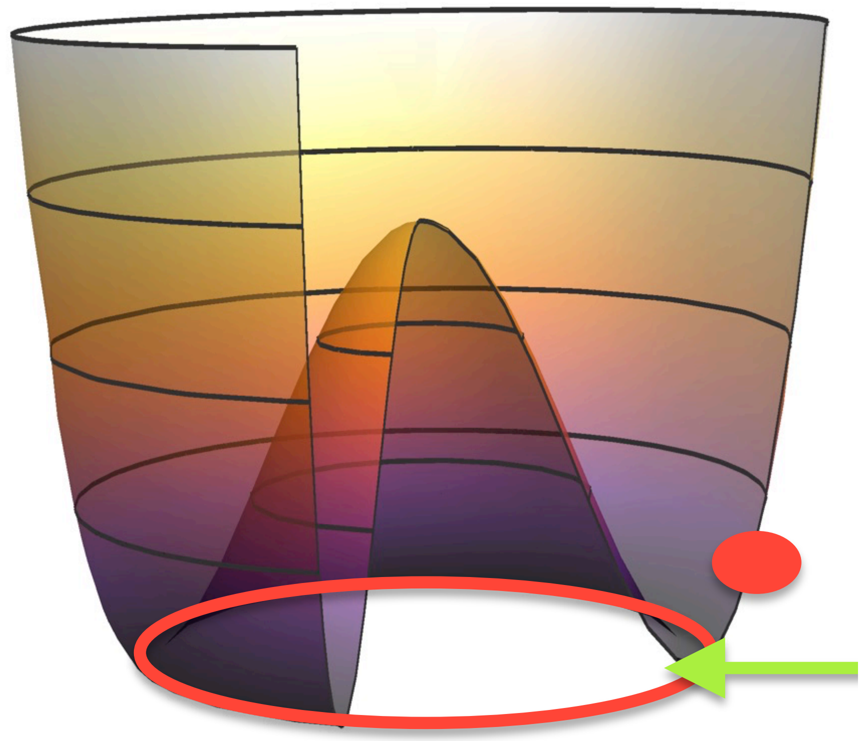
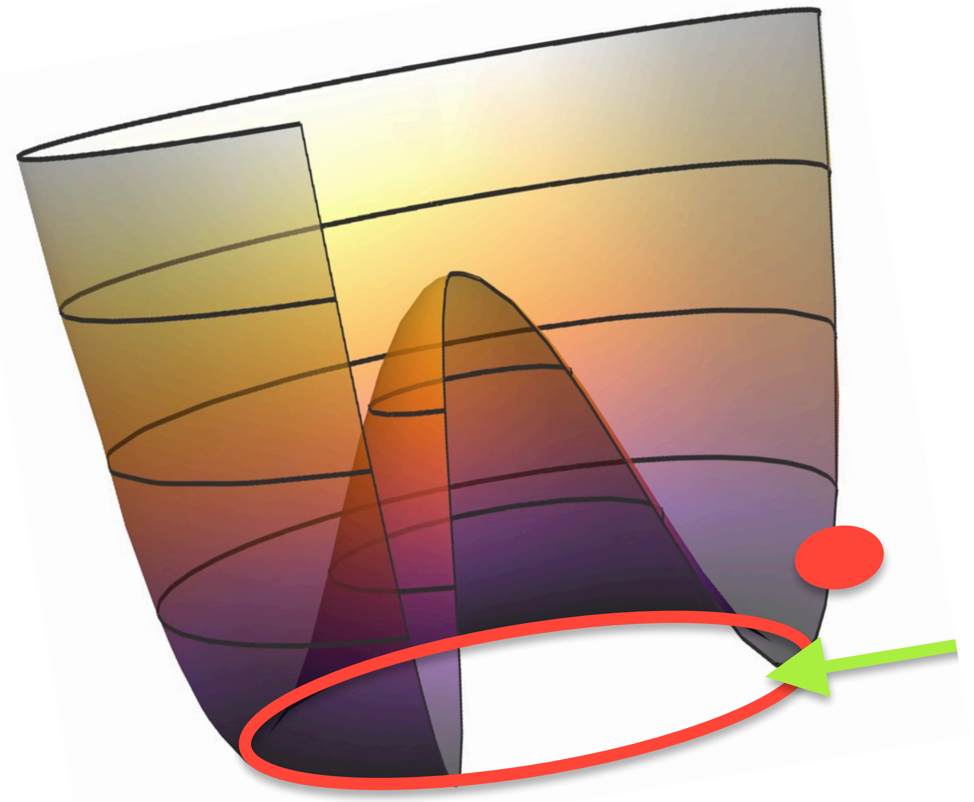
- Unobserved neutron electric dipole moment implies $|\theta| < 10^{-10}$. Why so small?
- A mechanism of spontaneous symmetry breaking would explain this, forcing $\theta \to 0$ with the **axion** as its light (would-be) Goldstone boson and "cleaning up" this problem Peccei Quinn, Weinberg, Wilczek

# Not in this talk: axion dark matter

- The axion can also "clean up" another problem: when $T \lesssim T_{\text{QCD}} \approx 200$ MeV chiral symmetry breaking "tilts" the potential



- The *misaligned* cosmological axion field starts oscillating* around this new non-degenerate minimum


**Cold dark matter** from ultralight bosons in the form of a *Bose condensate*


* Most scenarios feature non-trivial topology: strings and/or domain walls


# In this talk: hot axions


- DM is not the only expected axion population. Consider e.g. the so called invisible (KSVZ) axion Kim Shifman Vainshtein Zakharov
  $\mathscr{L}_{\text{int}} = -\frac{\alpha_s}{16\pi}\frac{a}{f_{\text{PQ}}}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$, with $a$ the axion field and $f_{\text{PQ}}$ the symmetry-breaking scale, $f_{\text{PQ}} > 4 \times 10^8$ **GeV from astrophysics.**
  $m_a \propto m_\pi f_\pi / f_{\text{PQ}} \approx 5.7\mu\text{eV}(10^{12}\text{GeV}/f_{\text{PQ}})$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- At sufficiently high temperatures *hot axions* would be in thermal equilibrium. They would later **freeze out** and contribute to *dark radiation*. $\Gamma_{\rm int} \sim T^3/f_{\rm PQ}^2$, Hubble rate $H \sim T^2/m_{\rm Pl}$. Hence $T_{\rm f.o.} \sim f_{\rm PQ}^2/m_{\rm Pl} \gtrsim 10$ MeV

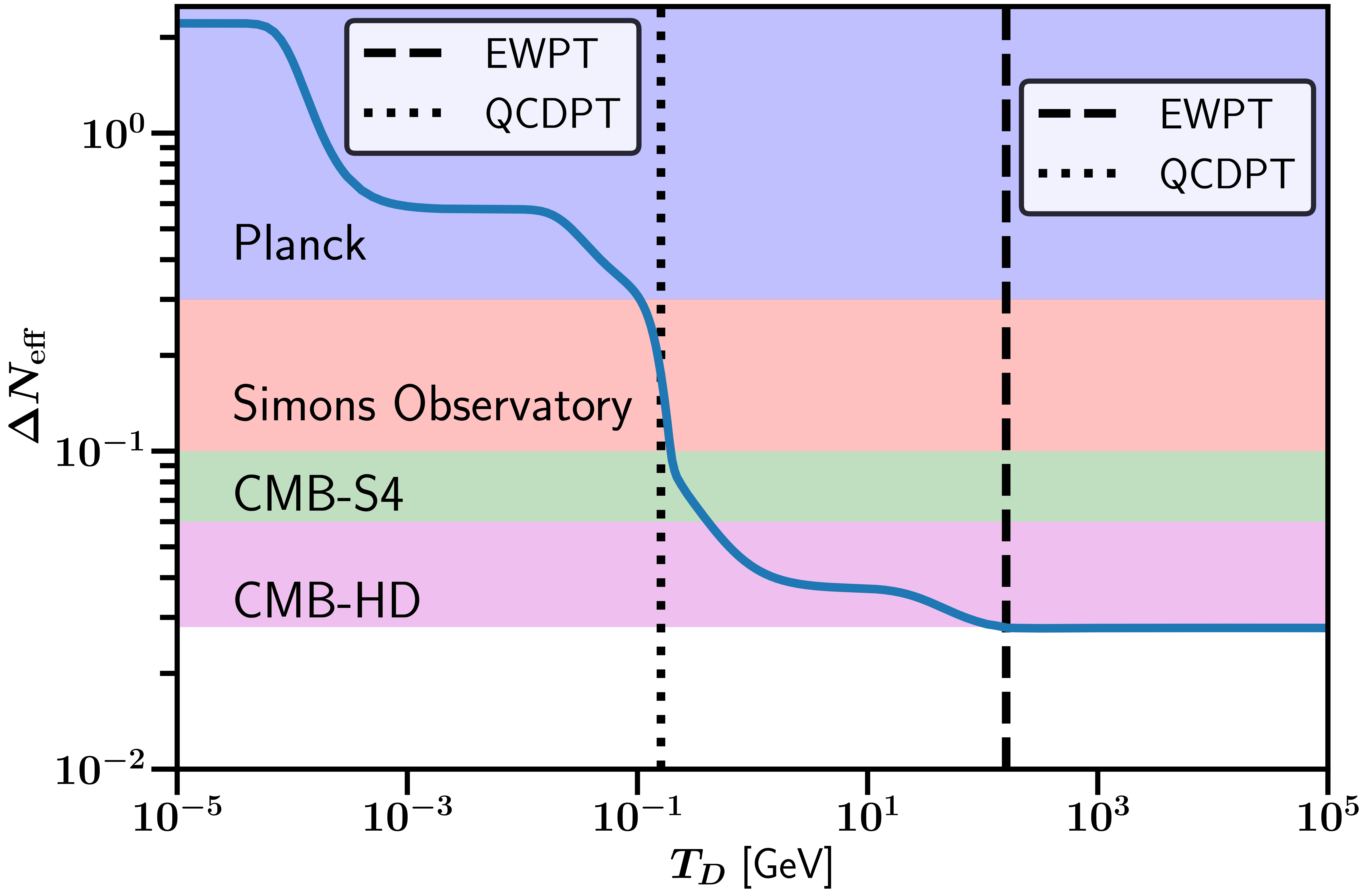
- This is constrained by BBN and CMB determinations of the so-called *effective number of neutrinos* $N_{\rm eff}$

# Dark radiation from hot axions

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{e_a}{e_\gamma}\bigg|_{\rm CMB}$$

Planck: $\Delta N_{\rm eff} < 0.3$ at $2\sigma$

- At $T > T_{\rm EW}$ the axion would be one in $O(100)$ light d.o.f.s in equilibrium. At $T < T_{\rm QCD}$ one in $\mathcal{O}(10)$
- The smaller $f_{PQ}$, the later the freeze out, the larger the contribution to $\Delta N_{\rm eff}$




- Future measurements will probe higher and higher freeze-out temperatures

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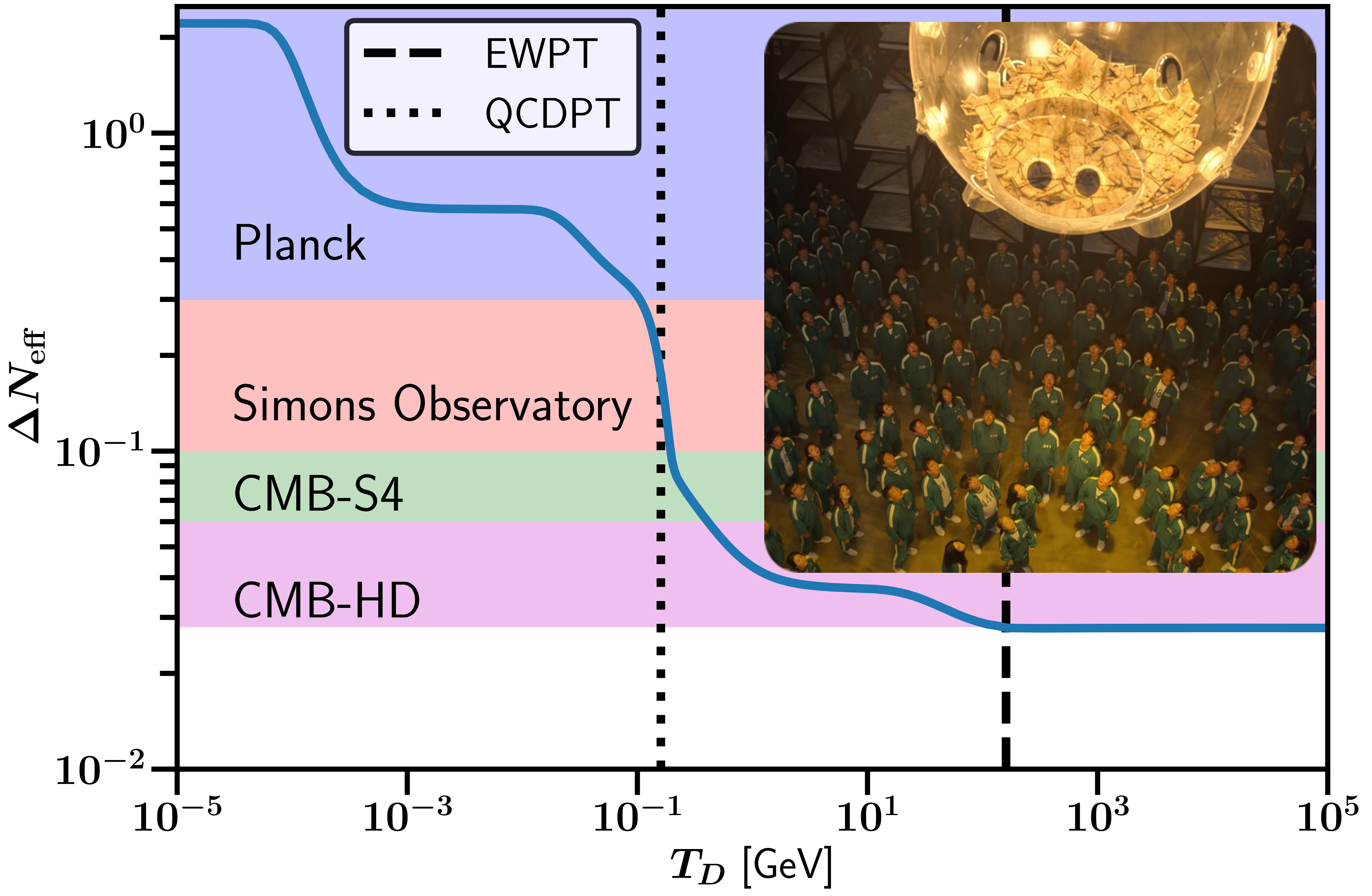
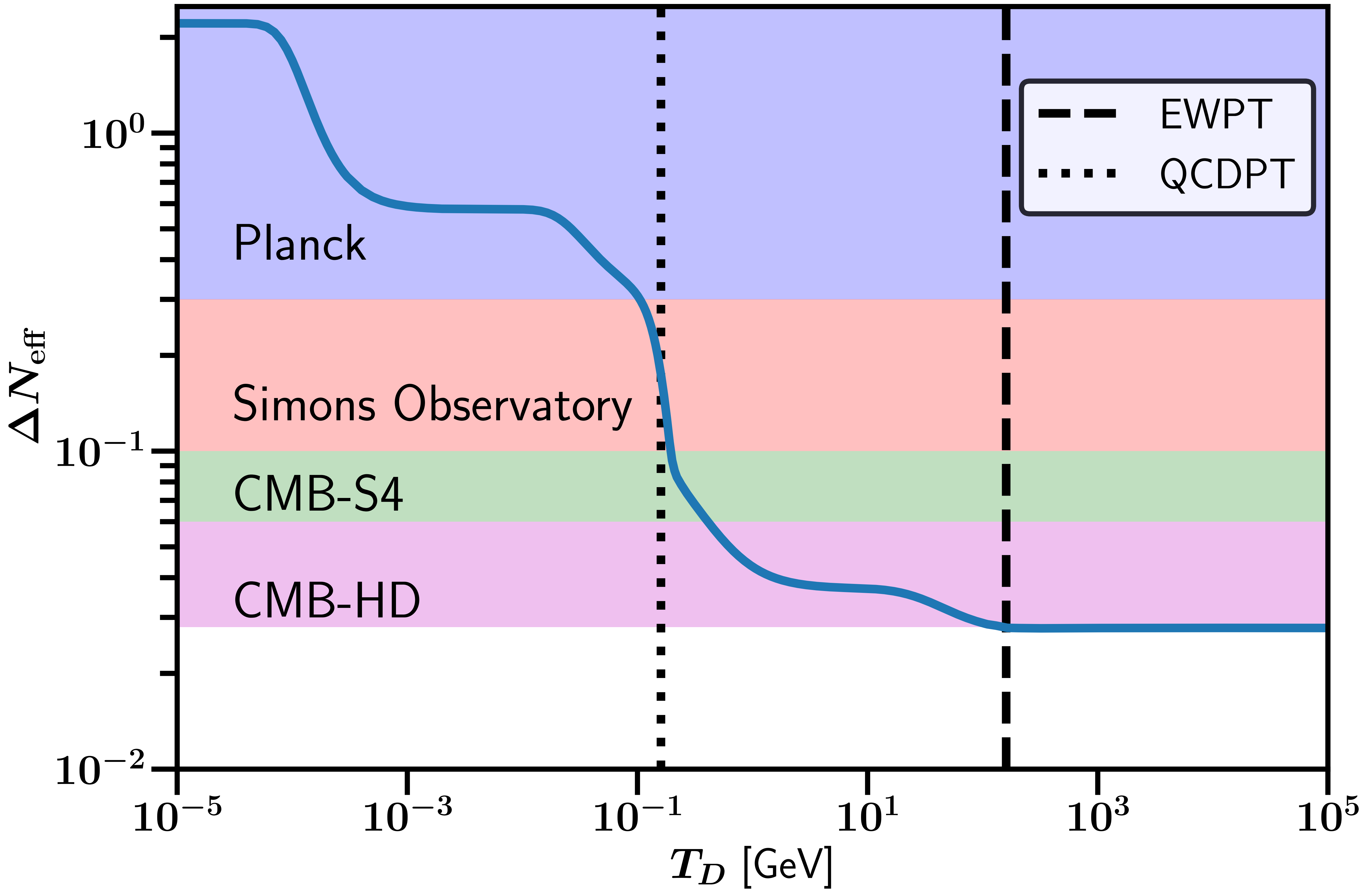



- Future measurements will probe higher and higher freeze-out temperatures

# Dark radiation from hot axions

- This is a "cartoon" for an instantaneous decoupling
- More quantitatively, how is the axion contribution to $N_{\text{eff}}$ computed as a function of $f_{\text{PQ}}$?
- How can we estimate and improve the theory uncertainty of this calculation?



# The axion rate

- To study its freeze out we then need to follow $f_a(t,\mathbf{k}) = (2\pi)^3 dN_a/d^3\mathbf{x}d^3\mathbf{k}$

$$\dot{f}_a(t,\mathbf{k}) \equiv \left(\partial_t - H\mathbf{k}\cdot\nabla_{\mathbf{k}}\right) f_a(t,\mathbf{k}) = \Gamma_a(k)\left[f_{\rm eq}(k^0) - f_a(t,\mathbf{k})\right] + \mathcal{O}(T^2/f_{\rm PQ}^2)$$

- Invisible (KSVZ) axion $\mathscr{L}_{\rm int} = -\dfrac{\alpha_s}{16\pi}\dfrac{a}{f_{\rm PQ}}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\, a}F^{\rho\sigma\, a}$

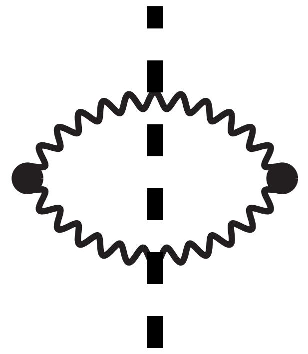
$$\Gamma_a = \frac{\alpha_s^2}{2k(8\pi f_{\rm PQ})^2}\int d^4X e^{iK\cdot X}\langle[J(X),J(0)]\rangle \qquad J = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\, a}F^{\rho\sigma\, a}$$

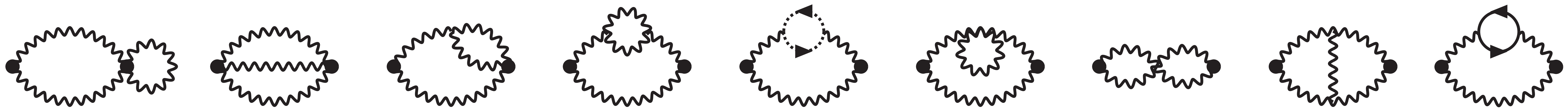
- Valid to first order in $(T/f_{\rm PQ})^2$ and to all orders in $\alpha_s = g_3^2/(4\pi)$ (QCD)
  Bödeker Sangel Wörmann **PRD93** (2016)

- $\Delta N_{\rm eff} \propto \int_{\mathbf{k}} k f_a(t_{\rm CMB},\mathbf{k})$. If $f_a$ remains close to eq. form, $k \gtrsim T$ **dominates**

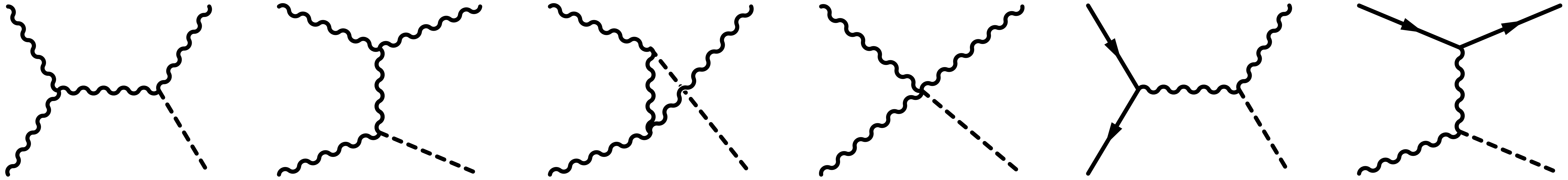
# The axion rate: naive leading order

- Spectral function implies taking **the cut**. Need to go to two loops, $\bullet = J$



- Cuts give naive tree-level diagrams for $2 \leftrightarrow 2$ processes for axions with $k \gtrsim T$



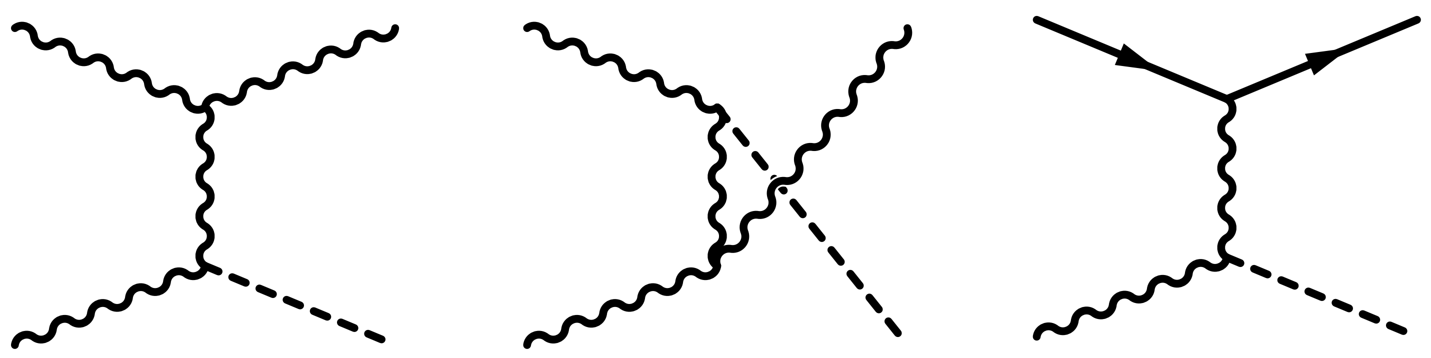
- They give phase space convolution of bare $|\mathscr{M}|^2$ and eq. statistical functions

$$\dot{f}_a(t,\mathbf{k}) = \Gamma_a(k) n_{\rm B}(k) = \frac{1}{4k}\int {\rm d}\Omega_{2\leftrightarrow 2}\sum_{bcd}\left|\mathcal{M}^{bc}_{da}(\boldsymbol{p}_1,\boldsymbol{p}_2;\boldsymbol{k}_1,\boldsymbol{k})\right|^2 f_b(p_1)\, f_c(p_2)\,[1\pm f_d(k_1)]$$

- (Unsurprisingly) find the well-known, simple Boltzmann picture

# The axion rate: naive leading order

- The well-known, simple Boltzmann picture has a well-known problem: the $t$ and $u$ channel diagrams are related to **Coulomb scattering**. $1/t^2$ turns into $1/t$ for this non-renormalizable coupling. **Naive rate is IR divergent**



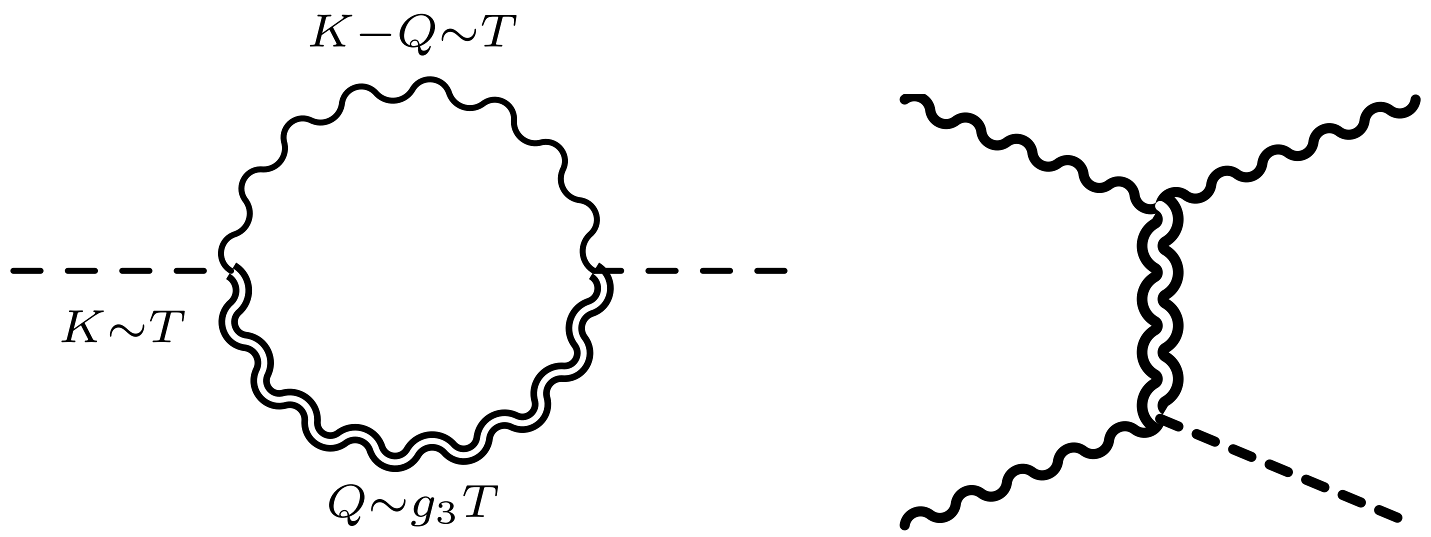
$$\frac{1}{4k}\int d\Omega_{2\leftrightarrow 2}\sum_{bcd}\left|\mathcal{M}^{bc}_{da}(\boldsymbol{p}_1,\boldsymbol{p}_2;\boldsymbol{k}_1,\boldsymbol{k})\right|^2 f_b(p_1)\, f_c(p_2)\,[1\pm f_d(k_1)]$$

- The solution is also well know: these processes are not taking place in a vacuum. A small-$t$ Coulomb gluon cannot resolve individual hard quarks and gluons, with typical wavelengths and separations of order $1/T$ and starts seeing their **collective behavior**

# The axion rate: towards leading order

- **Collective behavior** first emerges at $\lambda \sim 1/(gT)$: screening, plasma oscillations and Landau damping. Treated by **resumming Hard Thermal Loops (HTLs):** i.e. the gauge-invariant thermal amplitudes for $gT$ external momenta. Emergence of gluon screening mass $m_D^2 = g_3^2 T^2 (N_c/3 + N_f/6)$ Braaten Pisarski (1990)

- HTL resummation for small $t$ yields the **strict LO** result Braaten Yuan **PRL66** (1991) Graf Steffen **1008.4528**




$$\Gamma_a(k) = \frac{\alpha_s^3 T^3}{f_{\rm PQ}^2}\left[c_{LL}\ln\frac{k}{m_D} + f(k/T)\right]$$

$$\frac{1}{t^2} \to \frac{1}{(t - \mathrm{Re}\,\Pi_R)^2 + (\mathrm{Im}\,\Pi_R)^2}$$

- $\ln k/m_D$ reveals that underlying approximations will fail for $k \lesssim m_D \sim g_3 T$

# The axion rate: strict leading order

- **strict LO** rate Graf Steffen **1008.4528** $\Gamma_a(k) = \frac{\alpha_s^3 T^3}{f_{\rm PQ}^2}\left[c_{LL}\ln\frac{k}{m_D} + f(k/T)\right]$
- $\Delta N_{\rm eff} \propto e_a$: $\dot{e}_a + 4He_a = \int_{\mathbf{k}} k\,\Gamma_a(k)\,[f_{\rm eq}(k) - f_a(t,\mathbf{k})]$
  $\int_{\mathbf{k}}$ runs over $k \ll m_D$, where the rate **extrapolates out** of its validity region

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  $\int_{\mathbf{k}}$ runs over $k \ll m_D$, where the rate **extrapolates out** of its validity region
- Litmus test: **momentum-averaged rate**

  $\langle\Gamma_a\rangle \equiv \int_{\mathbf{k}} \Gamma_a(k) n_{\rm B}(k) / \int_{\mathbf{k}} n_{\rm B}(k)$

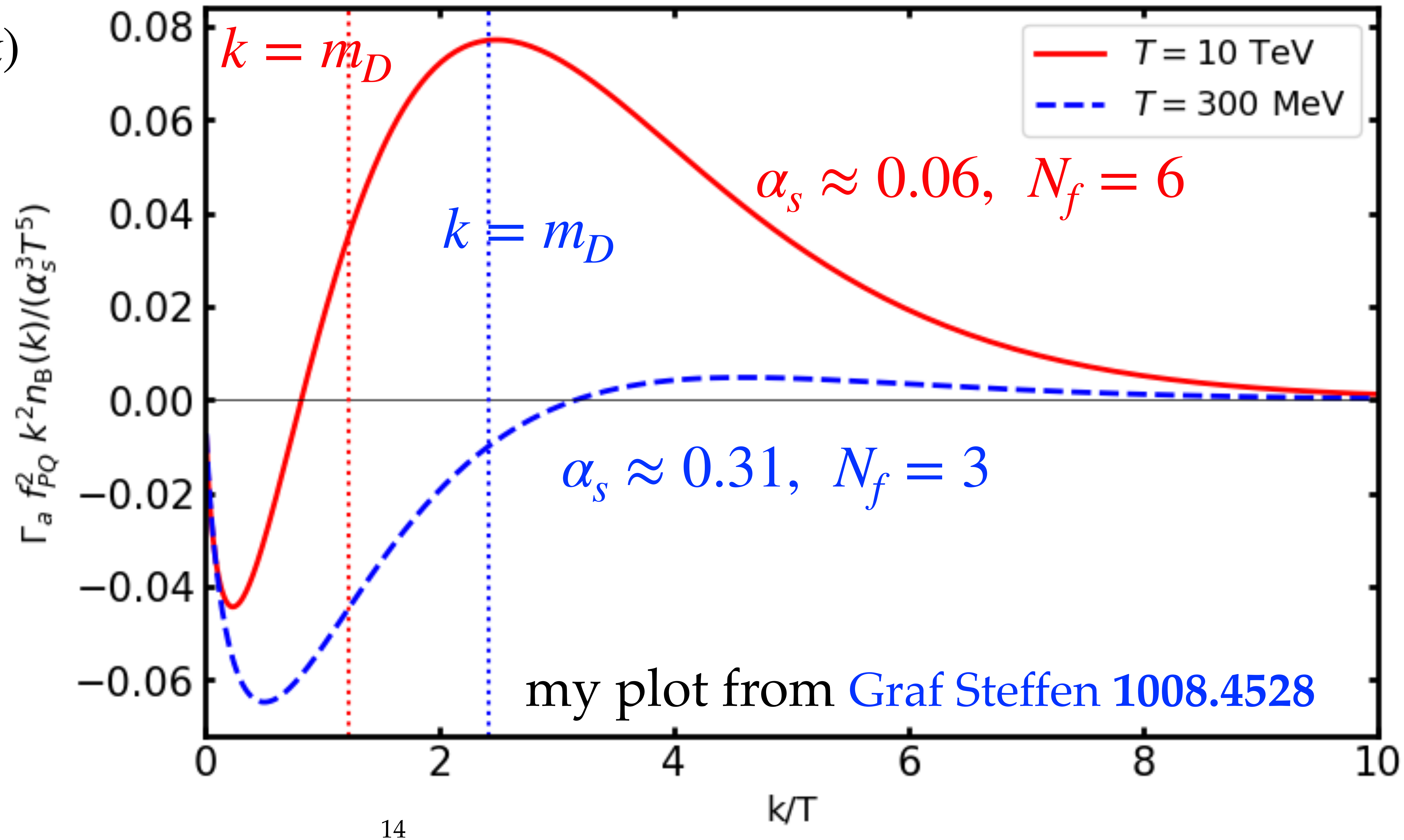
  may become problematic approaching the QCD crossover: $m_D/T \approx 2$, so the contribution of the **extrapolation regime becomes more and more important**

# The axion rate: strict leading order

$$\langle \Gamma_a \rangle \equiv \int_{\mathbf{k}} \Gamma_a(k) n_{\mathrm{B}}(k) \Big/ \int_{\mathbf{k}} n_{\mathrm{B}}(k)$$

**Negative contribution** completely overtakes the rate close to the QCD crossover

$$m_D = g_3 T \sqrt{N_c/3 + N_f/6}$$

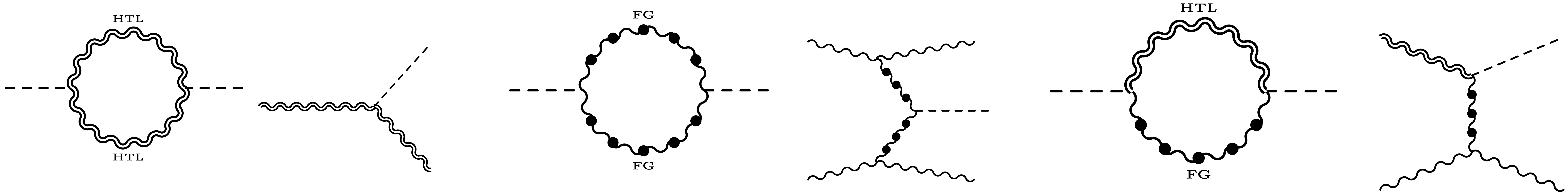


# The axion rate: beyond strict leading order

- How to address this failure and gauge the theory uncertainty? Dos and don’ts
- Main idea: resum a subset of higher-order contributions, going **beyond the strict LO**

# The axion rate: beyond strict leading order

- **Idea 1**: resum one-loop Feynman-gauge self-energies for all loop momenta, rather than gauge-invariant HTL for soft momenta Rychkov Strumia **hep-th/0701104**, Salvio Strumia Xue **1310.6983**

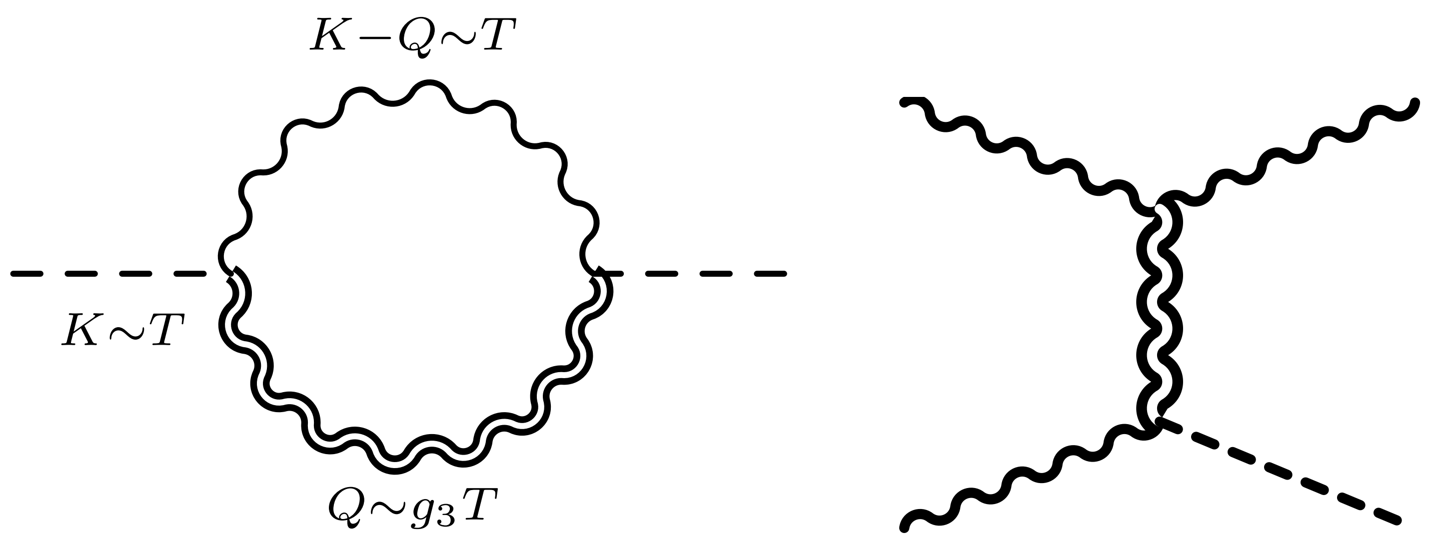



- Manifestly positive rate. Claimed to agree with strict LO at small $g$, with a claimed relative $\mathcal{O}(\alpha_s)$ gauge dependence

We show that **this rate is divergent**: gauge-dependent sensitivity to the $g^2T$ chromomagnetic scale. Finite results in the literature from numerical artifacts Bouzoud JG **2404.06113**

# The axion rate: beyond strict leading order

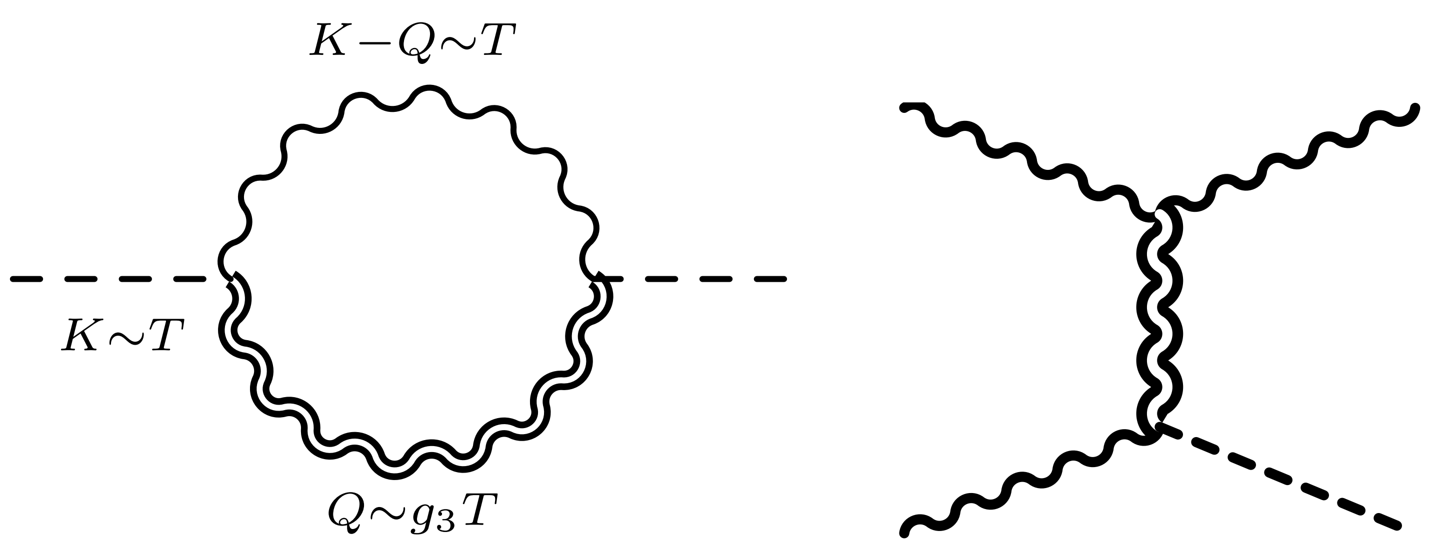
- **Ideas 2 and 3**: the analytical properties of thermal amplitudes at soft light-like momenta allow for a closed-form evaluation of the HTL-resummed part Aurenche Gelis Zaraket **hep-ph/0204146** Caron-Huot **0811.1603** JG *et al* **1302.5970**




- **Idea 2:** "*subtracted scheme*". We can now subtract the divergent limit from the naive form and add back its HTL-resummed analytical evaluation with one less approximation. Corresponds to a **resummation of some $\mathcal{O}(g^2)$ effects** JG Laine (2016) Bouzoud JG **2404.06113**

# The axion rate: beyond strict leading order

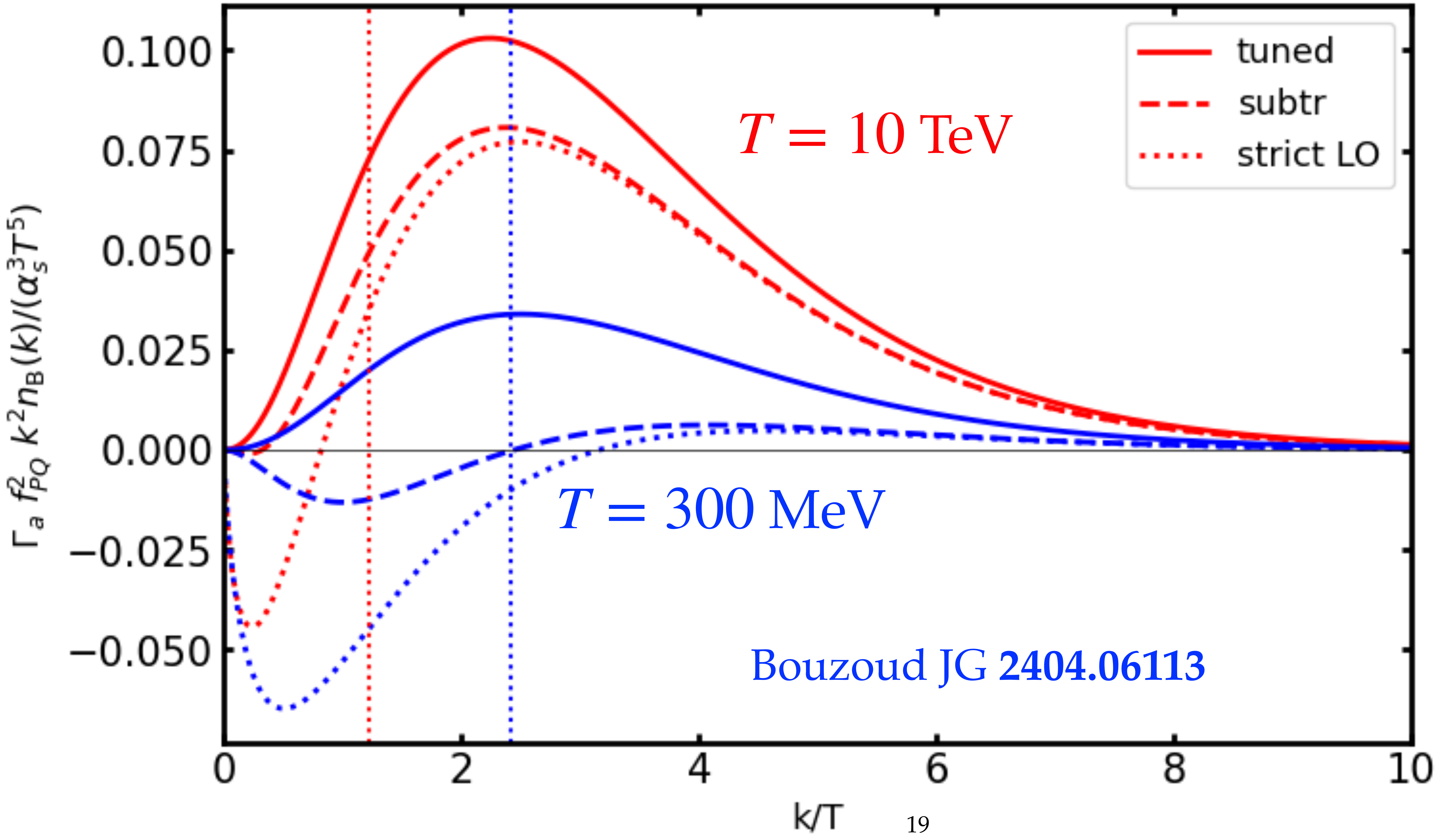
- **Ideas 2 and 3**: the analytical properties of thermal amplitudes at soft light-like momenta allow for a closed-form evaluation of the HTL-resummed part
  Aurenche Gelis Zaraket **hep-ph/0204146** Caron-Huot **0811.1603** JG *et al* **1302.5970**




$$\frac{1}{t^2} \to \left(\frac{q^2}{t(q^2+\xi^2 m_D^2)}\right)^2$$

- **Idea 3:** "*tuned scheme*". Add a mass to the Coulomb denominators and *tune* analytically the $\xi$ coefficient to reproduce the LO result at small $m_D/T$.
  **Resum some $\mathcal{O}(g)$ effects**
  Kurkela Lu Moore York (2014) Bouzoud JG **2404.06113**

# The axion rate: beyond strict leading order



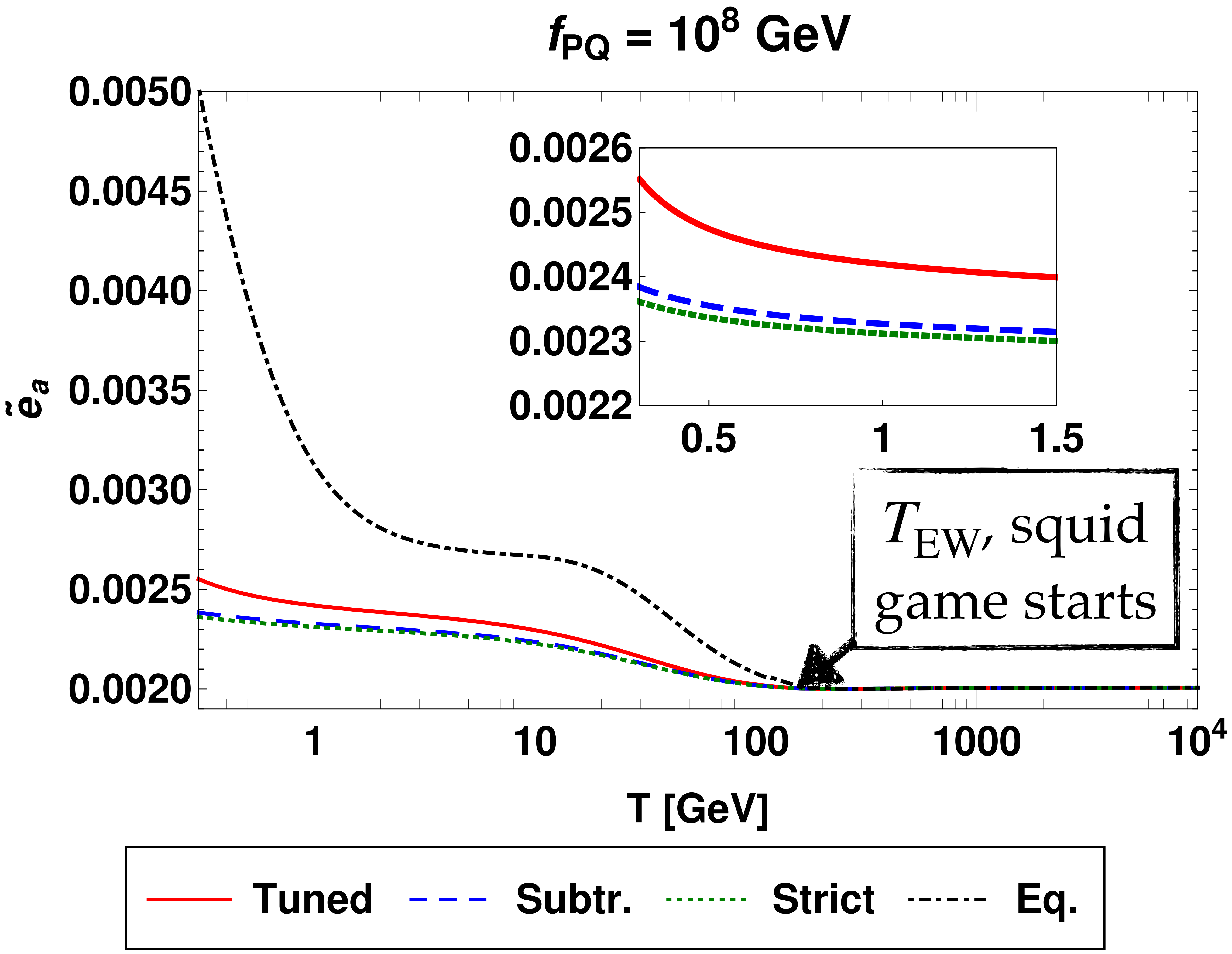

At $T = 300$ MeV our $\langle\Gamma_a\rangle_{\text{tun.}}$ is $\mathcal{O}(0.1)$ that from the divergent method in Salvio *et al* **1310.6983**, D'Eramo *et al* **2108.05371**

# Hot axion dark radiation

$$\partial_t f_a(t,\mathbf{k}) - Hk\partial_k f_a(t,\mathbf{k}) = \Gamma_a(k)\left[f_{\rm eq}(k^0) - f_a(t,\mathbf{k})\right]$$

- Solve the Boltzmann equation for $f_a$ for an high-T eq. initial condition and then obtain $e_a$ and $\Delta N_{\rm eff}$
- **Freeze-out visible in the broken phase** Close to the QCD transition we see the onset of *delayed production*, which is missing in our calculation of $\Delta N_{\rm eff}$

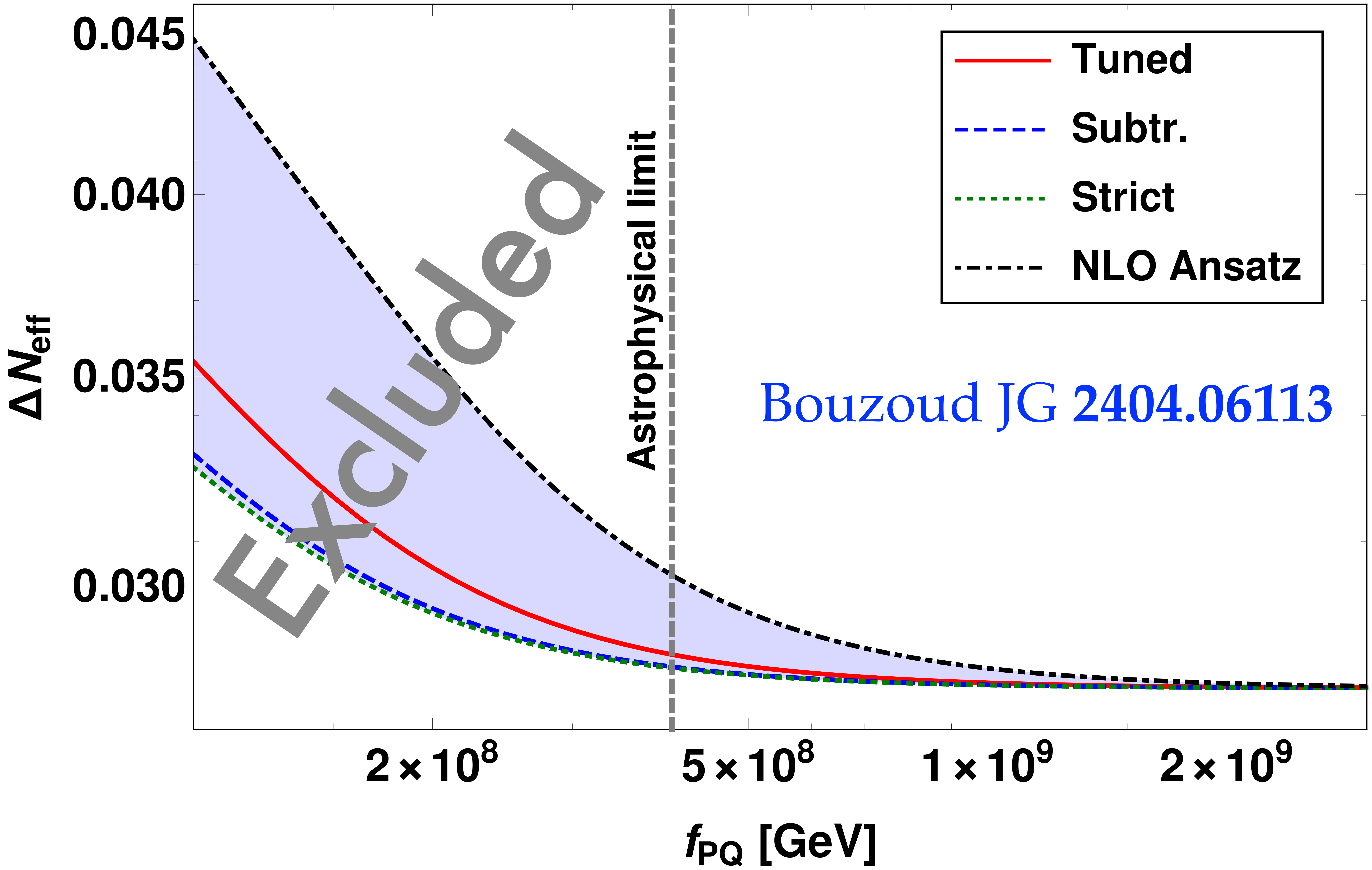
$$\tilde{e}_a \equiv e_a/s^{4/3}$$




Bouzoud JG **2404.06113**

# Hot axion dark radiation

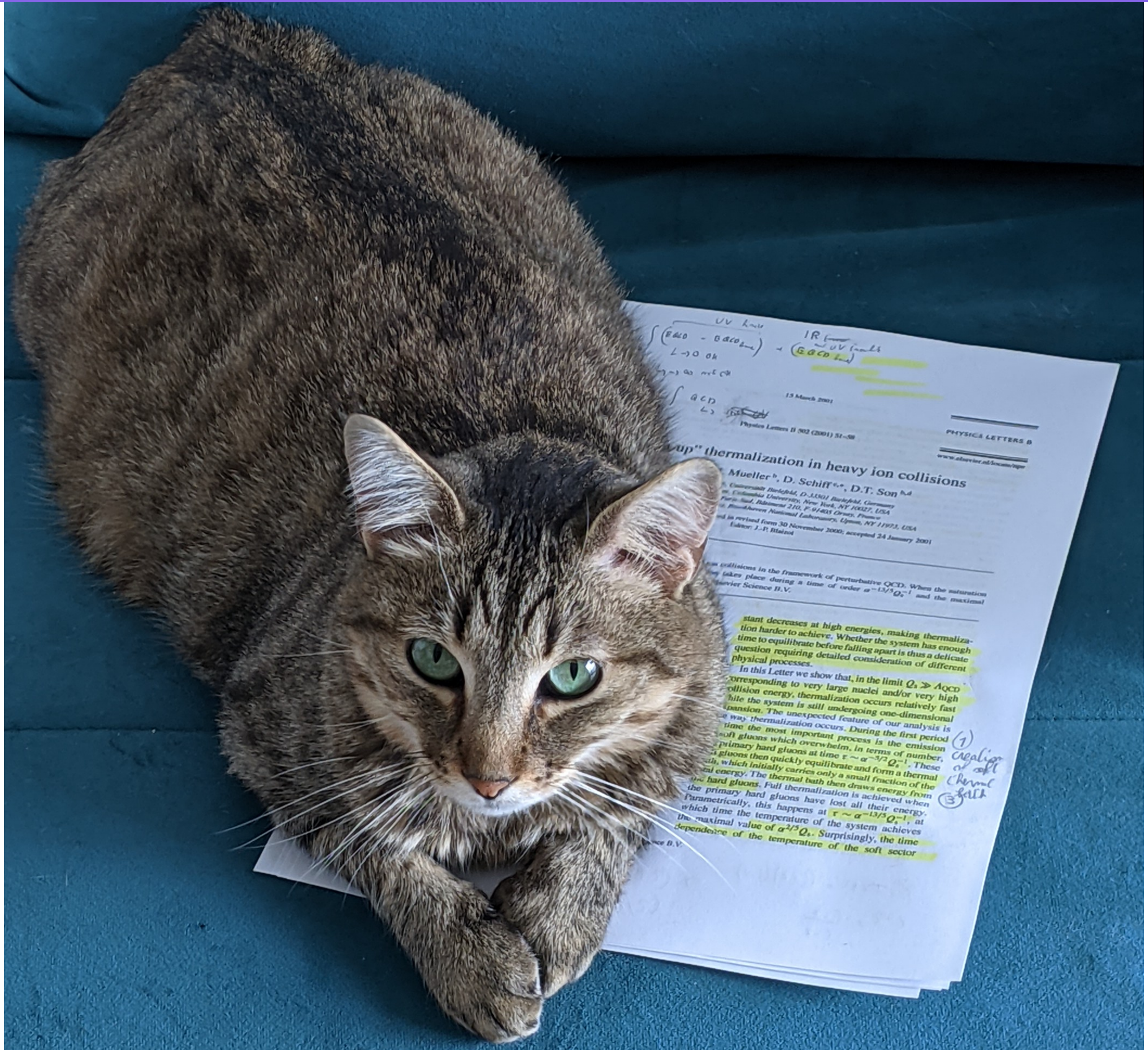
- NLO Ansatz includes known part of NLO rate, expected to be the dominant one.
- A **7% effect** on the observable at the smallest allowed value of $f_{\rm PQ}$. $\Delta N_{\rm eff}$ in reach of CMB-HD
- *Delayed production* at QCD transition will increase $\Delta N_{\rm eff}$ prediction



# Summary and outlook

- The thermal axion rate: a thermal QCD calculation applied to cosmology
- Quantification of theory uncertainty from different ways of handling collective effects at $T \gtrsim T_{\rm QCD}$, crucial for future CMB precision measurements Bouzoud JG 2024
- Calculation of the thermal rate from $2 \leftrightarrow 2$ scatterings with HTL resummation now fully automated, needing only a model file for $\mathscr{L}$
- Axion, graviton, gravitino, … rates can be obtained from a model file in $\mathcal{O}$(minute), from Feynman rule derivation, through computation of $|\mathscr{M}|^2$ and thermal masses to 2D numerics
  AUTOTHERM, Bouzoud JG Jackson, coming soon

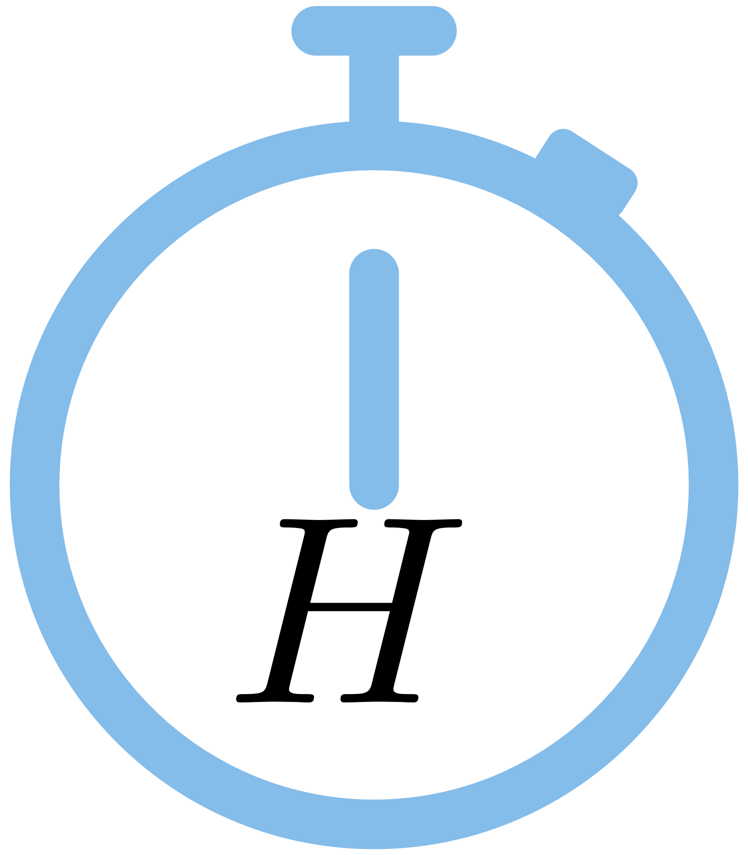
# Backup



# Production and interaction rates

## The basics

- Factor the system into "fast" and "slow" modes, and integrate out the former to obtain evolution eqs. for the latter



# Production and equilibration

- A particle $\phi$ is weakly coupled (coupling $h$) to an equilibrated bath with its internal couplings $g$

$$\mathcal{L} = \mathcal{L}_\phi + h\phi J + \mathcal{L}_{\text{bath}}$$

$J$ built of bath fields, one can prove to first order in $h$ and **all orders** in $g$
Bödeker Sangel Wörmann **PRD93** (2016)

$$\dot{f}_\phi(t,\mathbf{k}) = \Gamma(k)\left[f_{\text{eq}}(k^0) - f_\phi(t,\mathbf{k})\right] + \mathcal{O}(h^4)$$

$$\Gamma(k) = \frac{h^2}{2k^0}\int d^4X e^{iK\cdot X}\langle[J(X),J(0)]\rangle$$

- *Single-particle phase-space distribution*: $f(t,\mathbf{k}) = (2\pi)^3 dN_\phi/d^3\mathbf{x}d^3\mathbf{k}$, sensible only for sufficiently weakly interacting particles
- $\langle\hat{O}\rangle$ ensemble average

# Production and equilibration

$$\dot{f}_\phi(t,\mathbf{k}) = \Gamma(k)\left[f_{\rm eq}(k^0) - f_\phi(t,\mathbf{k})\right] + \mathcal{O}(h^4) \qquad \Gamma(k) = \frac{h^2}{2k^0}\int d^4X e^{iK\cdot X}\langle[J(X),J(0)]\rangle$$

- When using these equations in cosmology, the l.h.s is modified to include Hubble expansion

$$\dot{f}_\phi(t,\mathbf{k}) \to (\partial_t - H\mathbf{k}\cdot\nabla_{\mathbf{k}})f_\phi(t,\mathbf{k})$$

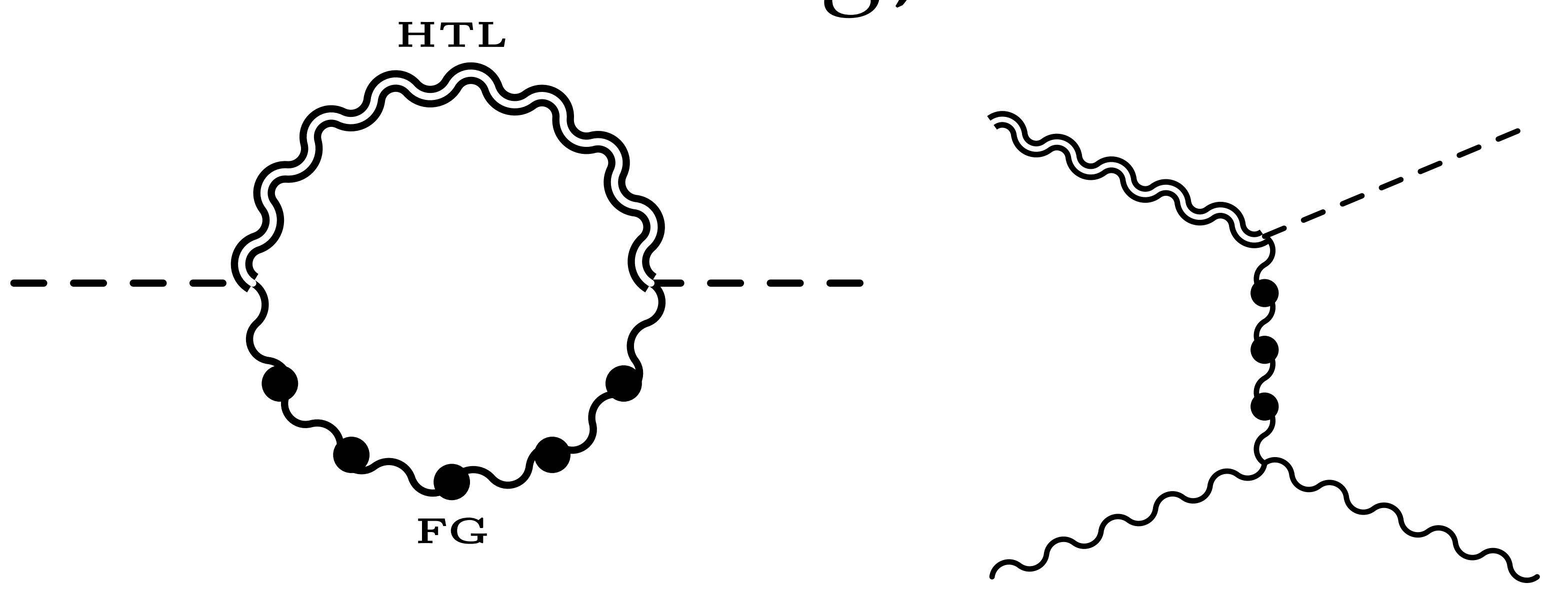
  and often (number, energy) densities are the quantity of interest, e.g. $n_\phi = \int_{\mathbf{k}} f_\phi$

$$\dot{n}_\phi + 3Hn_\phi = \int_{\mathbf{k}} \Gamma(k)[f_{\rm eq}(k^0) - f_\phi(t,k)]$$

- If **scale separation** is present and $g \ll 1$, perturbative expansion of $\Gamma(k \gtrsim T)$ can reproduce standard Boltzmann. But **quasiparticle picture is not necessary**!

# The axion rate: beyond strict leading order

- Manifestly positive rate. Claimed to agree with strict LO at small $g$, with a claimed relative $\mathcal{O}(\alpha_s)$ gauge dependence




- Where's the catch? Look at Landau damping sector $t = q_0^2 - q^2$ is the space-like momentum of the FG-resummed gluon

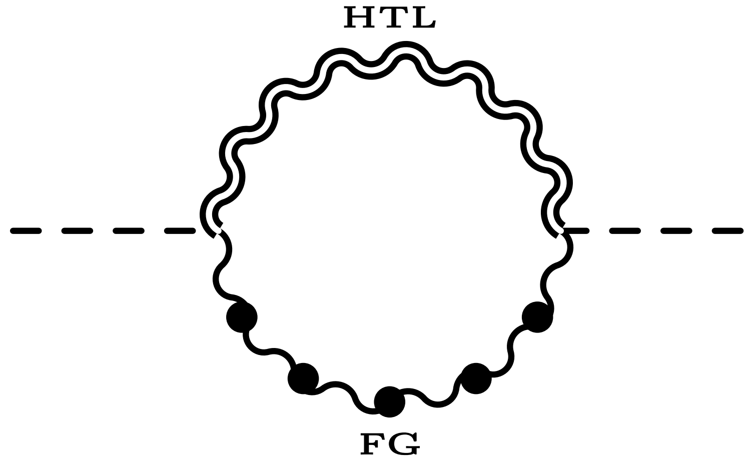
$$\frac{1}{t^2} \to \frac{1}{(t - \mathrm{Re}\,\Pi_R)^2 + (\mathrm{Im}\,\Pi_R)^2}$$

$$\Gamma_a(k) \propto \int dq^0 dq\, \theta(-t) \frac{f(q^0, q, k)}{(t - \mathrm{Re}\,\Pi_R)^2 + (\mathrm{Im}\,\Pi_R)^2}$$

- Two independent polarisations in medium, $L$ and $T$ wrt $\mathbf{q}$
- At soft $Q$, $\Pi_R^{\mathrm{FG}}(Q) = \Pi_R^{\mathrm{HTL}}(Q) + \mathcal{O}(g^2 T Q)$. So agree with HTL?
- Subtlety: for $q_0 \ll q \sim gT$ the transverse HTL vanishes: no perturbative one-loop magnetic mass. The subleading gauge-dependent term takes over!

# The axion rate: beyond strict leading order

$$\Gamma_a(k) \propto \int dq^0 dq \theta(-t) \frac{f(q^0, q, k)}{(t - \mathrm{Re}\,\Pi_R)^2 + (\mathrm{Im}\,\Pi_R)^2}$$




- For $q_0 \ll q \sim gT$ the transverse HTL vanishes: no perturbative one-loop magnetic mass. The subleading gauge-dependent term takes over!
- $\Pi^{FG}_{R,T}(0,q) = -3g_3^2 N_c qT/16 + \mathcal{O}(g_3^2 q^2)$ is negative, and remains negative in all gauges Kalashnikov Klimov (1980) Linde (1980) Kajantie Kapusta (1982, 1985)
- At zero frequency the imaginary part vanishes by causality. Gauge-dependent double pole at the chromomagnetic scale $g_3^2 T$

$$\Gamma_a(k) \supset \int_{q_0 \ll q} dq dq^0 \theta(-t) \frac{f(0, q, k)}{(q^2 - 3g_3^2 T N_c q/16)^2} =$$

# The axion rate: beyond strict leading order

$$\Gamma_a(k) \propto \int dq^0 dq \theta(-t) \frac{f(q^0, q, k)}{(t - \mathrm{Re}\,\Pi_R)^2 + (\mathrm{Im}\,\Pi_R)^2}$$

higher-order corrections to $G^{-1}(k)$ [3]. A general analysis similar to our analysis of infrared divergences of $\Omega(T)$ shows that in higher orders of perturbation theory the terms $\sim k^2(g^2T/k)^N$ appear in the expansion of $G^{-1}(k)$, so that at small $k$

$$G^{-1}(k) = k^2 + a_1 g^2 Tk + a_2 g^4 T^2 + a_3 g^6 T^3/k + \ldots, \quad (3)$$

Linde (1980)

where $a_i$ are some constants $\approx 1$ (in the Feynman gauge $a_1 = -9/16$ for the group SU(3) [10]). This series is divergent for $k \lesssim g^2T$. Analogously it can be shown that the series for $\Pi_{00}(k)$ is *also* divergent at $k < g^2T$. Therefore the perturbation theory can give us no information about the behaviour of the Green function $G^{ab}_{\mu\nu}(k_0 = 0, k)$ at $k < g^2T$. One can show on-

- For $q_0 \ll q \sim gT$ the transverse HTL vanishes: n magnetic mass. The subleading gauge-depende
- $\Pi^{\mathrm{FG}}_{R,T}(0,q) = -3g_3^2 N_c qT/16 + \mathcal{O}(g_3^2 q^2)$ is negative gauges Kalashnikov Klimov (1980) Linde (1980) Kaja
- At zero frequency the imaginary part vanishes b dependent double pole at the chromomagnetic scale $g_3^2 T$

$$\Gamma_a(k) \supset \int_{q_0 \ll q} dq dq^0 \theta(-t) \frac{f(0, q, k)}{(q^2 - 3g_3^2 T N_c q/16)^2} =$$ 💥

# The axion rate: beyond strict leading order

- Idea 1: resum one-loop Feynman-gauge self-energies for all loop momenta, rather than gauge-invariant HTL for soft momenta
  Rychkov Strumia **hep-th/0701104**, Salvio Strumia Xue **1310.6983**

- Manifestly positive rate. Claimed to agree with strict LO at small $g$, with a claimed relative $\mathcal{O}(\alpha_s)$ gauge dependence

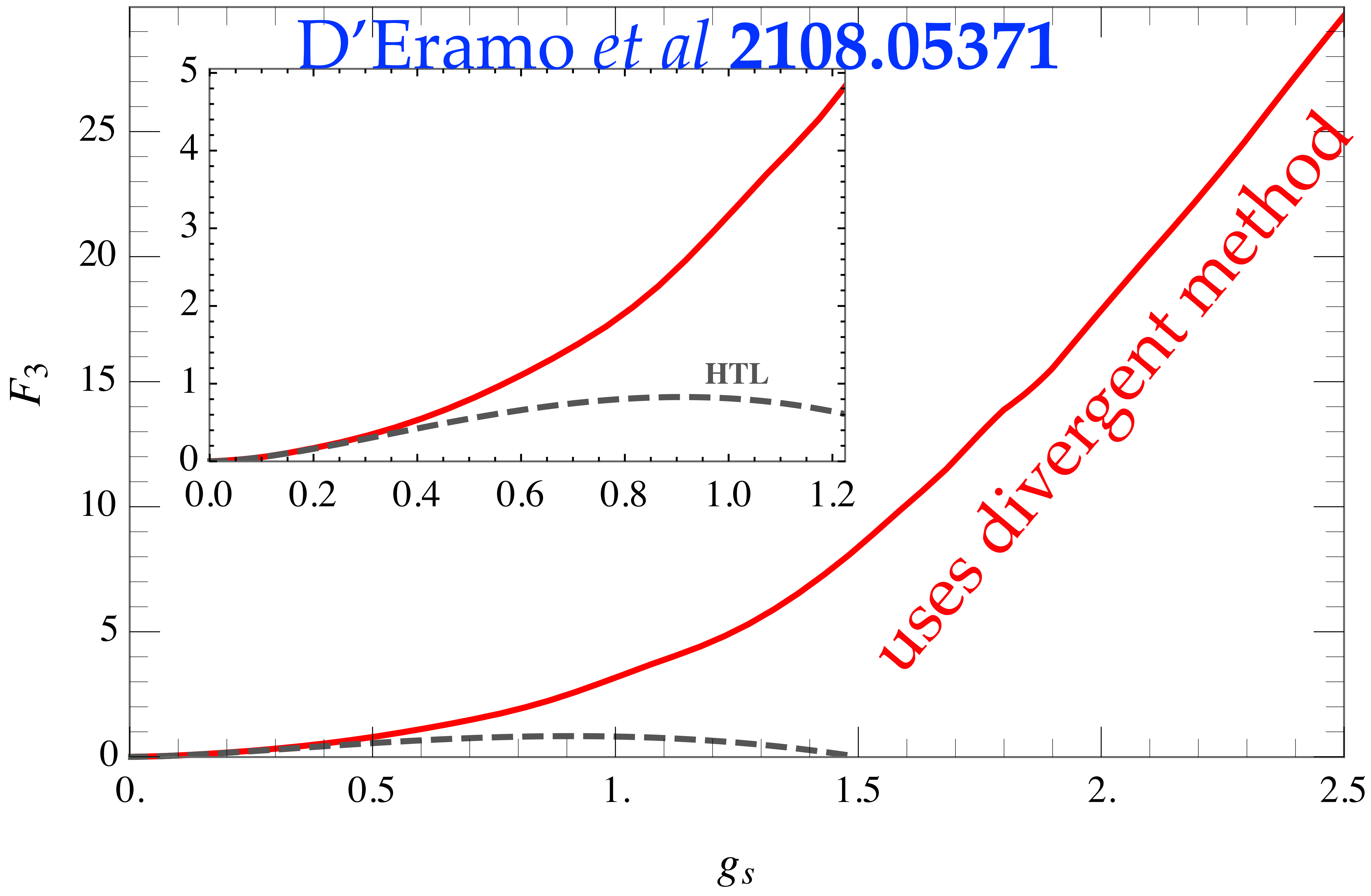
- Shown to give rise to a divergent rate, due to the incorrect handling of the chromomagnetic sector where perturbation theory breaks down

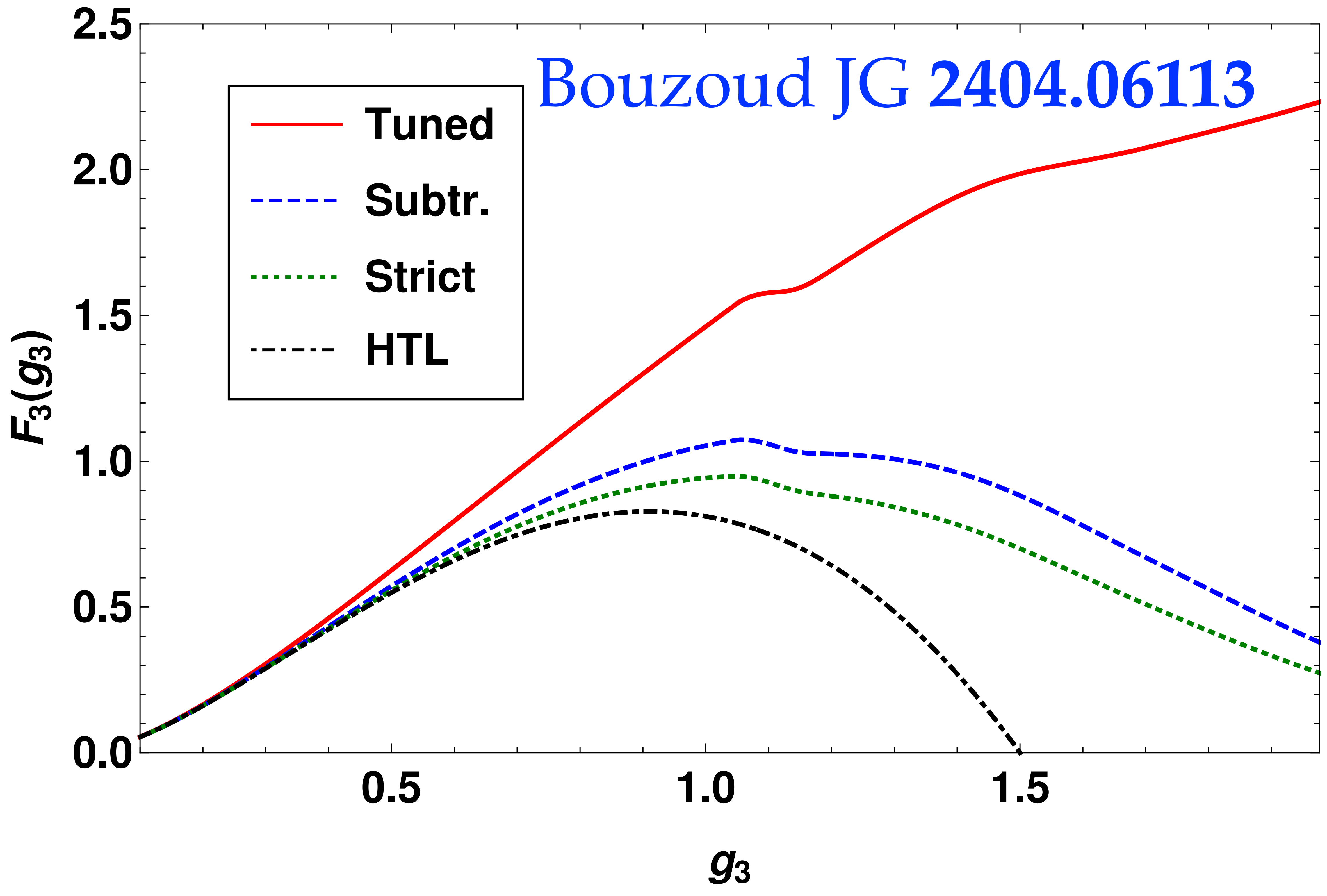
- Finite numerical results in original papers and in works by other authors implementing this method likely due to a finite numerical imaginary part at zero frequency
  Bouzoud JG **2404.06113**

# The integrated axion rate

$$F_3(T) = \frac{512\pi^5 f_{\rm PQ}^2 \langle\Gamma\rangle}{(N_c^2-1)g_3^4(T)T^3}$$

$$\langle\Gamma_a\rangle \equiv \int_{\mathbf{k}} \Gamma_a(k) n_{\rm B}(k) / \int_{\mathbf{k}} n_{\rm B}(k)$$

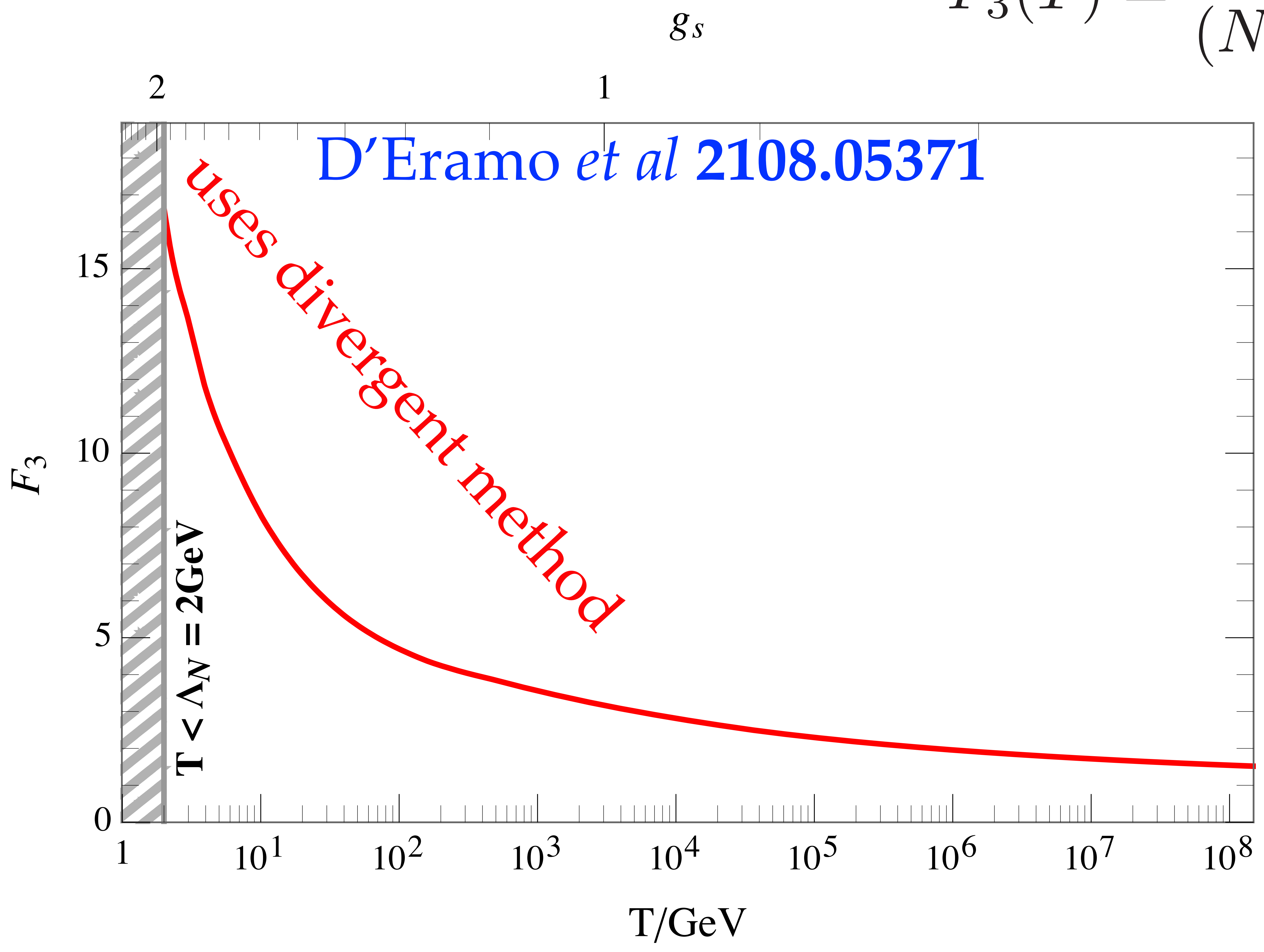


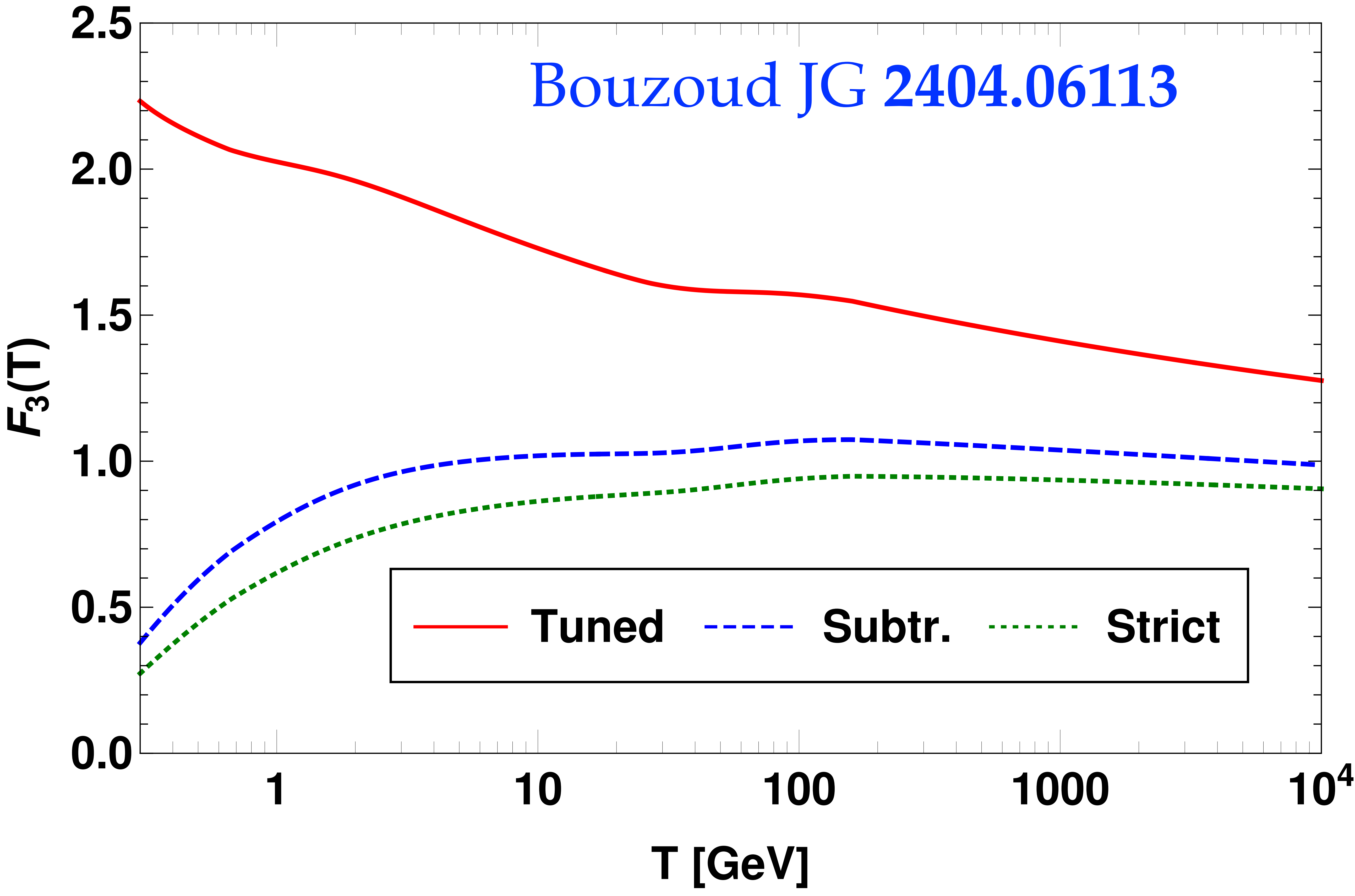




HTL: strict LO with negative contrib

# The integrated axion rate

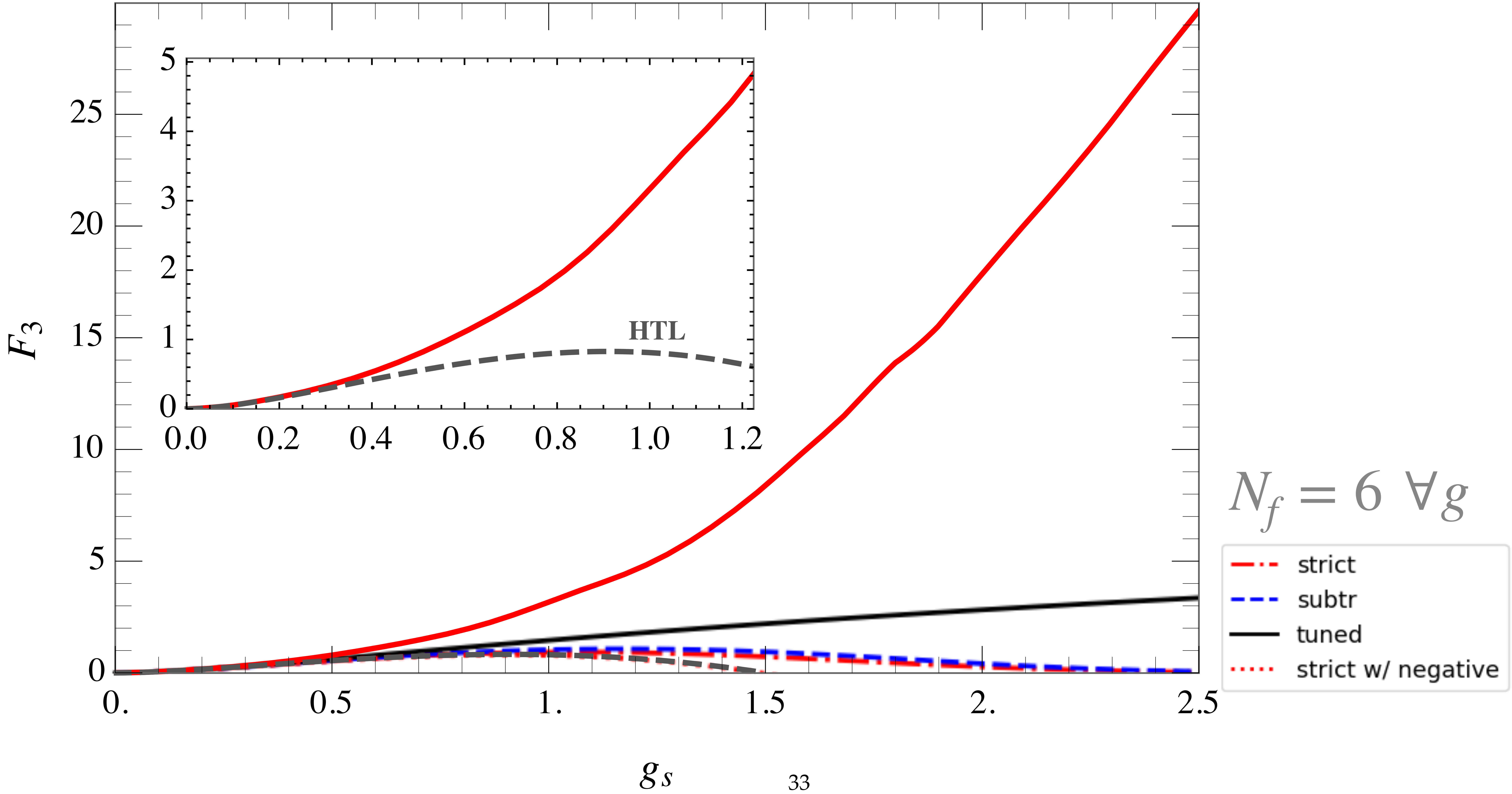
$$F_3(T) = \frac{512\pi^5 f_{\rm PQ}^2 \langle\Gamma\rangle}{(N_c^2-1)g_3^4(T)T^3}$$



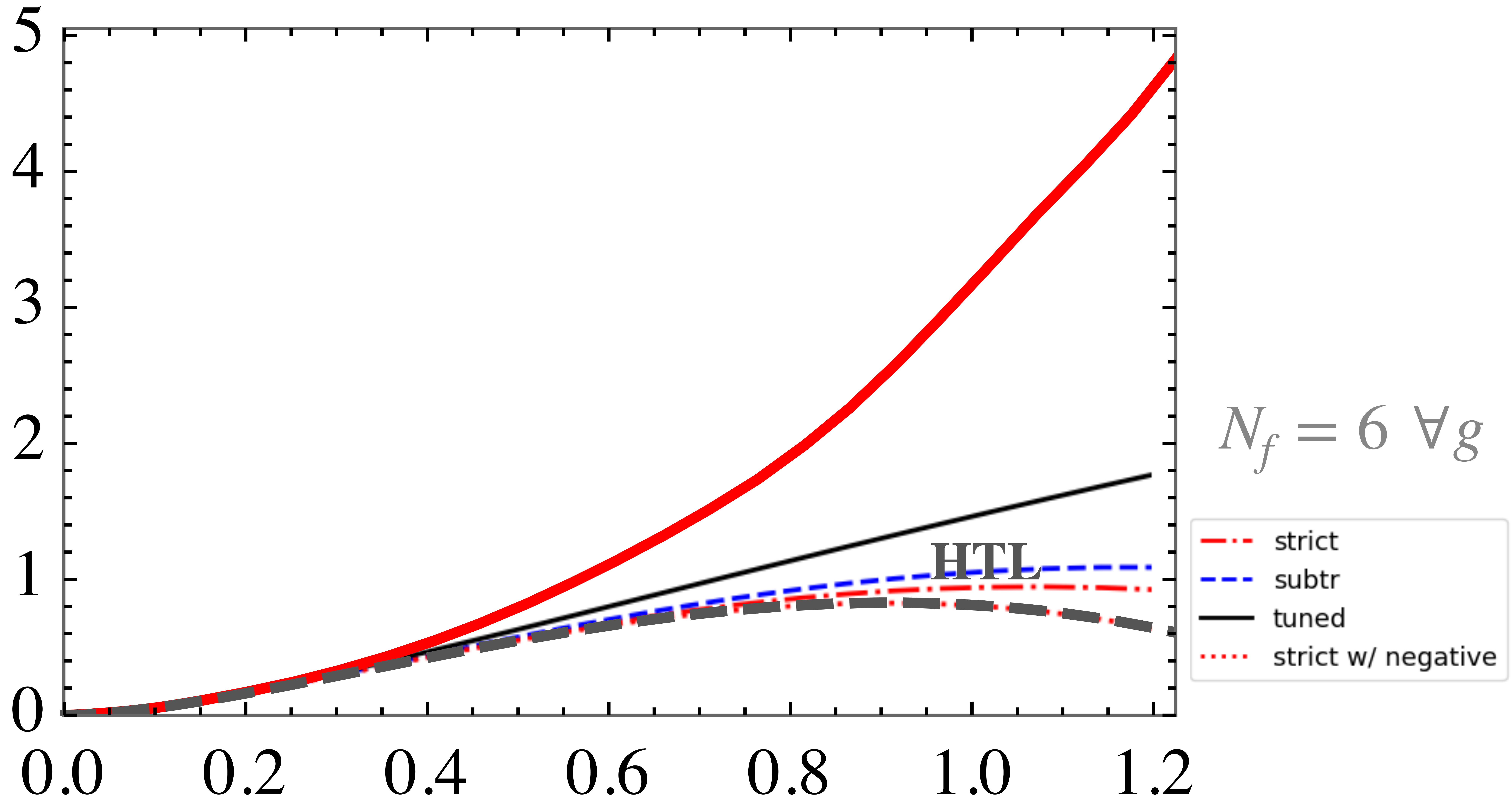




- We use $\alpha_s(T)$ and $m_q(T)$ from Laine Schicho Schröder **1911.09123**

# The axion energy density, redux



# The axion energy density, redux



# Teaser: what about NLO?

- First estimate of NLO $\mathcal{O}(g)$ corrections to the $k \gtrsim T$ rate by *assuming* they are equal to those to $\hat{q}$, a jet quenching quantity sharing the same soft-gluon sector Caron-Huot **0811.1603**

- Full-fledged NLO calculation for $k \gtrsim T$ and LO for $k \sim gT$ coming Bouzoud JG Laine