

BREAKING THE “DISTANCE DUALITY RELATION” TO EXPLAIN COSMIC TENSIONS



ELSA M. TEIXEIRA

(elsa.teixeira@umontpellier.fr)

Based on:

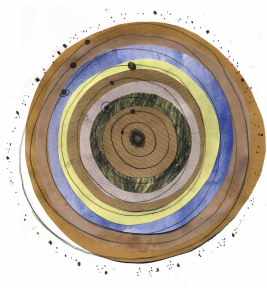
• [[arxiv:2504.10464](https://arxiv.org/abs/2504.10464)] with:
William Giarè, Natalie Hogg,
Thomas Montandon, Adèle Poudou
and Vivian Poulin

• [[arxiv:2505.02909](https://arxiv.org/abs/2505.02909)] with:
Ruchika, William Giarè, and
Alessandro Melchiorri

Illustrations: Inês Viegas Oliveira
(ivoliveira.com)

The Hubble Tension

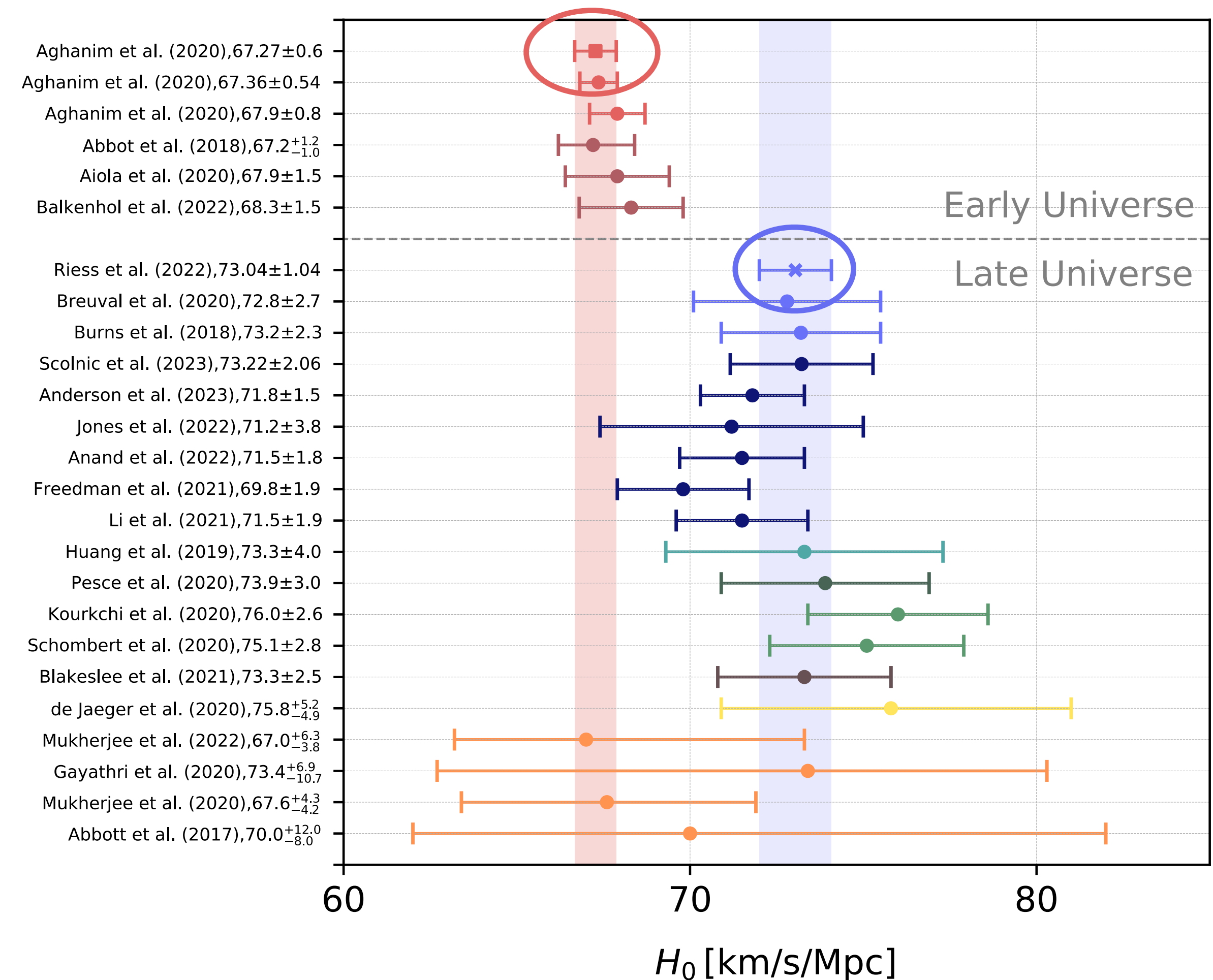


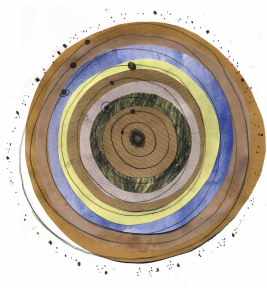


The “Hubble Tension”

Unreconcilable values for H_0 from the CMB and from direct local distance ladder measurements

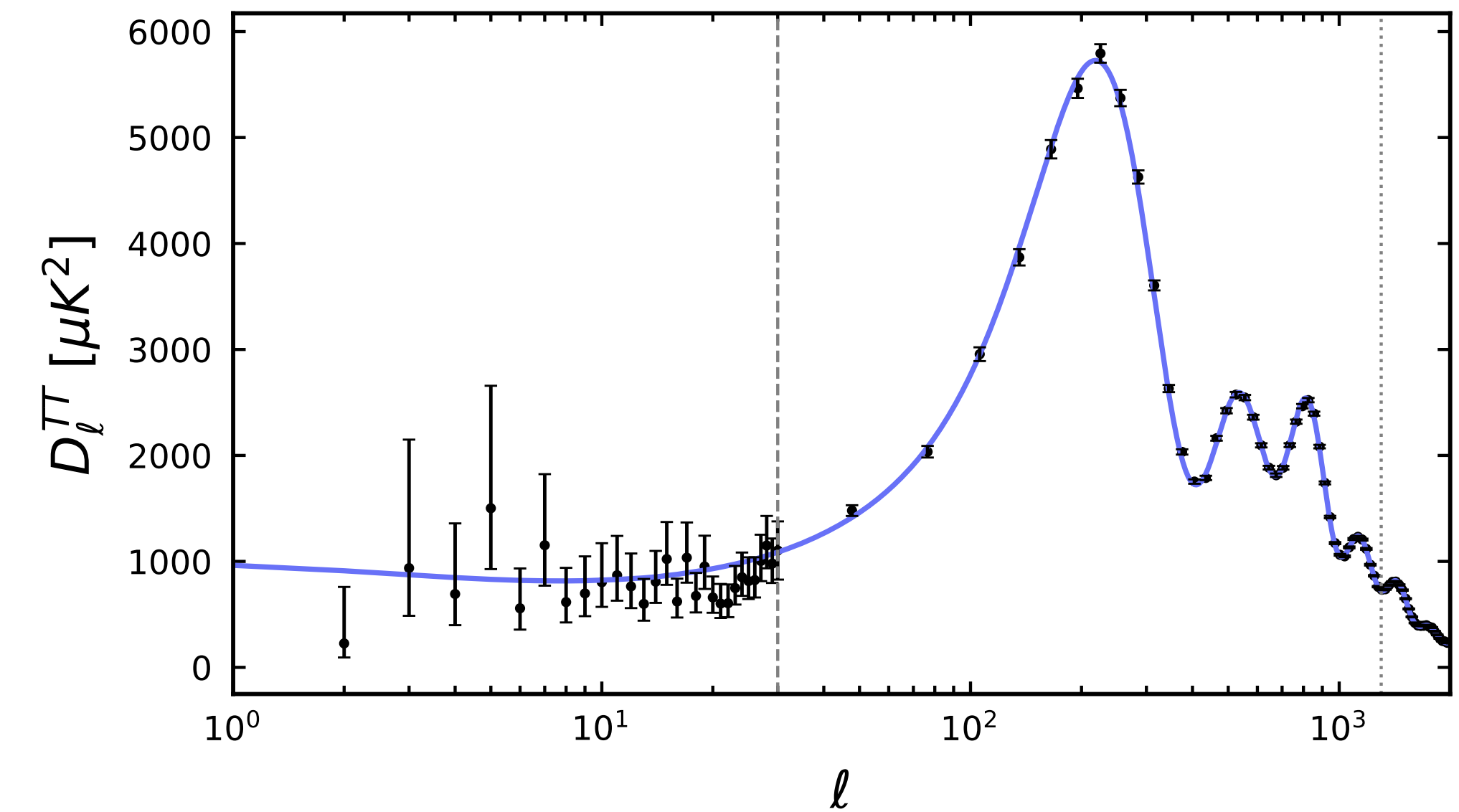
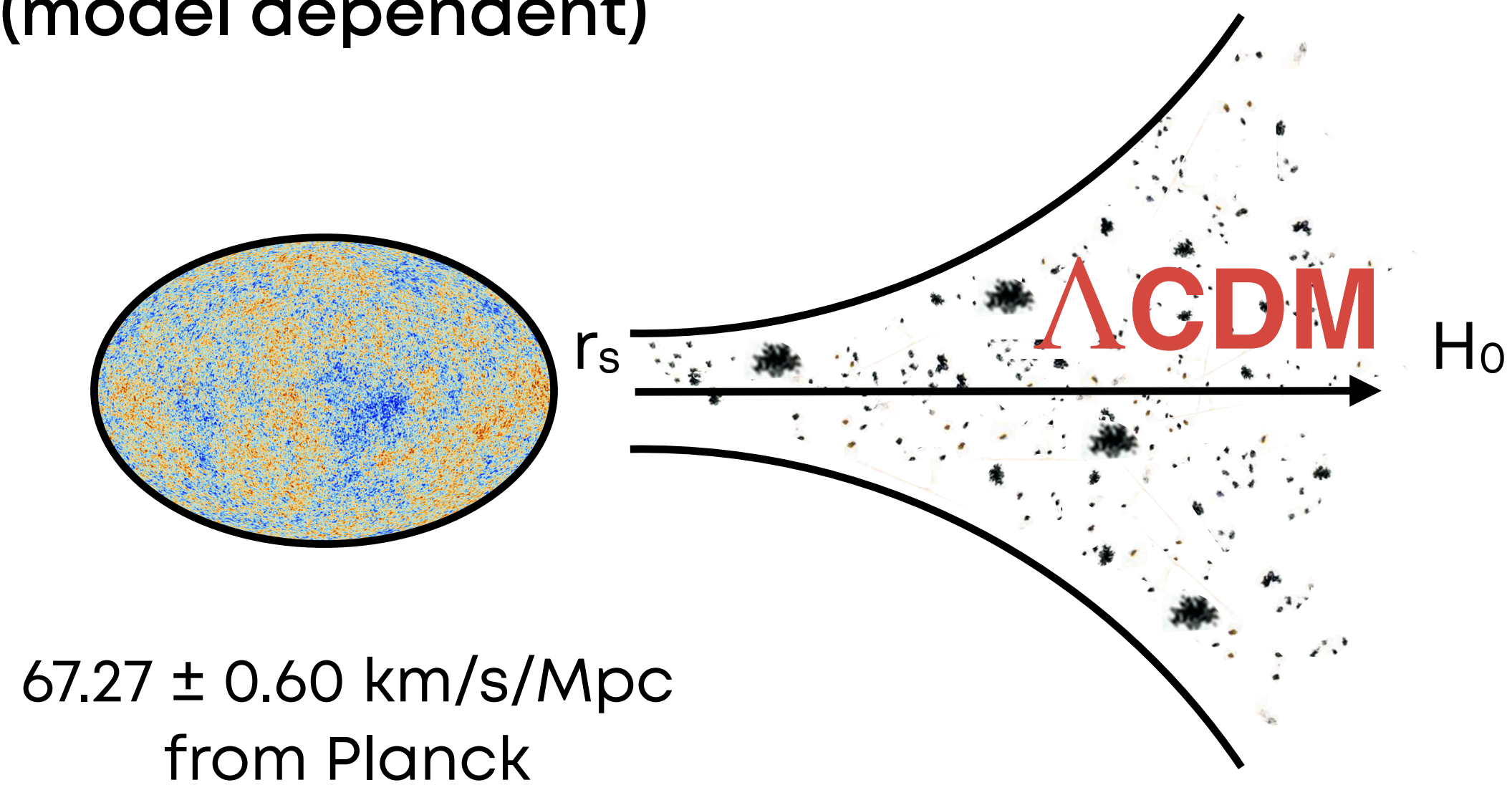
- ~5 σ tension between Planck 2018 and SH₀ES:
 - CMB (Planck): $H_0 = 67.27 \pm 0.60$ km/s/Mpc
 - SNe (R22): $H_0 = 73.04 \pm 1.04$ km/s/Mpc
- The CMB data assumes the Λ CDM model
- DESI BAO (+BBN+CMB): $H_0 = 68.45 \pm 0.47$ km/s/Mpc [DESI Collaboration DR2 2025: arXiv:2503.14738]
- Compilation of early vs late time data that disagree
- But how do we measure H_0 in each case?





The “Hubble Tension”

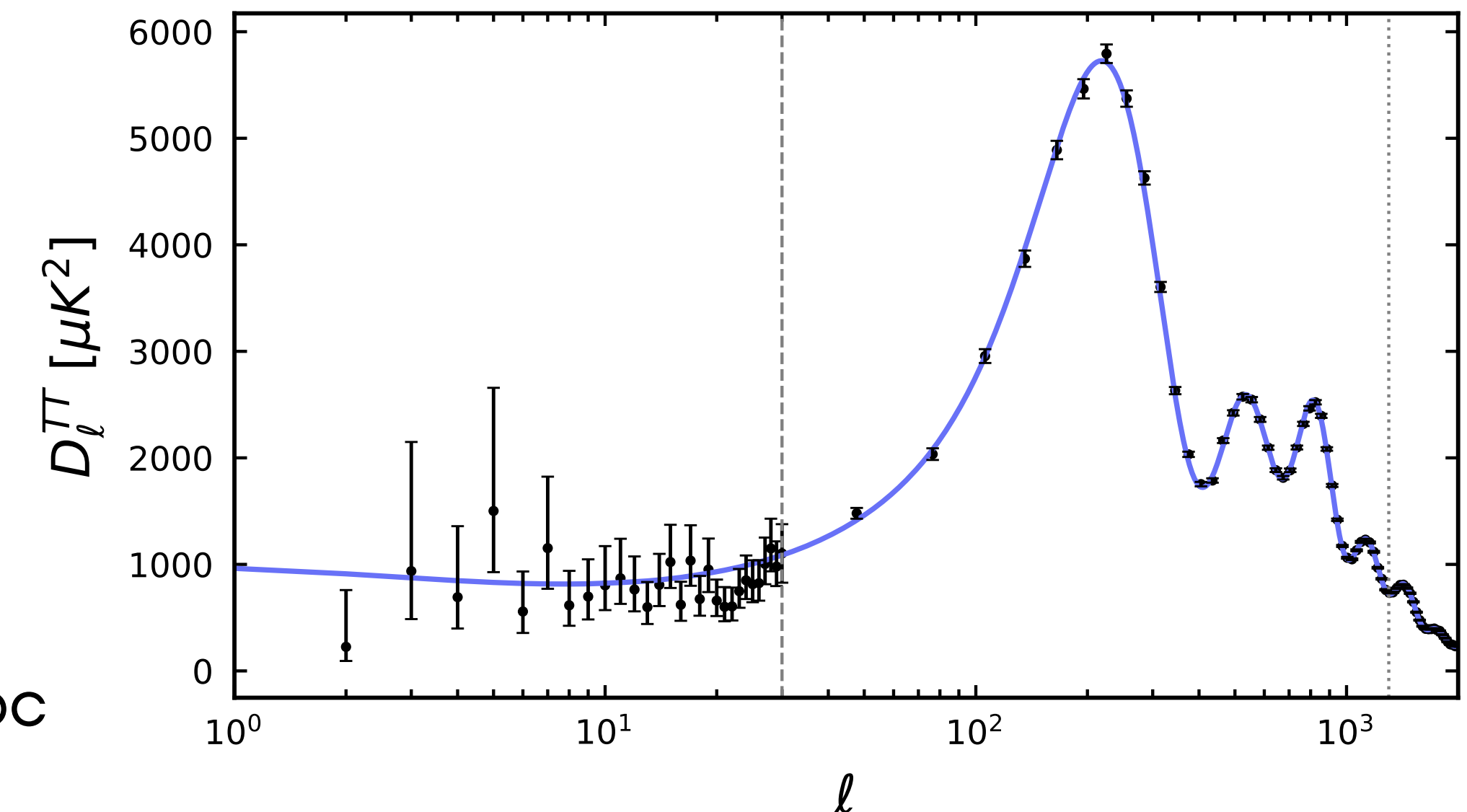
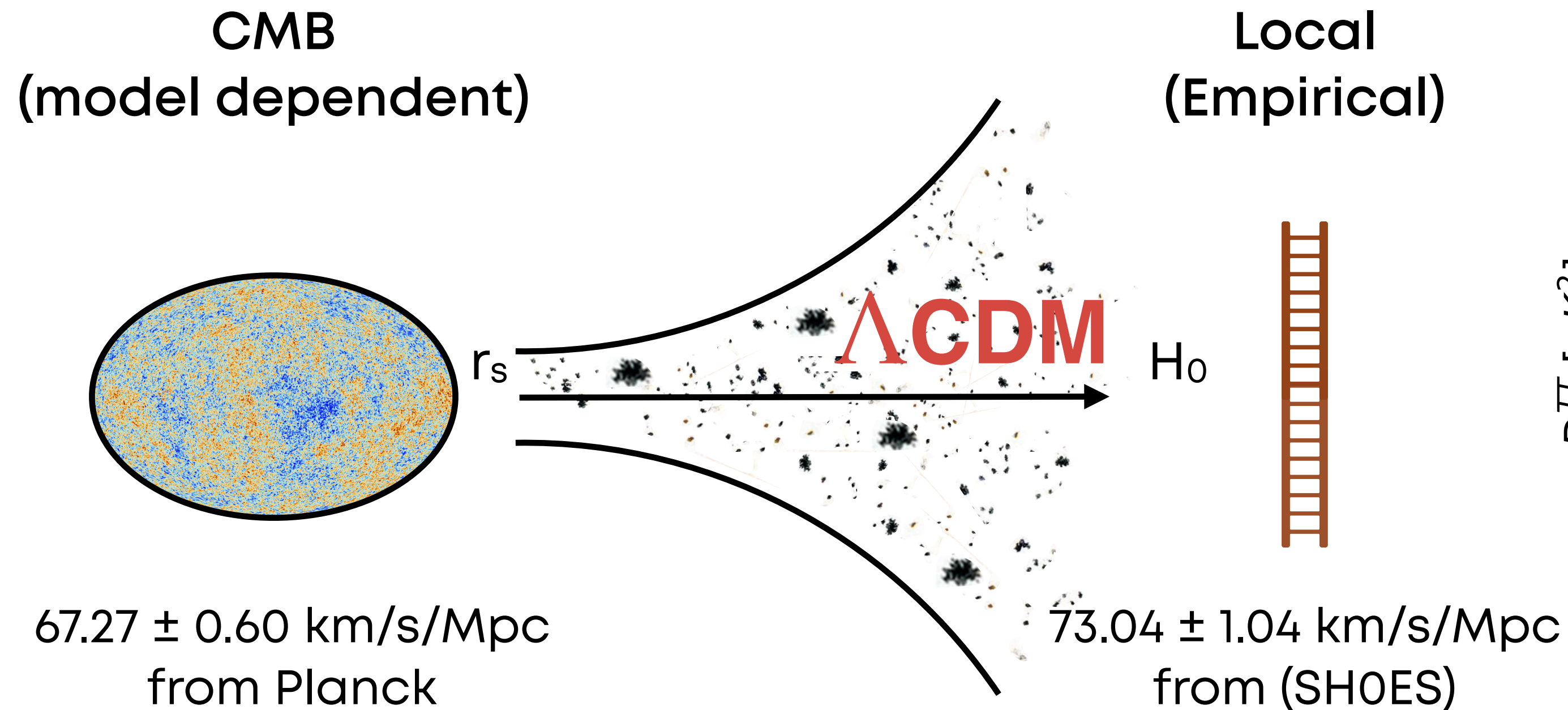
CMB
(model dependent)



[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]



The “Hubble Tension”

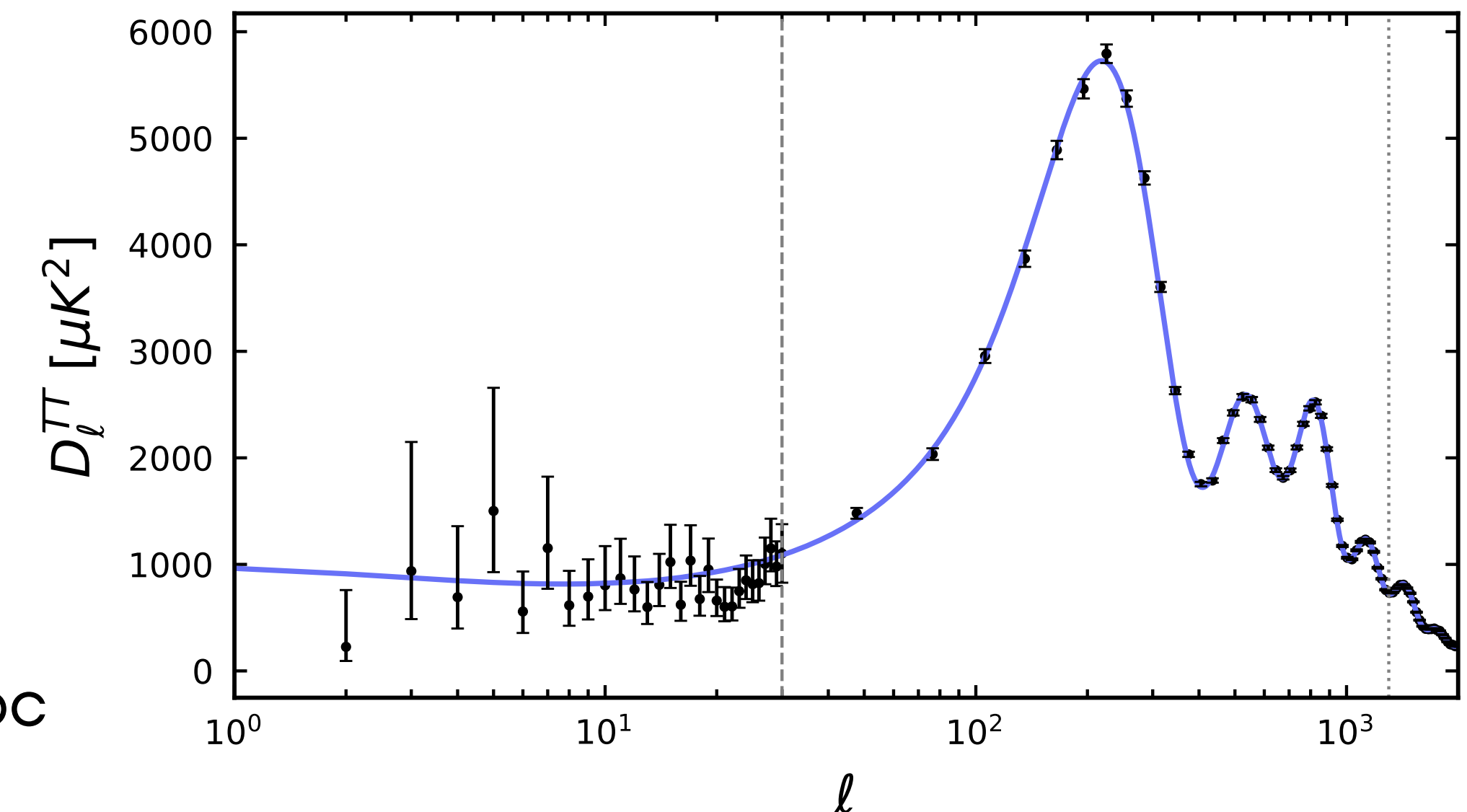
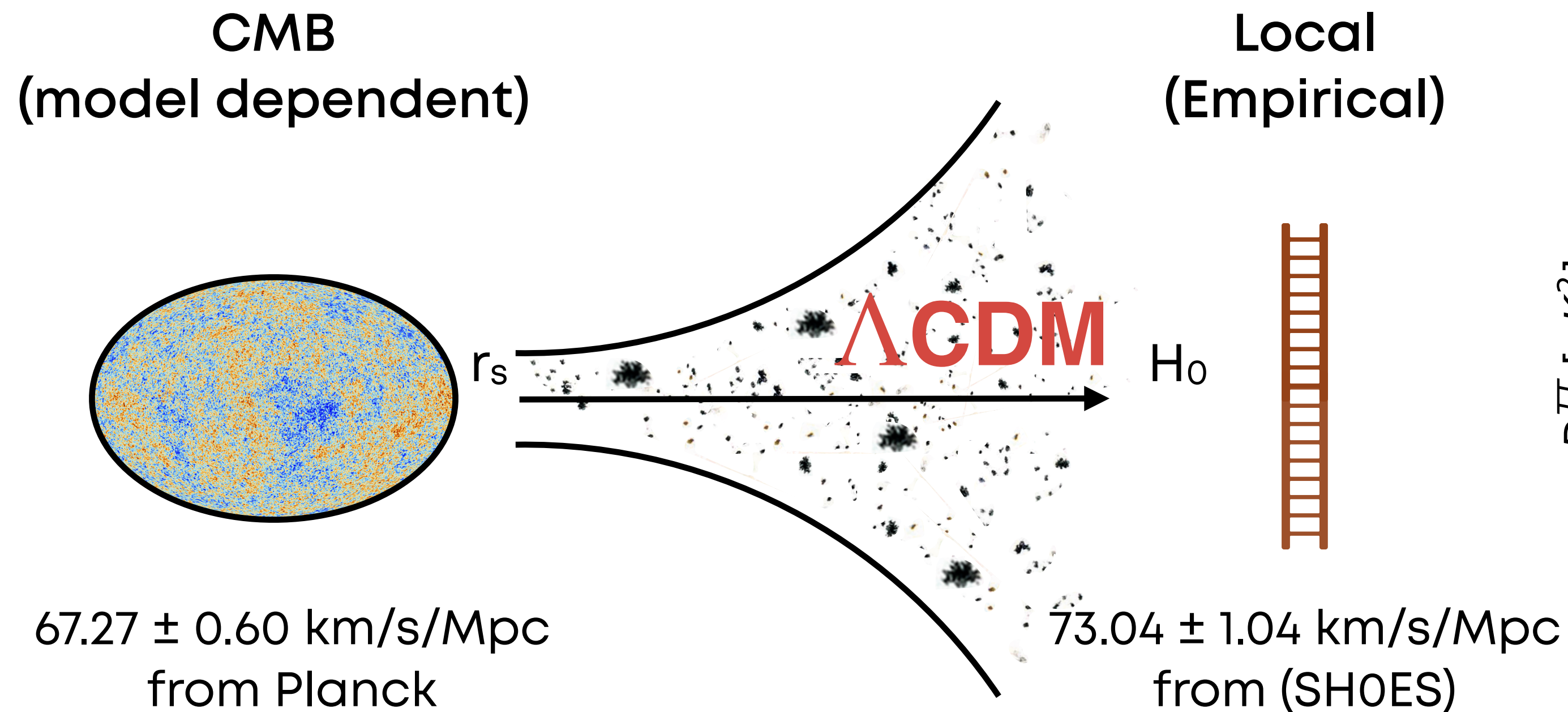


- Infer H_0 from the cosmological distance ladder
- Based on local distance measurements and astrophysical observables/calibrations

[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]



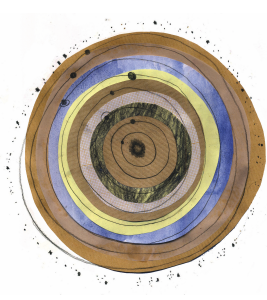
The “Hubble Tension”



- Infer H_0 from the cosmological distance ladder
- Based on local distance measurements and astrophysical observables/calibrations

The distance duality relation



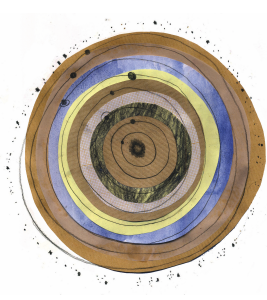


D_A from BAO

D_L from SN1a

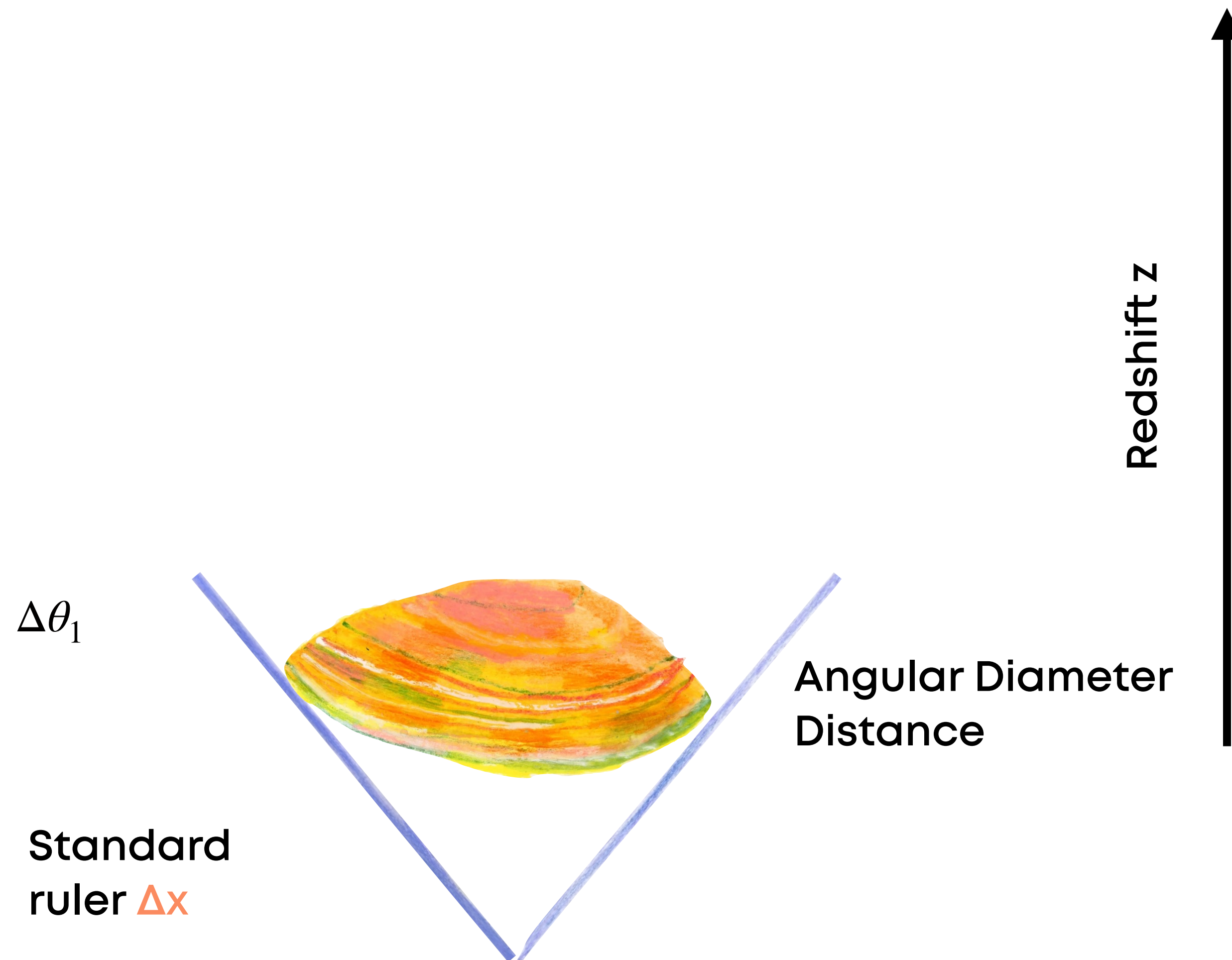
Redshift z

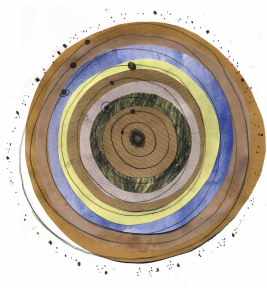




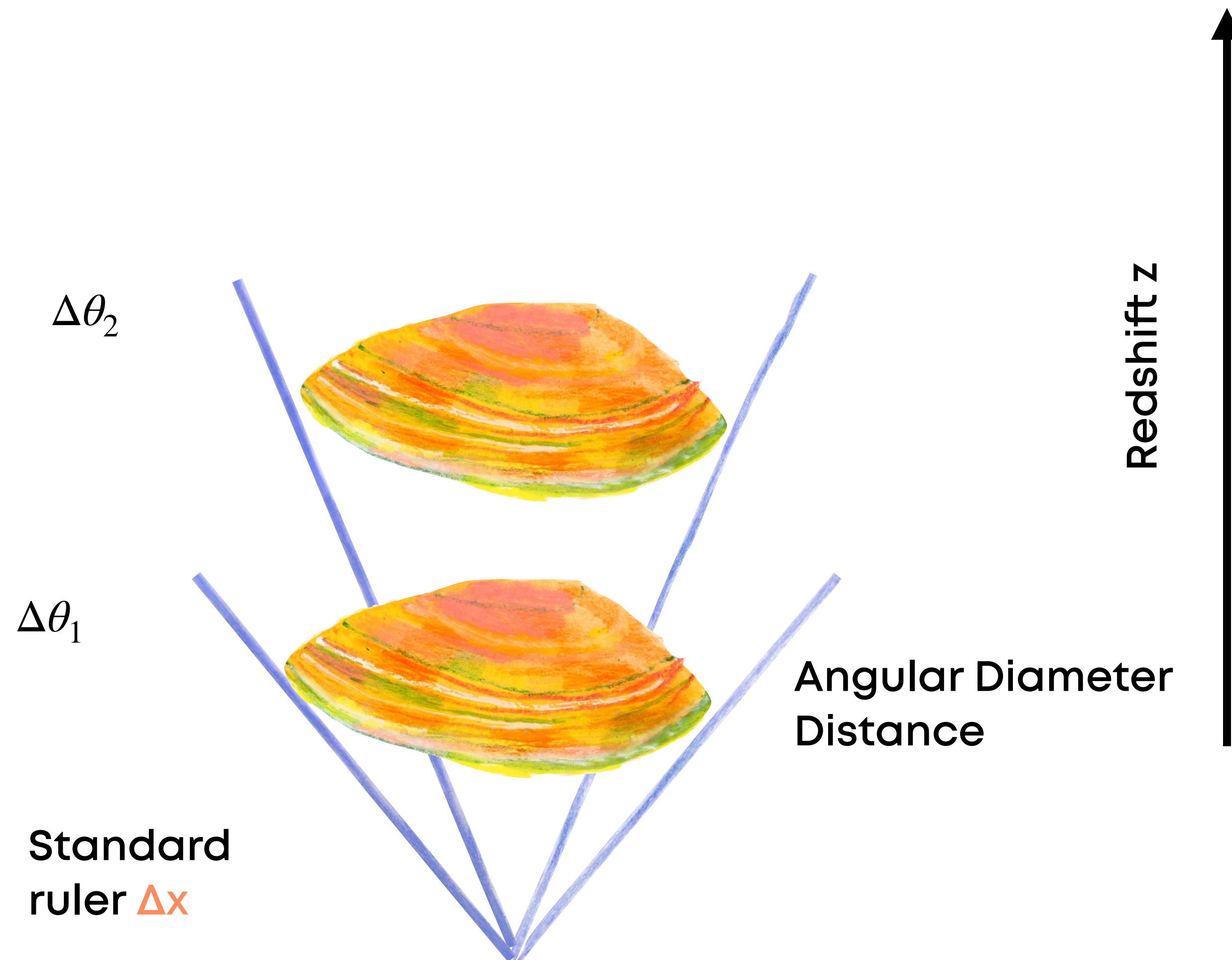
D_A from BAO

D_L from SN1a

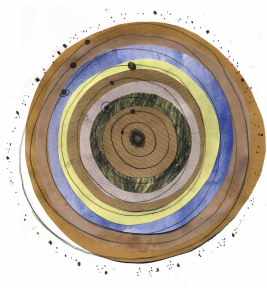




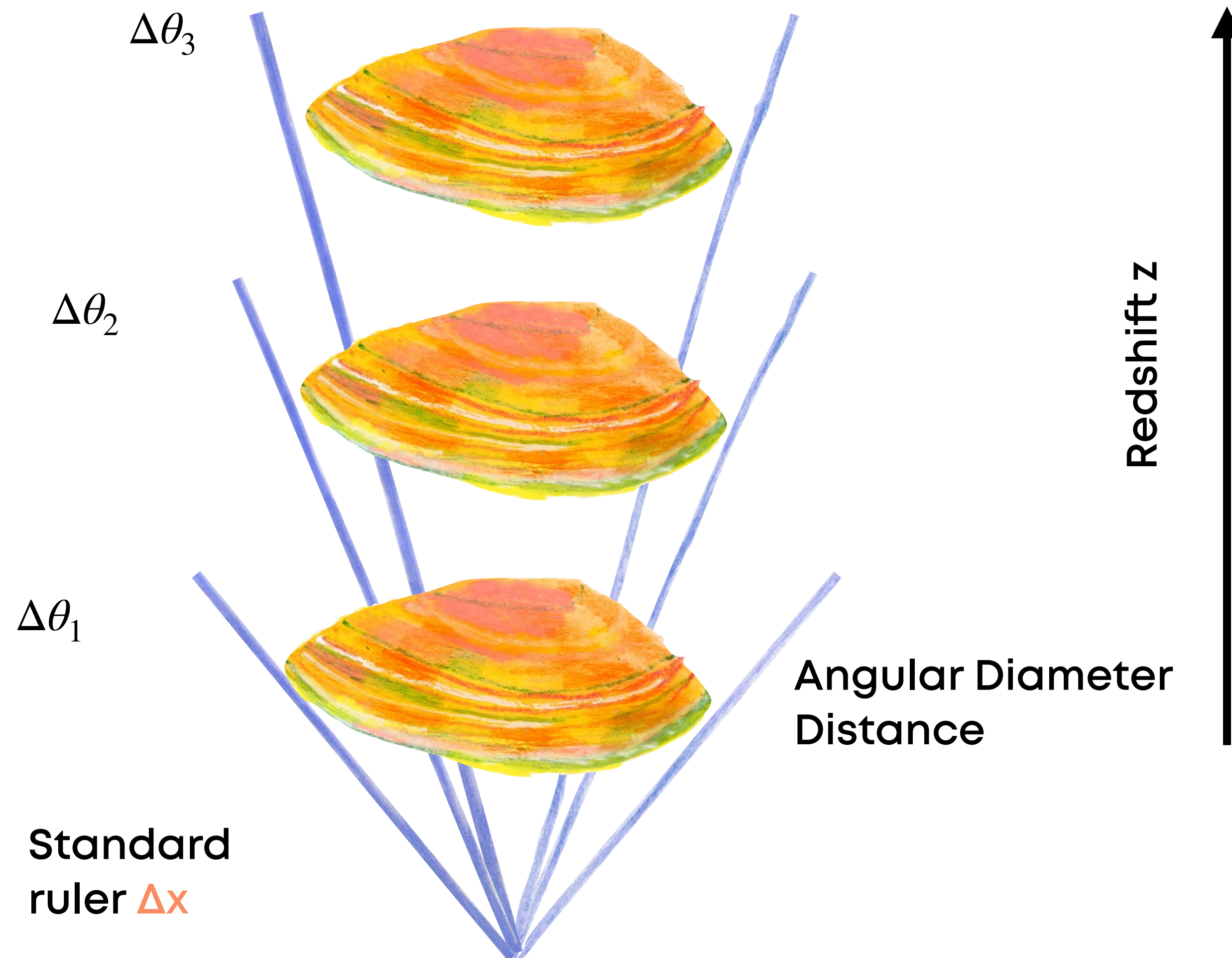
D_A from BAO



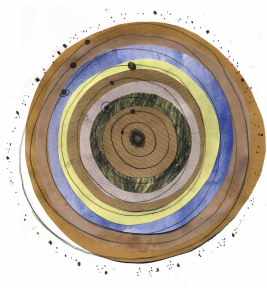
D_L from SN1a



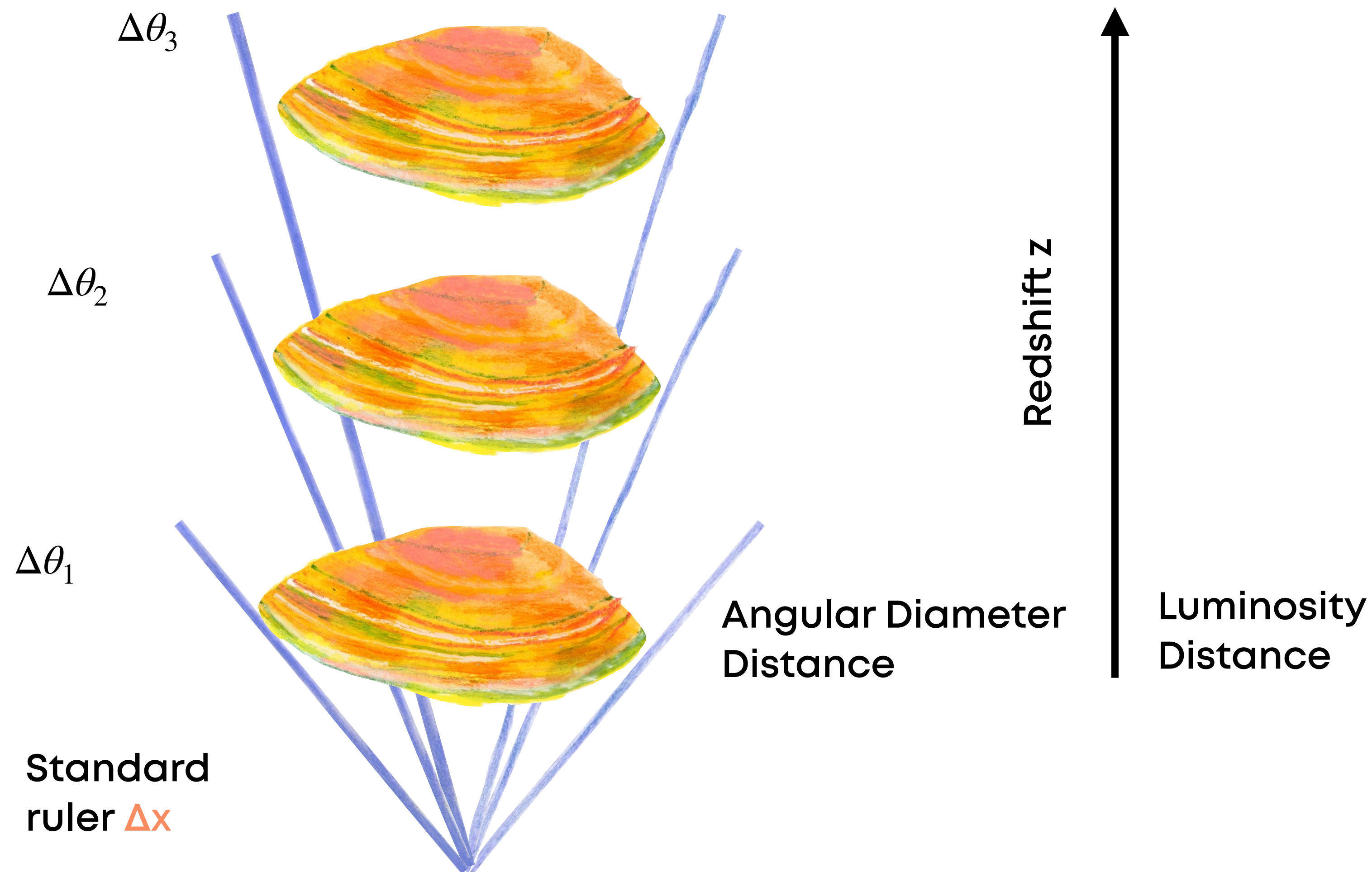
D_A from BAO



D_L from SN1a

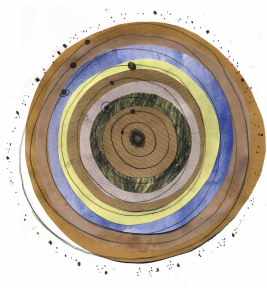


D_A from BAO

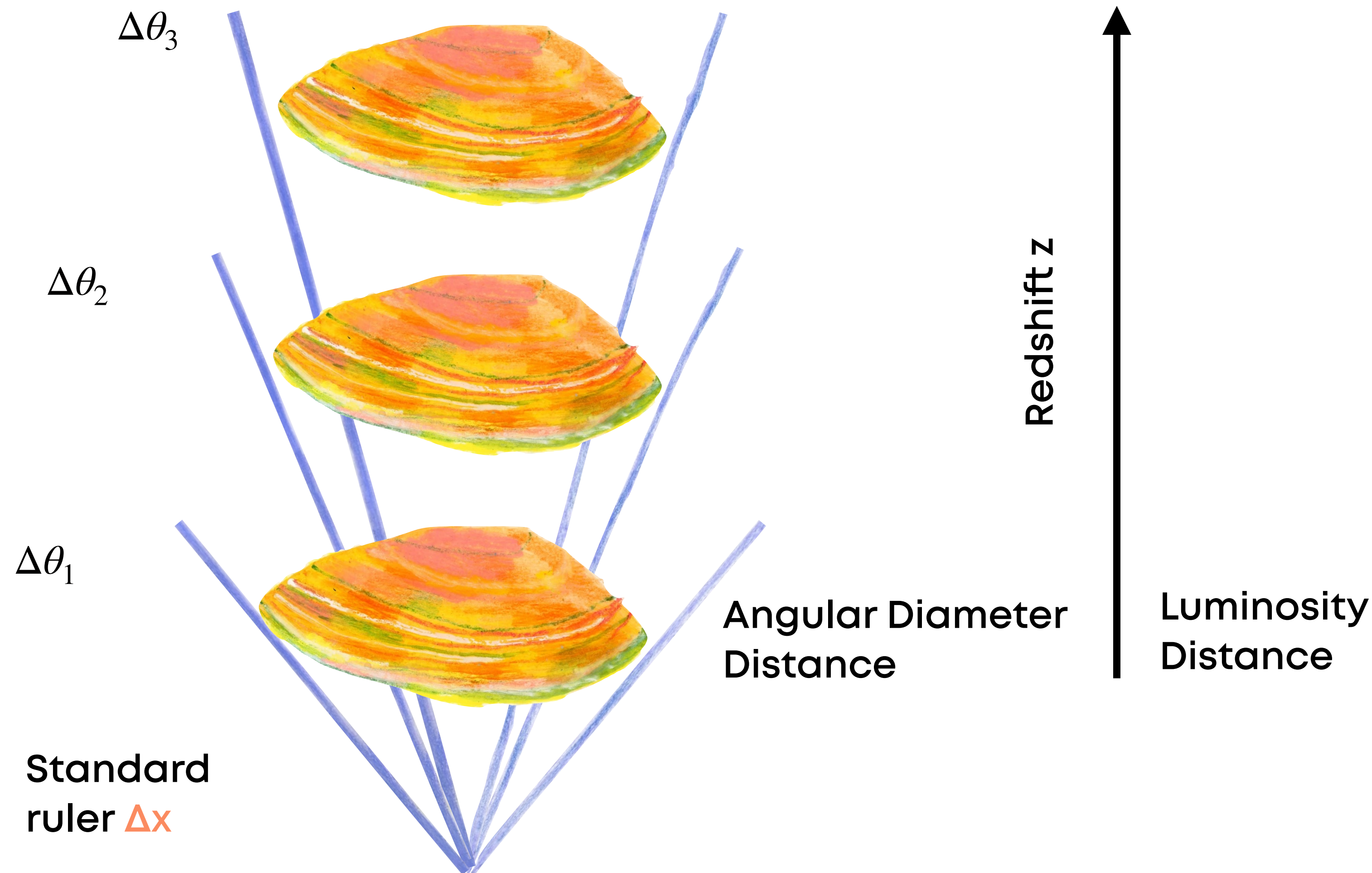


D_L from SN1a

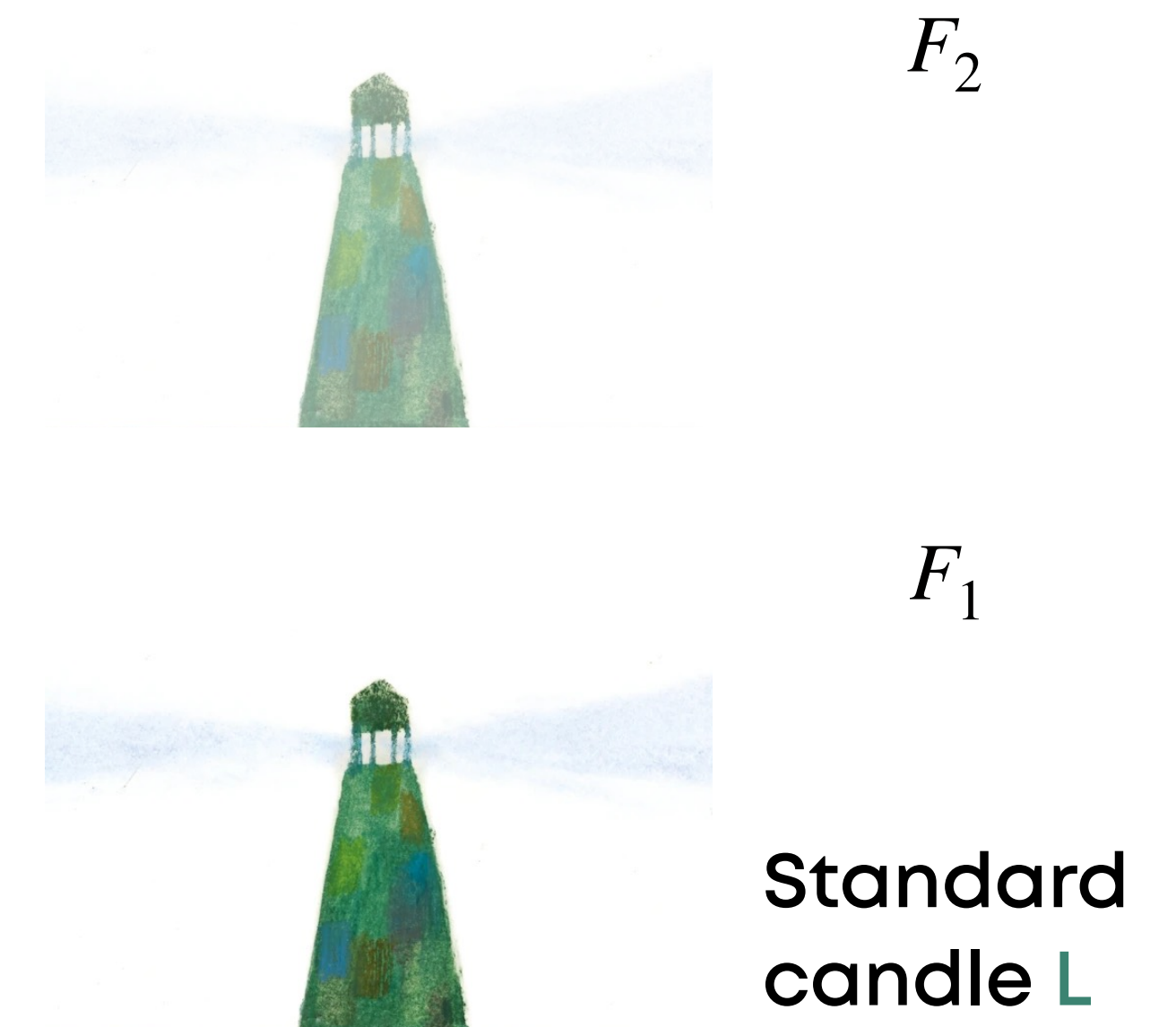




D_A from BAO

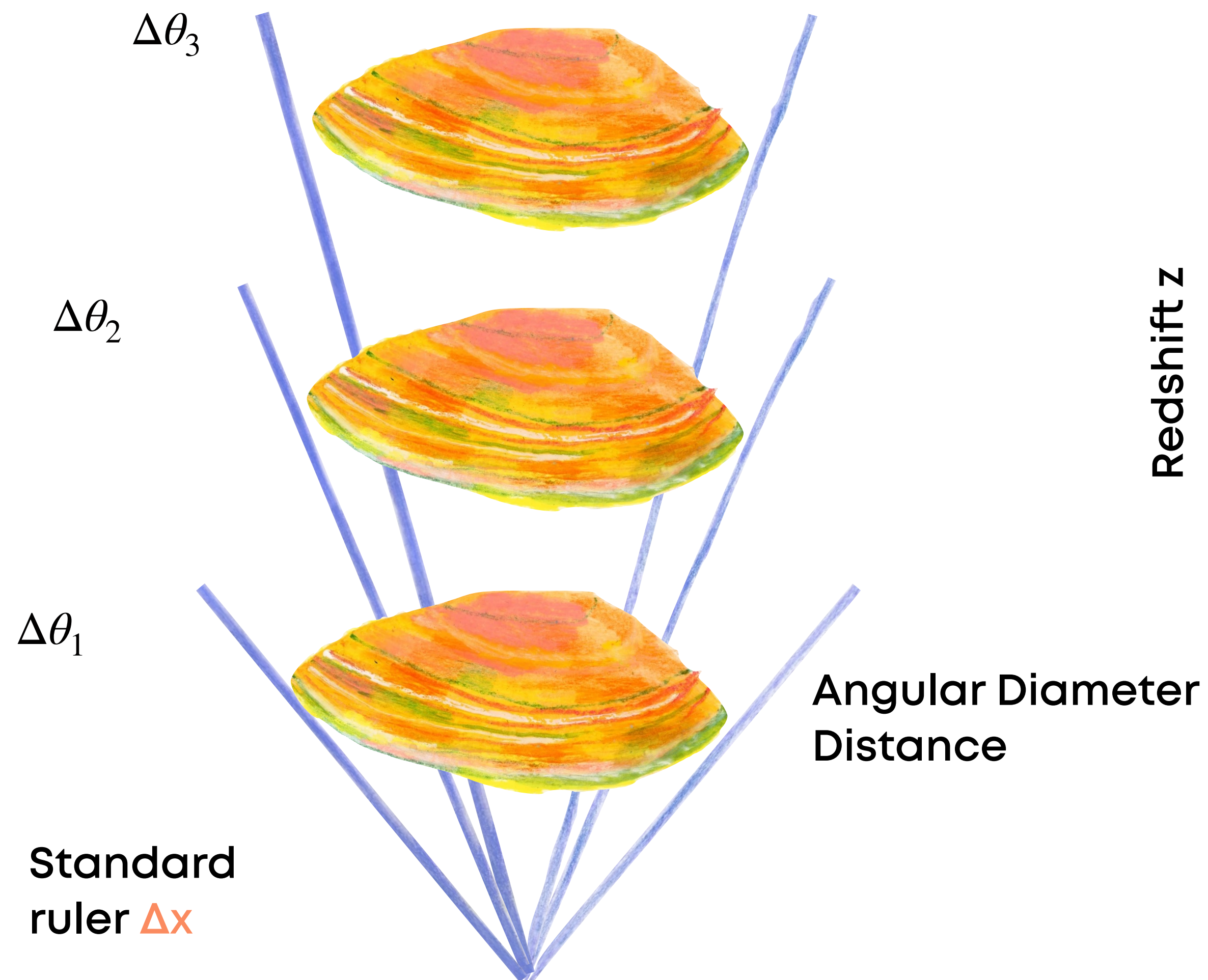


D_L from SN1a

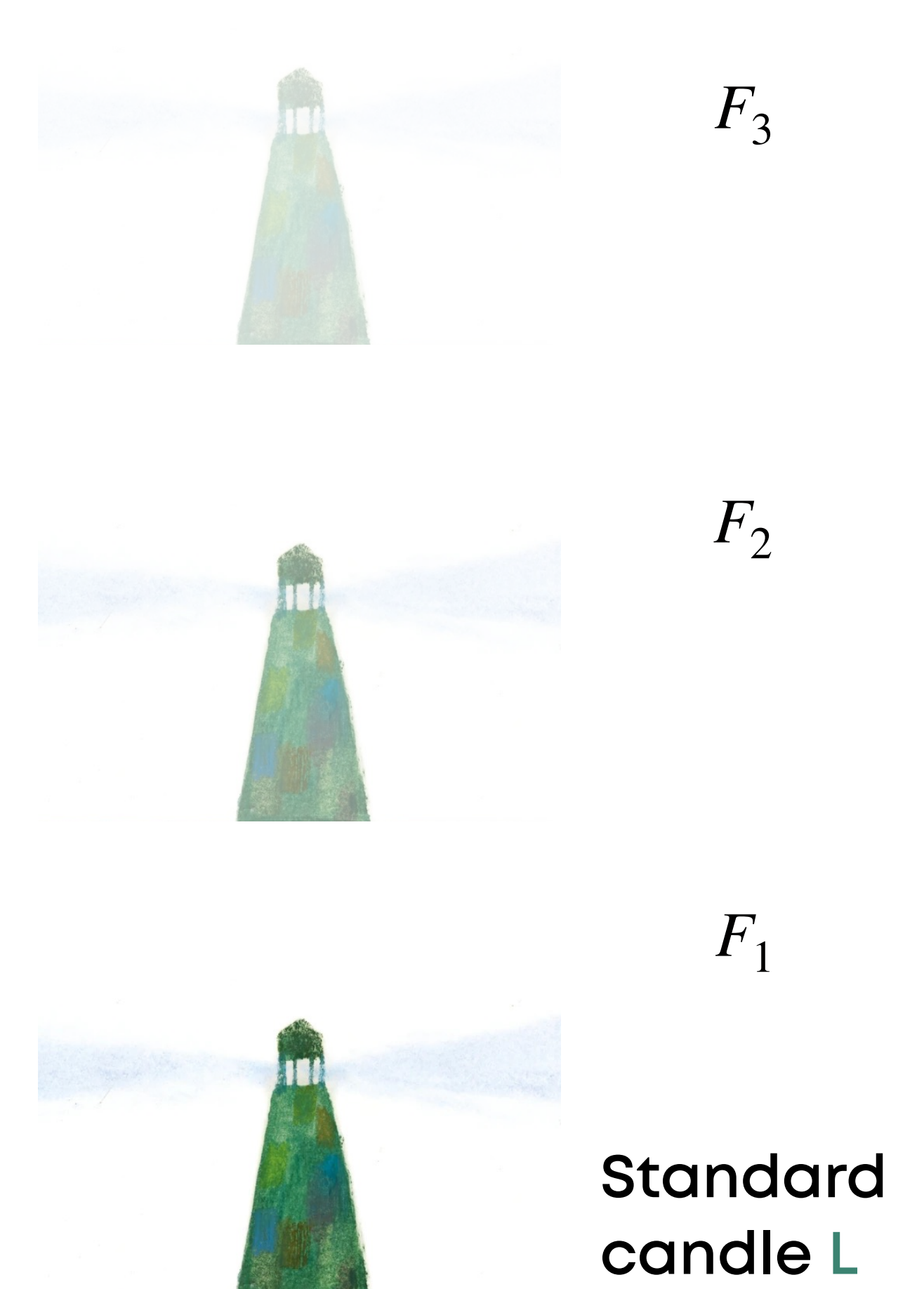




D_A from BAO



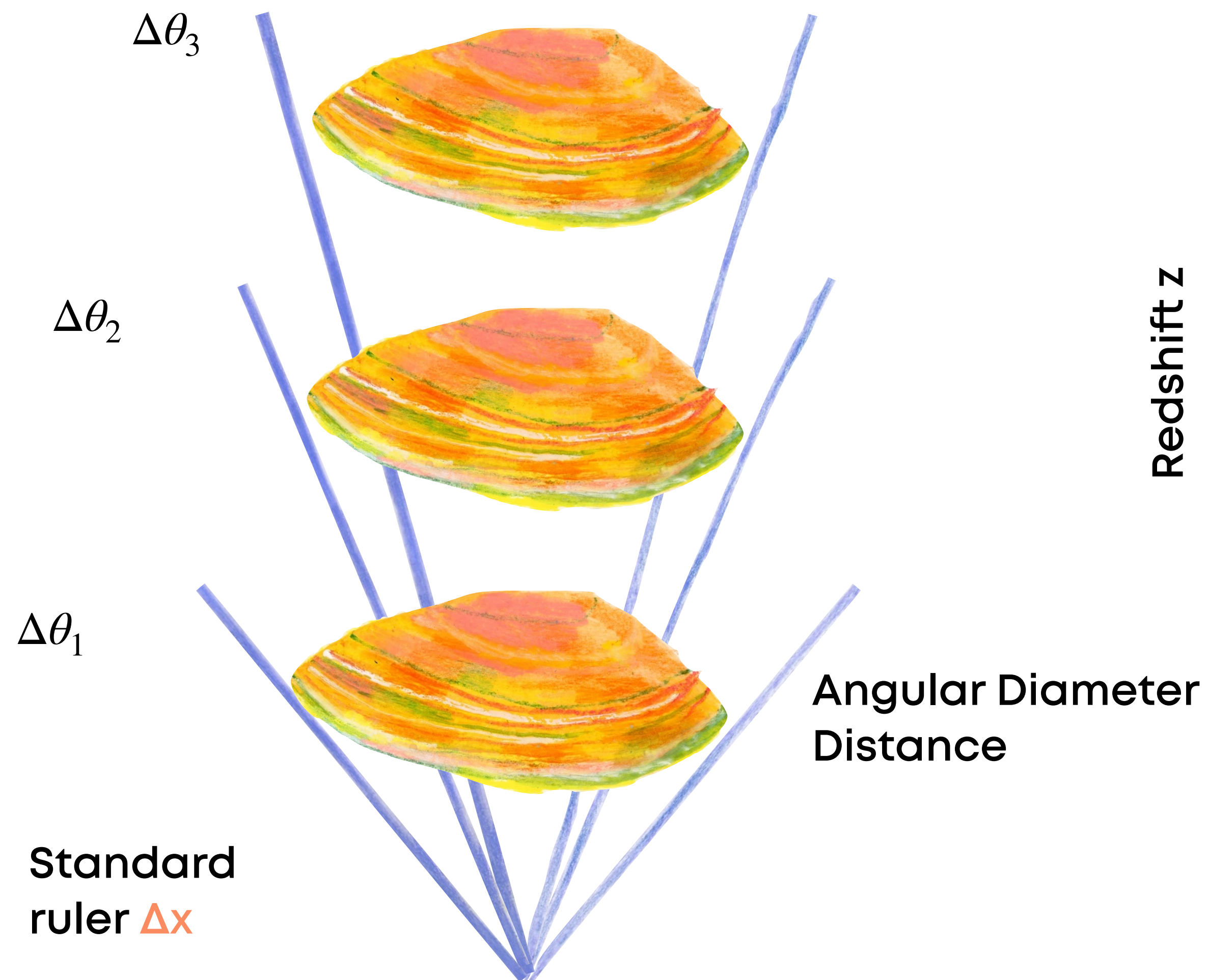
D_L from SN1a





Distance Duality Relation (DDR)

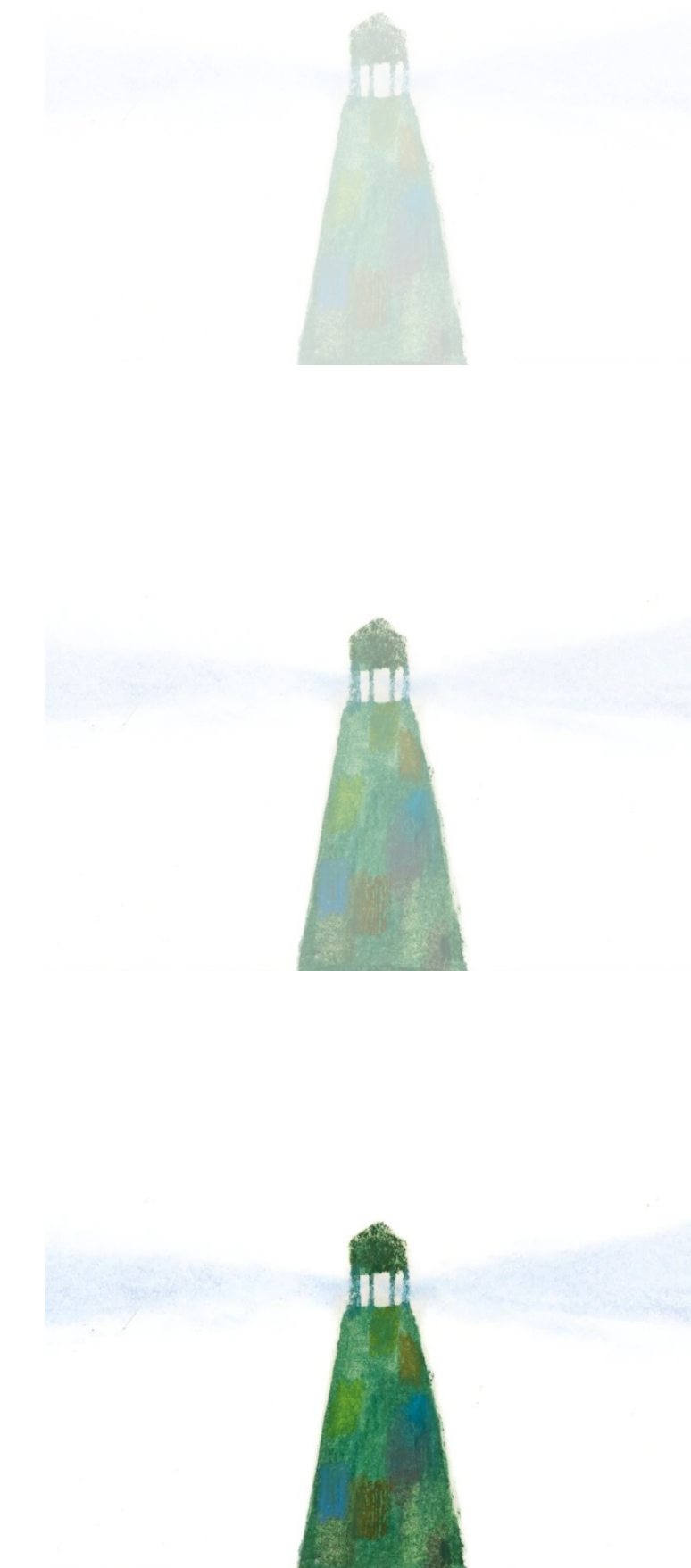
D_A from BAO



Redshift z

Luminosity Distance

D_L from SN1a

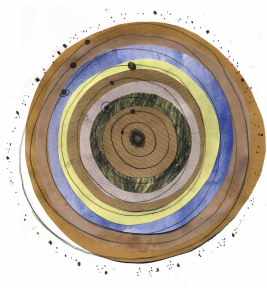


F_3

F_2

F_1

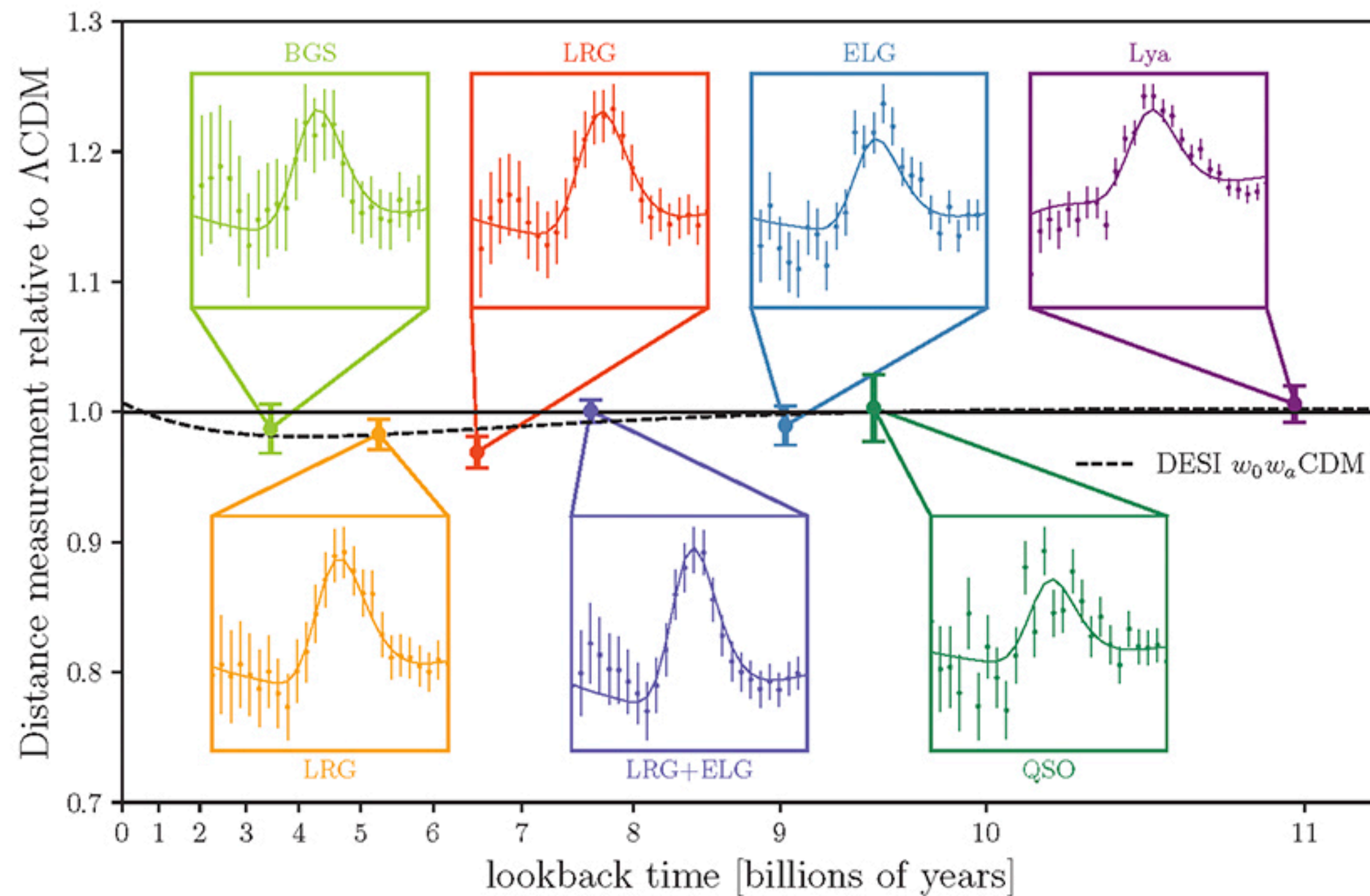
Standard candle L



Distance Duality Relation (DDR)

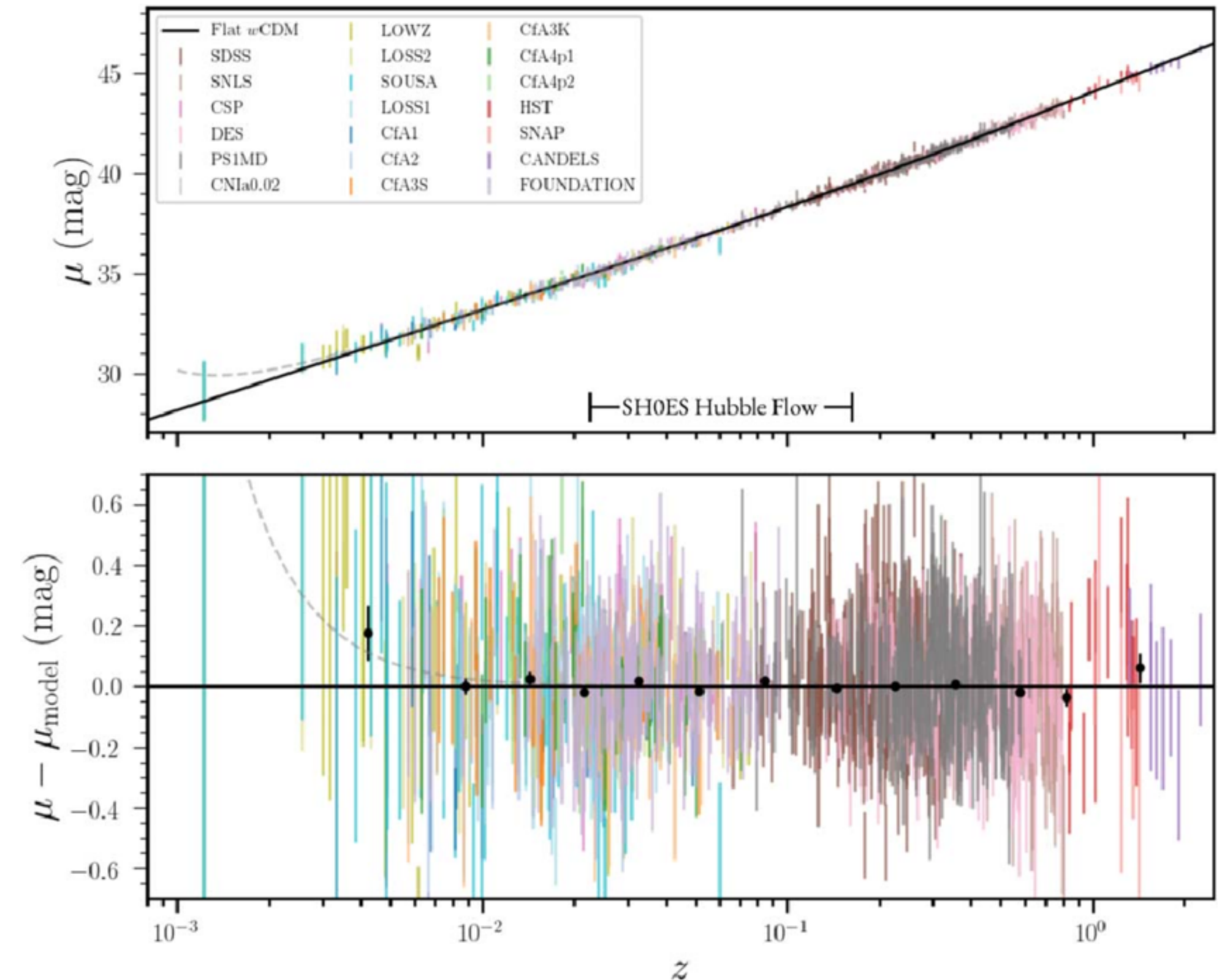
D_A from BAO by DESI

[I. M. H. Etherington (1933)]

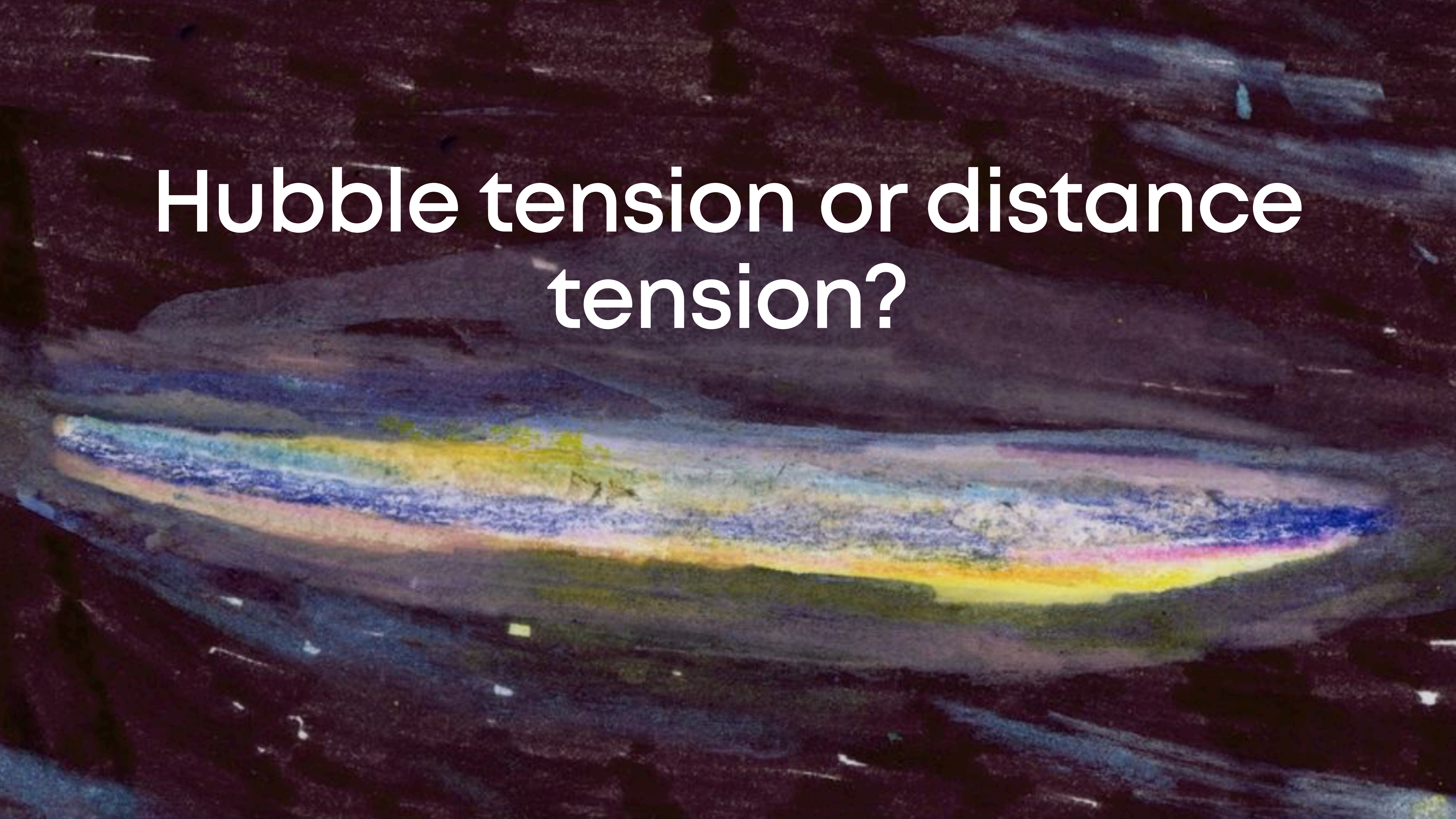


[DESI Collaboration 2024, arxiv:2404.03002]

D_L from SN1a by Pantheon+



[The Pantheon+ Analysis 2022, arxiv:2202.04077]



Hubble tension or distance
tension?



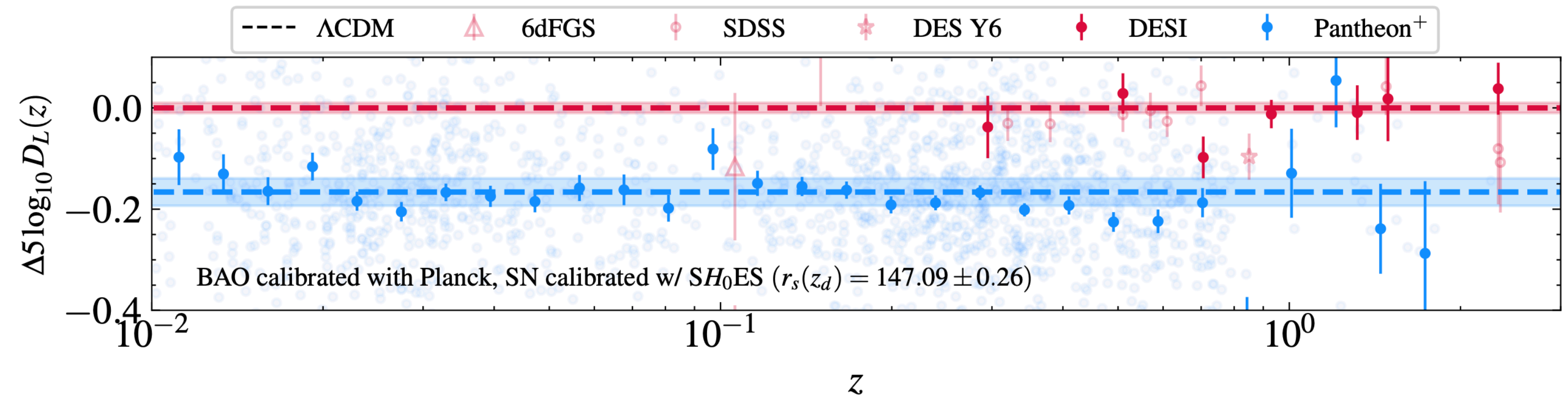
Hubble Tension or Distance Tension?

From *Planck*: $r_s \sim 147$ Mpc:

[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]

From *SHOES*: $M_b \sim -19.25$

[Riess et. al: Astrophys. J. Lett. 934 (2022) 1 L7]



[Poulin et al.: arXiv: 2407.18292]

[Camarena et al.: arXiv: 2101.08641]

[Efsthathiou: arXiv: 2103.08723]

[Raveri: arXiv: 2309.06795]

[Tutusaus et al.: arXiv: 2311.16862]



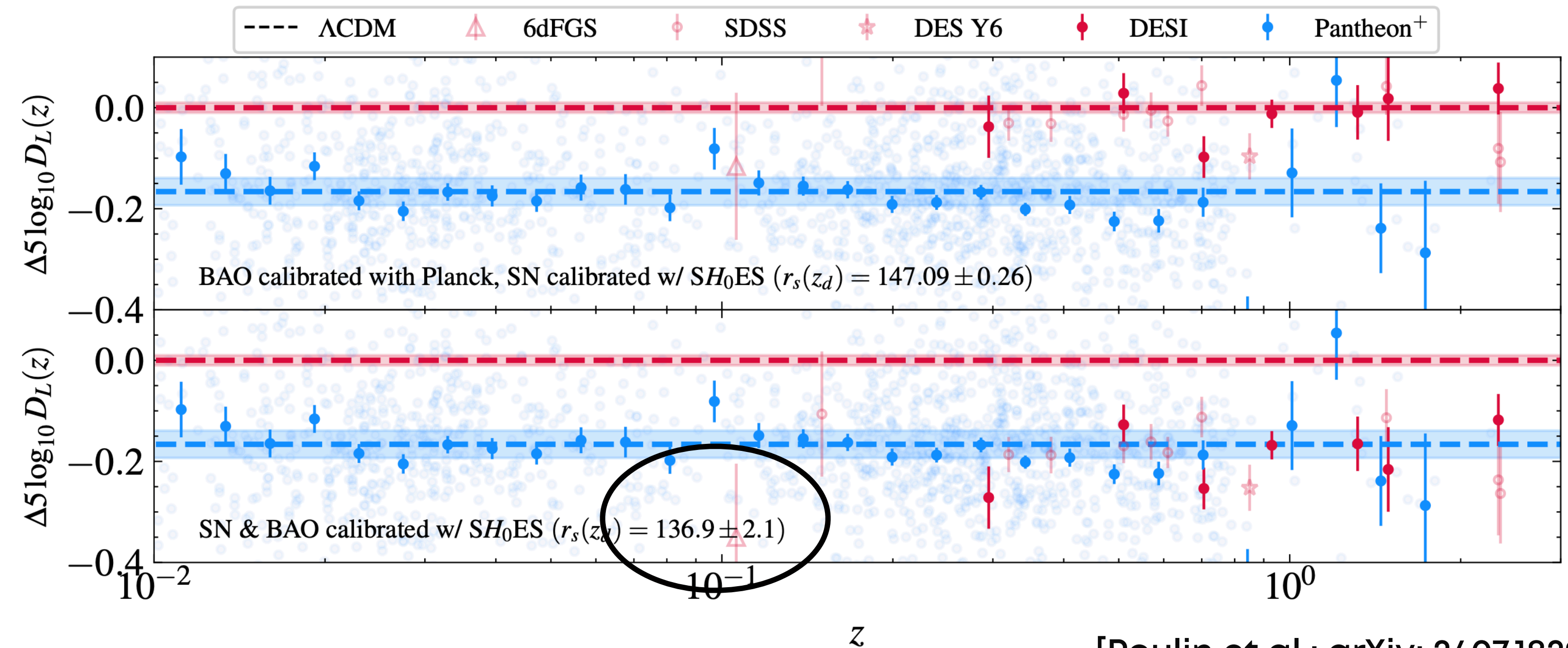
Hubble Tension or Distance Tension?

From *Planck*: $r_s \sim 147$ Mpc:

[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]

From *SHOES*: $M_b \sim -19.25$

[Riess et. al: Astrophys. J. Lett. 934 (2022) 1 L7]



[Poulin et al.: arXiv: 2407.18292]

Bring the data sets together:

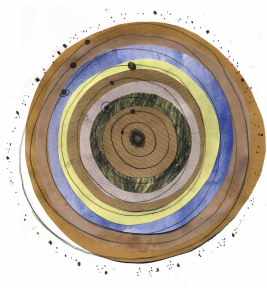
- Change calibrators, e.g. change r_s (constant overall shift)

[Camarena et al.: arXiv: 2101.08641]

[Efsthathiou: arXiv: 2103.08723]

[Raveri: arXiv: 2309.06795]

[Tutusaus et al.: arXiv: 2311.16862]



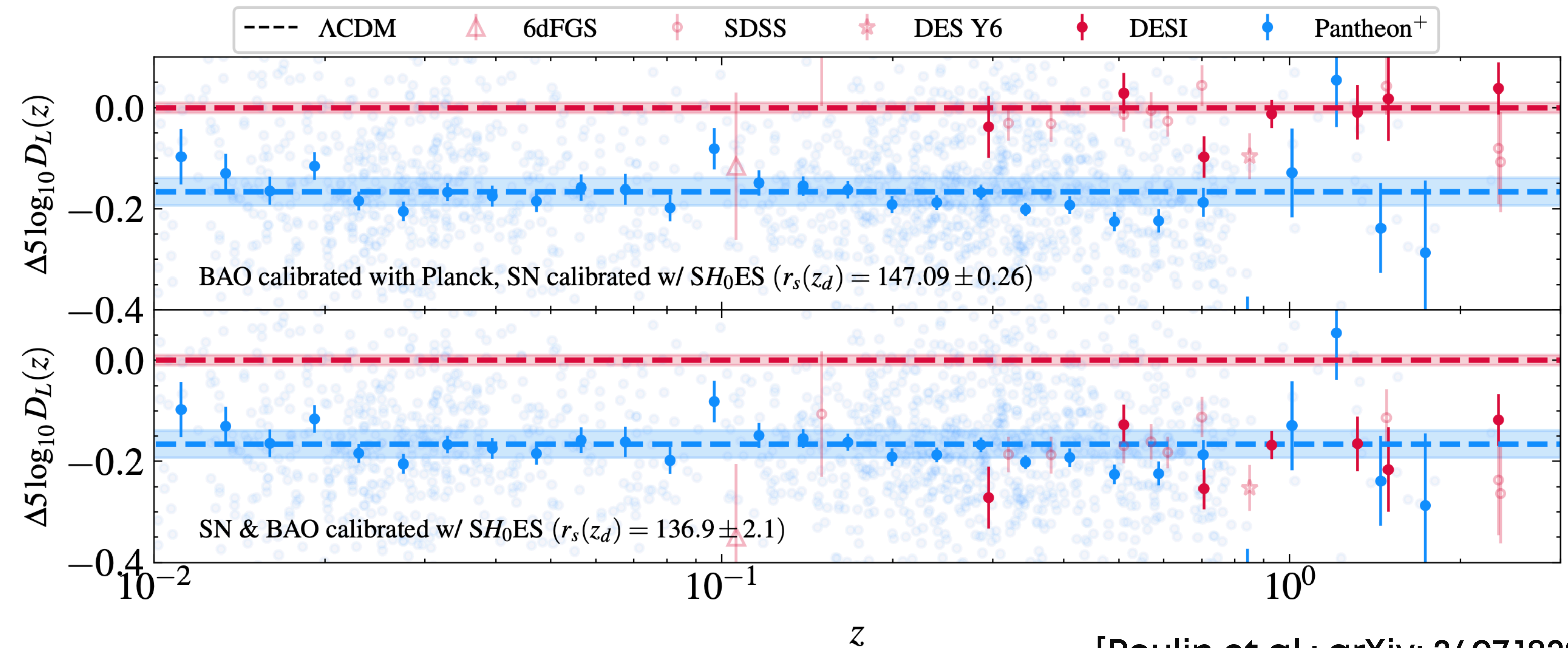
Hubble Tension or Distance Tension?

From *Planck*: $r_s \sim 147$ Mpc:

[Aghanim et al.: Astron.Astrophys. 641 (2020) A6]

From *SHOES*: $M_b \sim -19.25$

[Riess et. al: Astrophys. J. Lett. 934 (2022) 1 L7]



[Poulin et al.: arXiv: 2407.18292]

Bring the data sets together:

- Change calibrators, e.g. change r_s (constant overall shift)

[Camarena et al.: arXiv: 2101.08641]

[Efsthathiou: arXiv: 2103.08723]

- Break the “distance duality relation” (possible redshift dependance)

[Raveri: arXiv: 2309.06795]

[Tutusaus et al.: arXiv: 2311.16862]

The DDR and evidence for Dynamical Dark Energy

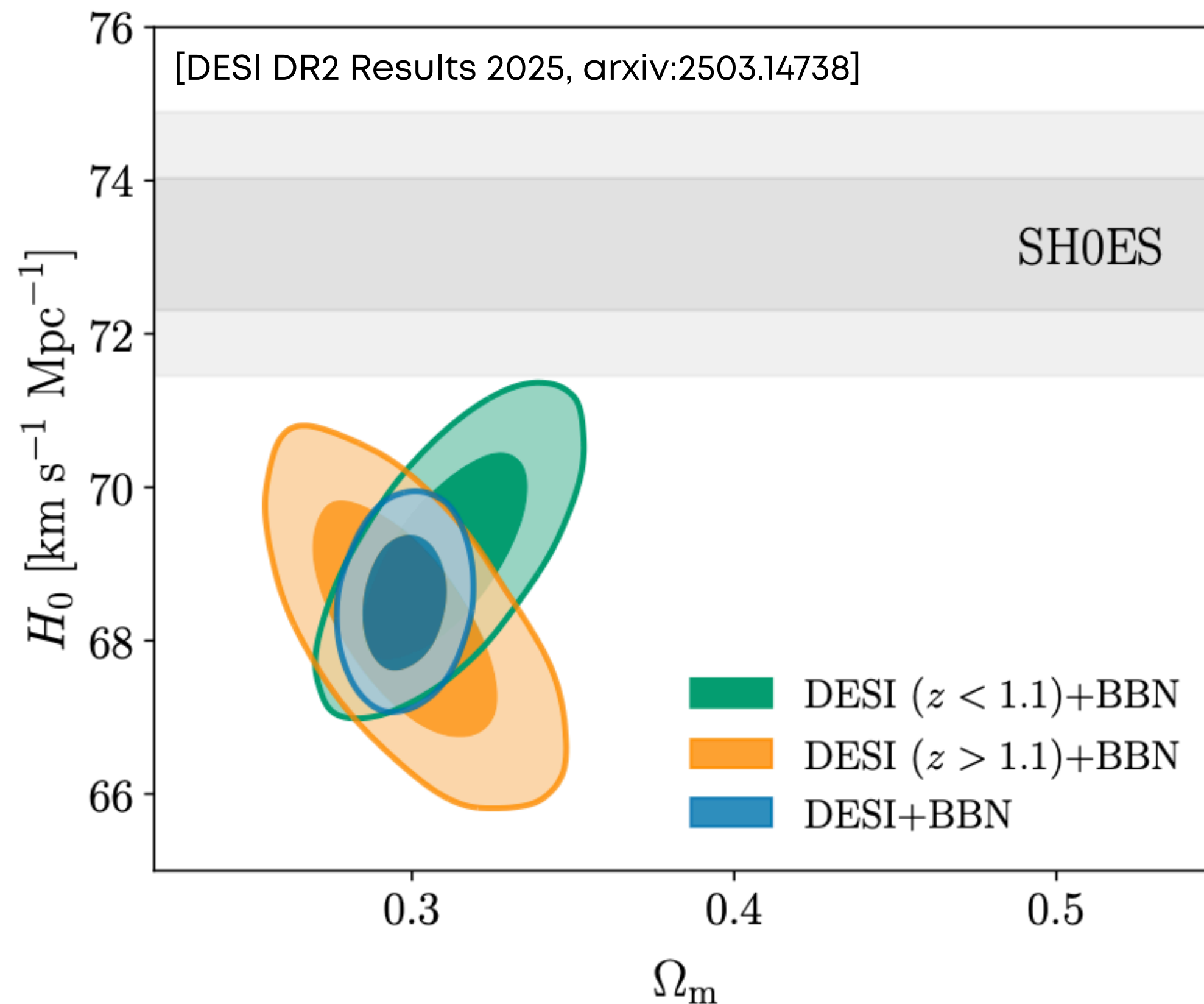
Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: [arxiv:2504.10464](https://arxiv.org/abs/2504.10464)]



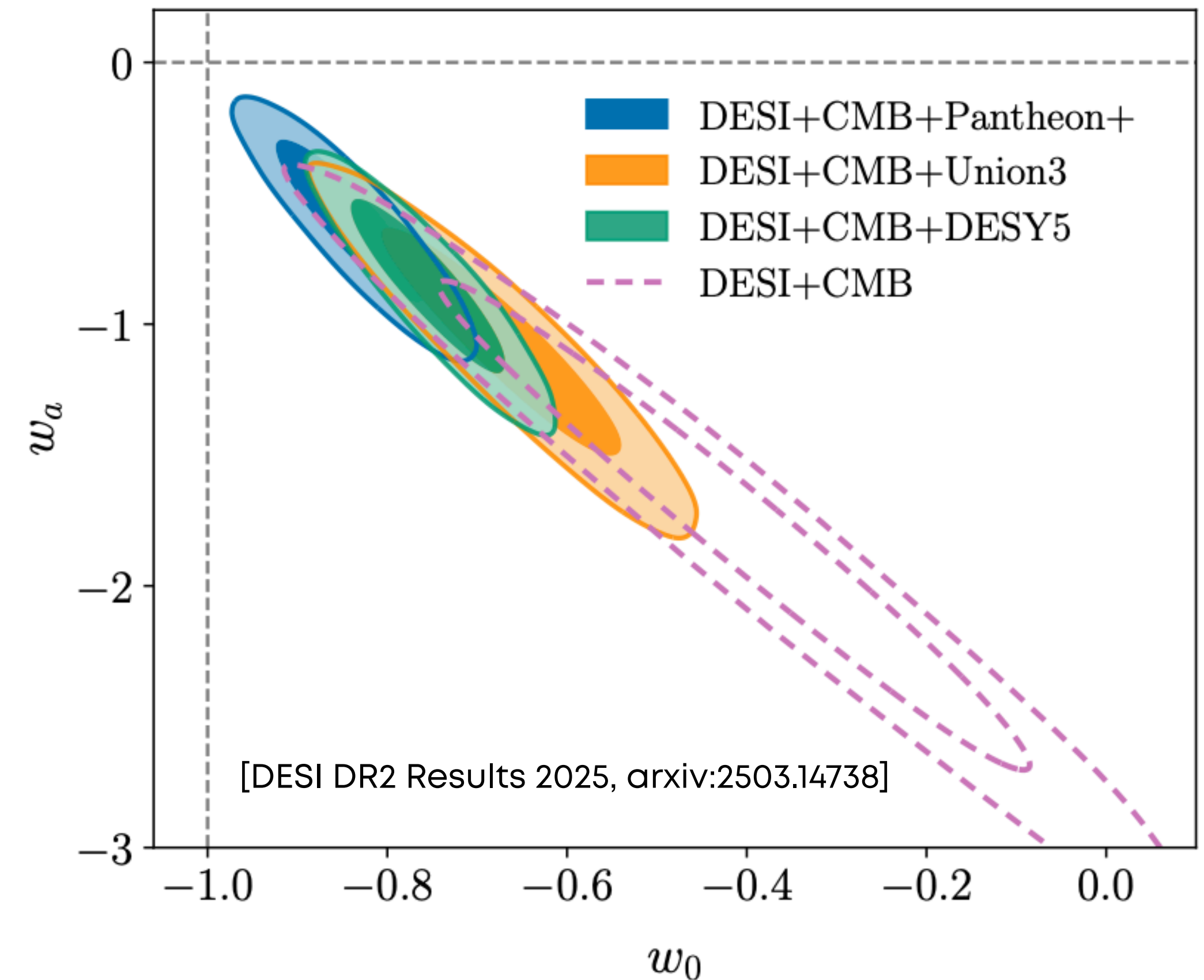
The H_0 tension at late times

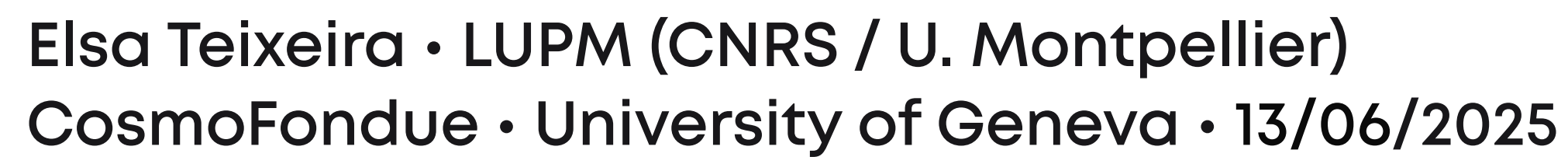
● Tension of more than 3σ with SH0ES

● Hints of dynamical DE in DESI data



$$\dot{\rho}_{\text{DE}} + 3H\rho_{\text{DE}}(1 + w) = 0, \quad w = w_0 + w_a(1 - a)$$

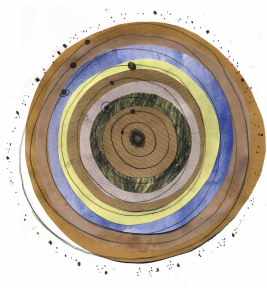




Reconciling the cosmological distances between DESI BAO and Pantheon+ SN

- DDR is assumption of Λ CDM - holds for metric theories of gravity with photons travelling on null geodesics + their number conservation
- Violation encoded in $\eta(z)$: e.g. photons interacting with BSM particles or astrophysical absorption/opacity
- Proof of concept: effect of geometrical breaking of DDR for SN and BAO (no thermal evolution of CMB)
- If $\eta(z)$ is just a constant then we are probably dealing with calibration issues [Poulin et al.: arXiv: 2407.18292]
- Is there evidence for more than 1 dof and/or redshift dependence? Mechanisms and physical implications





Breaking the DDR

Reconciling the cosmological distances between DESI BAO and Pantheon+ SN

1. Can simple phenomenological parameterisations of **DDR violation in a Λ CDM background cosmology** **reduce/eliminate the tension** between calibrated SNIa and BAO?





Breaking the DDR

Reconciling the cosmological distances between DESI BAO and Pantheon+ SN

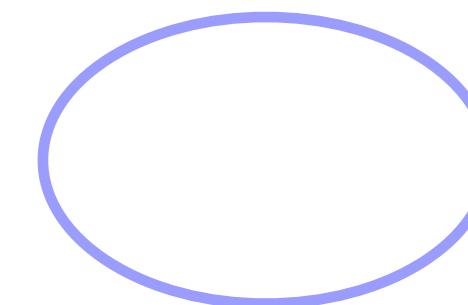
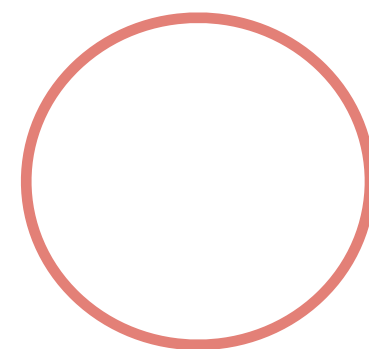
1. Can simple phenomenological parameterisations of **DDR violation in a Λ CDM background cosmology reduce/eliminate the tension** between calibrated SNIa and BAO?
2. Is there evidence for a **violation of the DDR that changes over cosmic history?**
3. Can a **DDR violation alter the preference for dynamical dark energy** observed in the combination of current BAO and SNIa data?





Data Sets

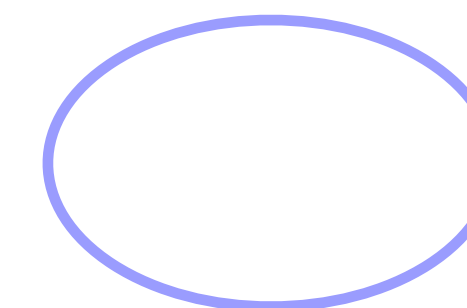
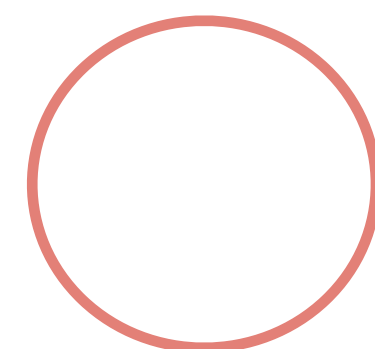
- **Pantheon-plus (SN)**: measurements of $\mu(z, D_L)$ from spectroscopically detected Type Ia supernovae in the redshift range $0.001 < z < 2.26$ [Brout et al.: ApJ 938 110 (2022)]
- **DESI Y1 BAO**: BAO measurements of $H(z)$ and $D_A(z)$ in the redshift range $z \sim 0.1 - 4.1$ [Adame et al.: arxiv:2404.03002]
- **SHOES** prior on M_B : $M_B \sim -19.25$ [Riess et. al: Astrophys. J. Lett. 934 (2022) 1 L7]
- **Planck 2018 CMB** data: (high- ℓ TTTEEE ‘Plik-lite’, ‘Plik’ low- ℓ TT and EE) [Aghanim et al.: Astron.Astrophys. 641 (2020) A5]





Data Sets

- **Pantheon-plus (SN)**: measurements of $\mu(z, D_L)$ from spectroscopically detected Type Ia supernovae in the redshift range $0.001 < z < 2.26$ [Brout et al.: ApJ 938 110 (2022)]
- **DESI Y1 BAO**: BAO measurements of $H(z)$ and $D_A(z)$ in the redshift range $z \sim 0.1 - 4.1$ [Adame et al.: arxiv:2404.03002]
- **SHOES** prior on M_B : $M_B \sim -19.25$ [Riess et. al: Astrophys. J. Lett. 934 (2022) 1 L7]
- **Planck 2018 CMB** data: (high- ℓ TTTEEE ‘Plik-lite’, ‘Plik’ low- ℓ TT and EE) [Aghanim et al.: Astron.Astrophys. 641 (2020) A5]



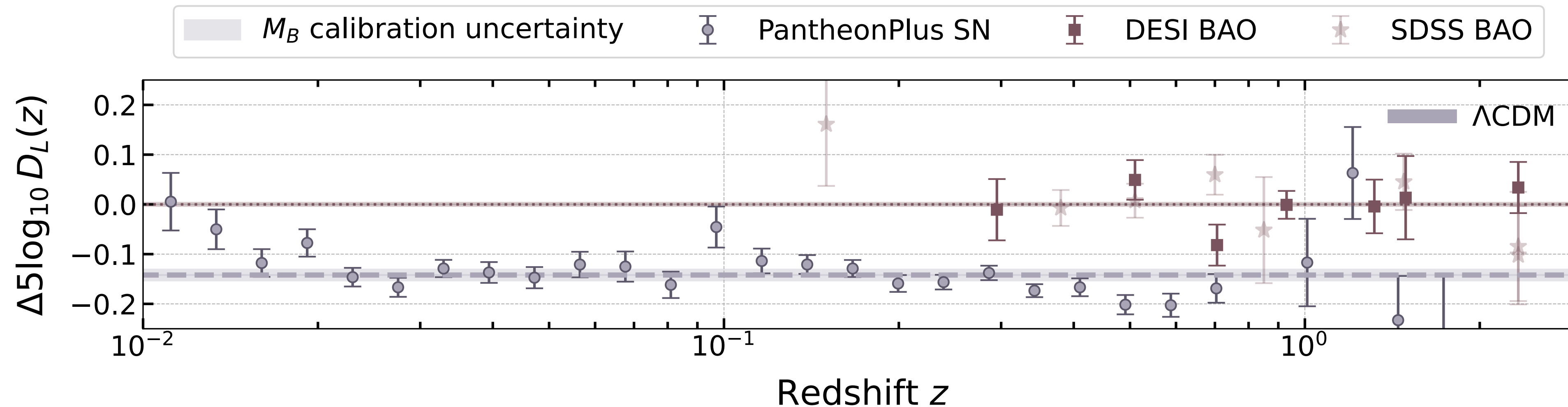
1. Breaking the DDR as a solution to the Hubble tension

Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: [arxiv:2504.10464](https://arxiv.org/abs/2504.10464)]



Combine the data

- In Λ CDM with no DDR violation the various **BAO estimates of $D_L(z)$** are systematically larger than the **SN Ia estimates**
- Tries to accommodate both data sets - bad overall fit reflecting tension

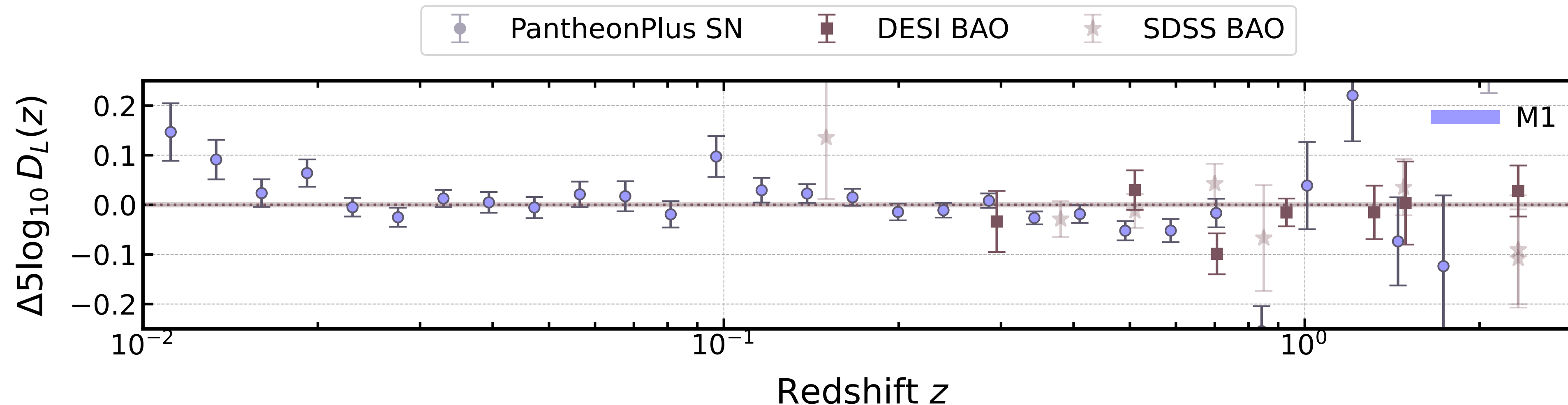


$$T = 5.7\sigma$$



M1 - Constant DDR

- A phenomenological violation of the DDR can indeed **resolve the Hubble tension**
- The model with the largest evidence is **compatible with a simple rescaling of the calibration** of either SNIa or BAO
- Reinforces the interpretation of the Hubble tension as a ‘**cosmic calibration tension**’

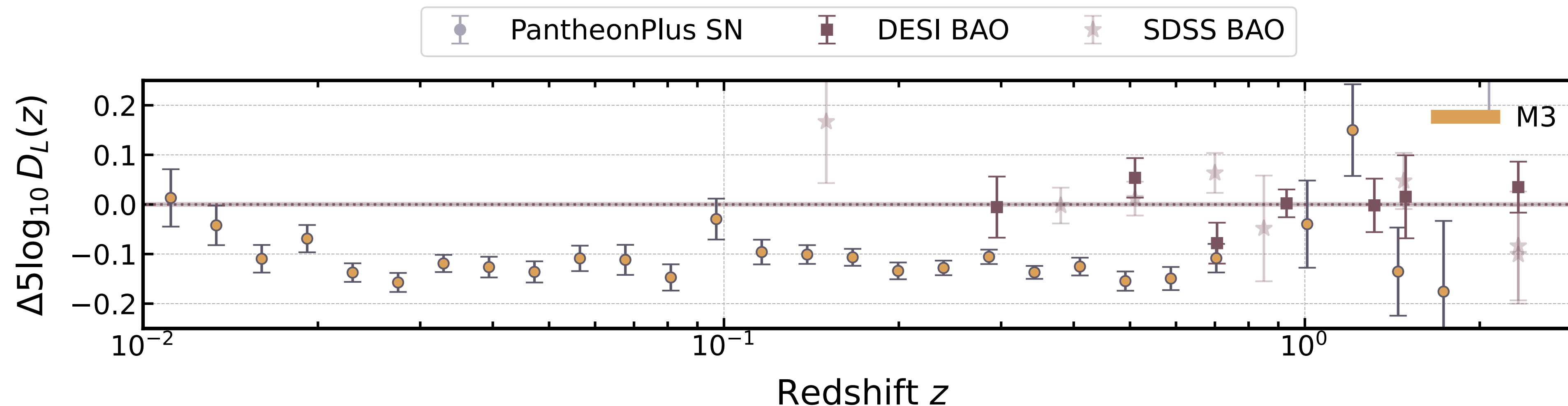


$$T = 0.2\sigma$$



M3 - Exponent DDR

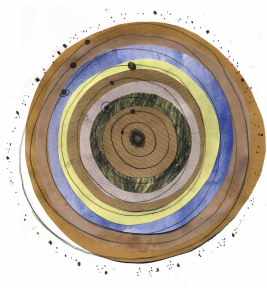
- A power-law provides a natural framework for describing DDR violations that may arise from a **violation of photon number conservation**
- For the case of CMB photons, this effect can be related to **changes in the temperature-redshift relation** $T(z) = T_0(1 + z)$ [Ruchika, W Giarè, EMT, A. Melchiorri.: arXiv: [2505.02909](#)]
- **Cannot resolve the Hubble tension**



$$T = 5.0\sigma$$

2. A DDR deviation or a change in calibration?

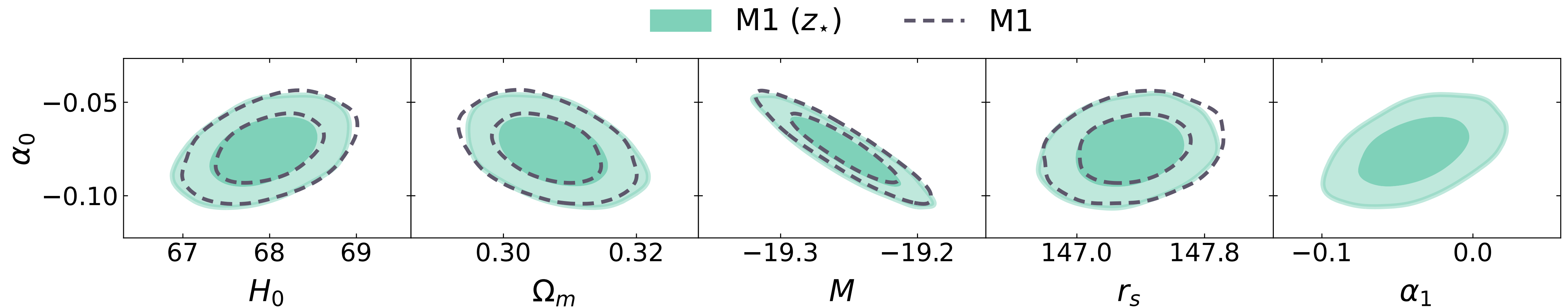
Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: [arxiv:2504.10464](https://arxiv.org/abs/2504.10464)]



$M1(z^*)$ - Double Constant DDR

- No evidence for extra degree of freedom for redshift dependance
- Very similar fit with one extra parameter

$$T = 0.08\sigma$$

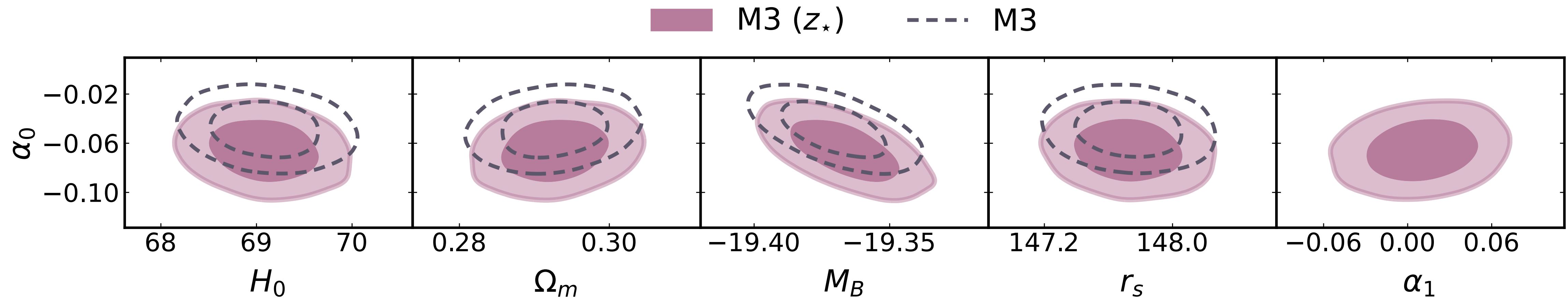


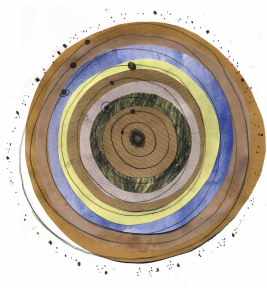


M3(z^*) - Double Exponent DDR

- No evidence for extra degree of freedom for redshift dependance
- Very similar fit with one extra parameter and no tension resolution

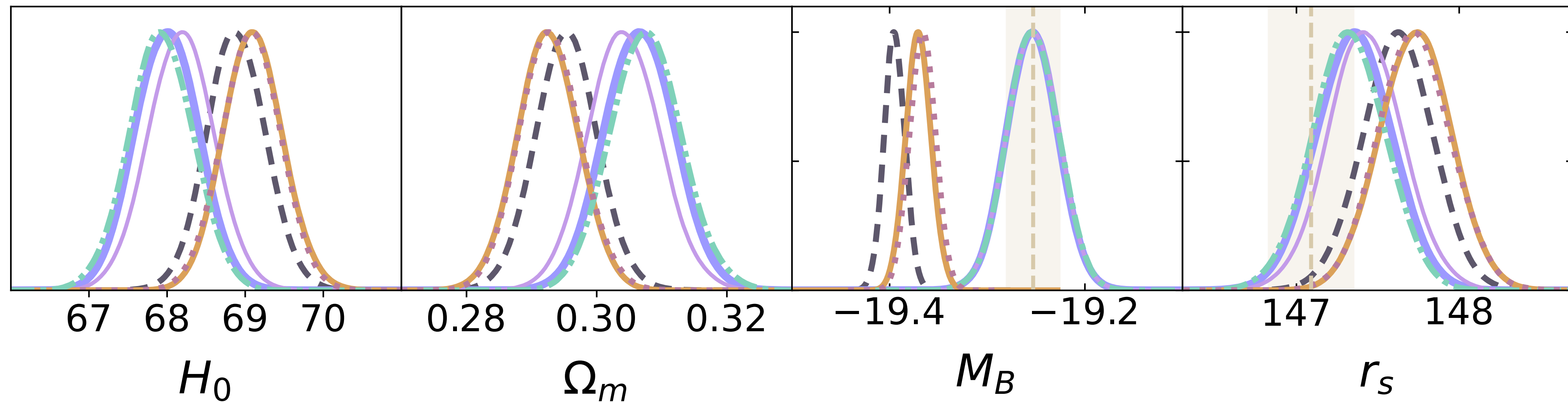
$$T = 4.8\sigma$$





Evidence for constant DDR

--- Λ CDM — M1 — M2 — M3 -.- M1 (z_*) ... M3 (z_*)



3. Degeneracy between DDR and dynamical dark energy

Based on: [E. M. Teixeira, W. Giarè, N. B. Hogg, T. Montandon, A. Poudou, and V. Poulin: [arxiv:2504.10464](https://arxiv.org/abs/2504.10464)]

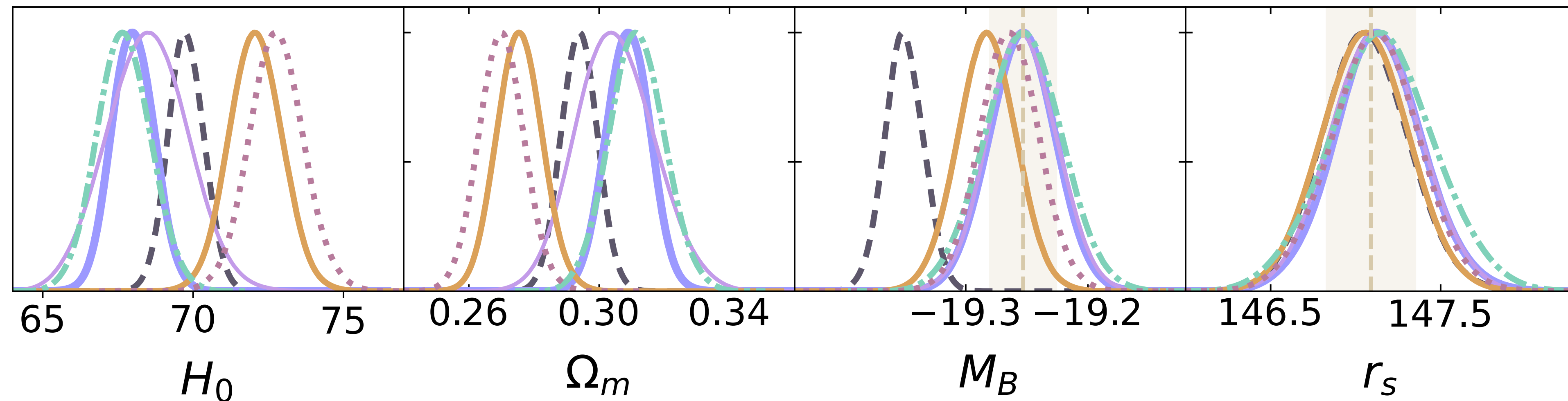


Evidence for dynamical DE

$$\dot{\rho}_{\text{DE}} + 3H\rho_{\text{DE}}(1 + w) = 0, \quad w = w_0 + w_a(1 - a)$$

- The model that performs better is **the double exponential DDR (M3(z*))** with **phantom dark energy** with $w \sim -1.155$ and a deviation in the DDR affecting the data at $z \lesssim 0.9$ with $\alpha_0 \sim -0.134$
- Definite **preference over a constant deviation (M1)** for both a Λ CDM or w_0w_a CDM background

-- w_0w_a CDM — M1 — M2 — M3 -.- M1 (z_*) M3 (z_*)



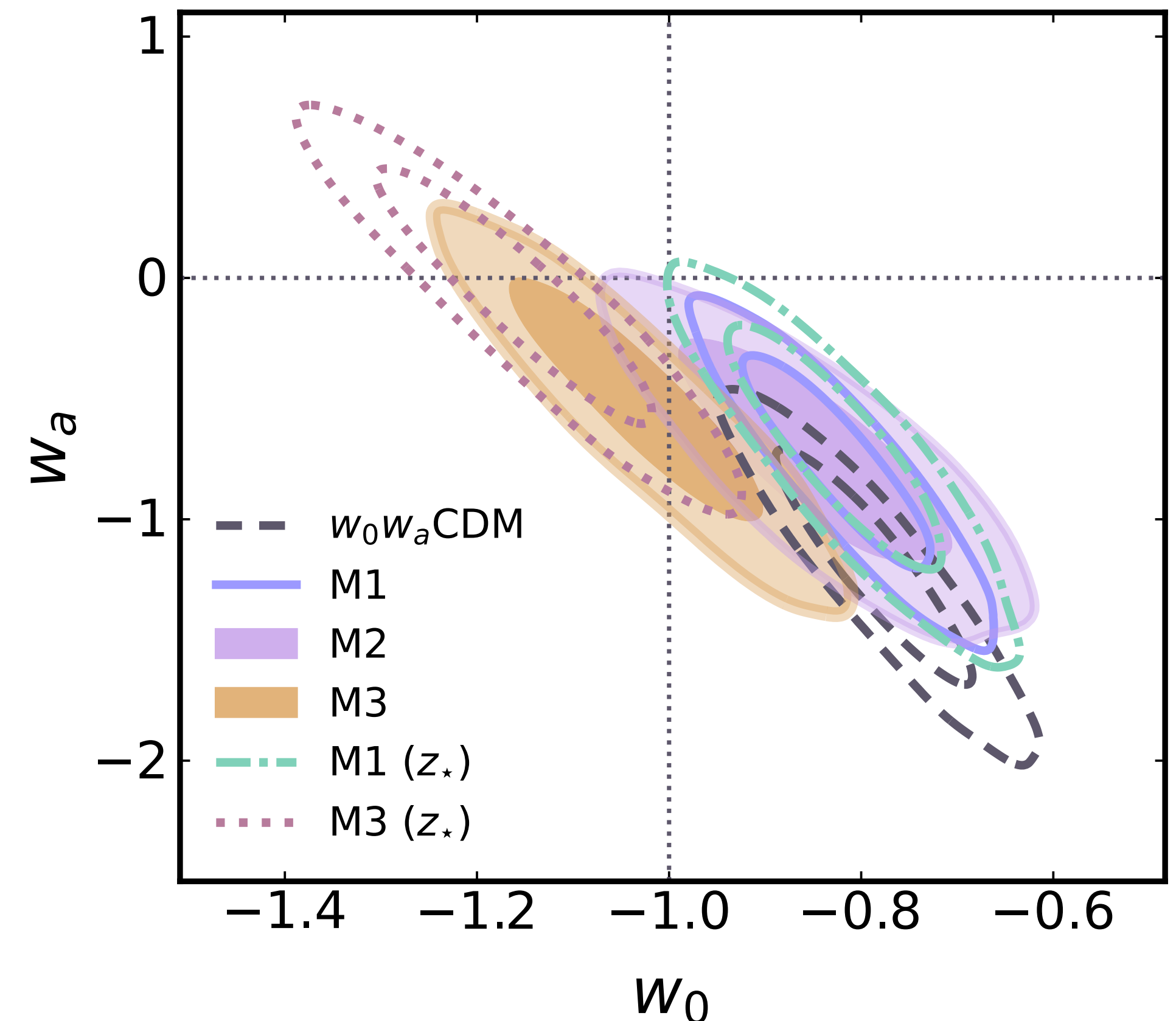


Evidence for dynamical DE

- Degeneracy between a violation of the DDR and a change in the background cosmology that affects the expansion history $H(z)/H_0$

$$\dot{\rho}_{\text{DE}} + 3H\rho_{\text{DE}}(1 + w) = 0, \quad w = w_0 + w_a(1 - a)$$

Planck18 + DESI + SN + SH_0ES



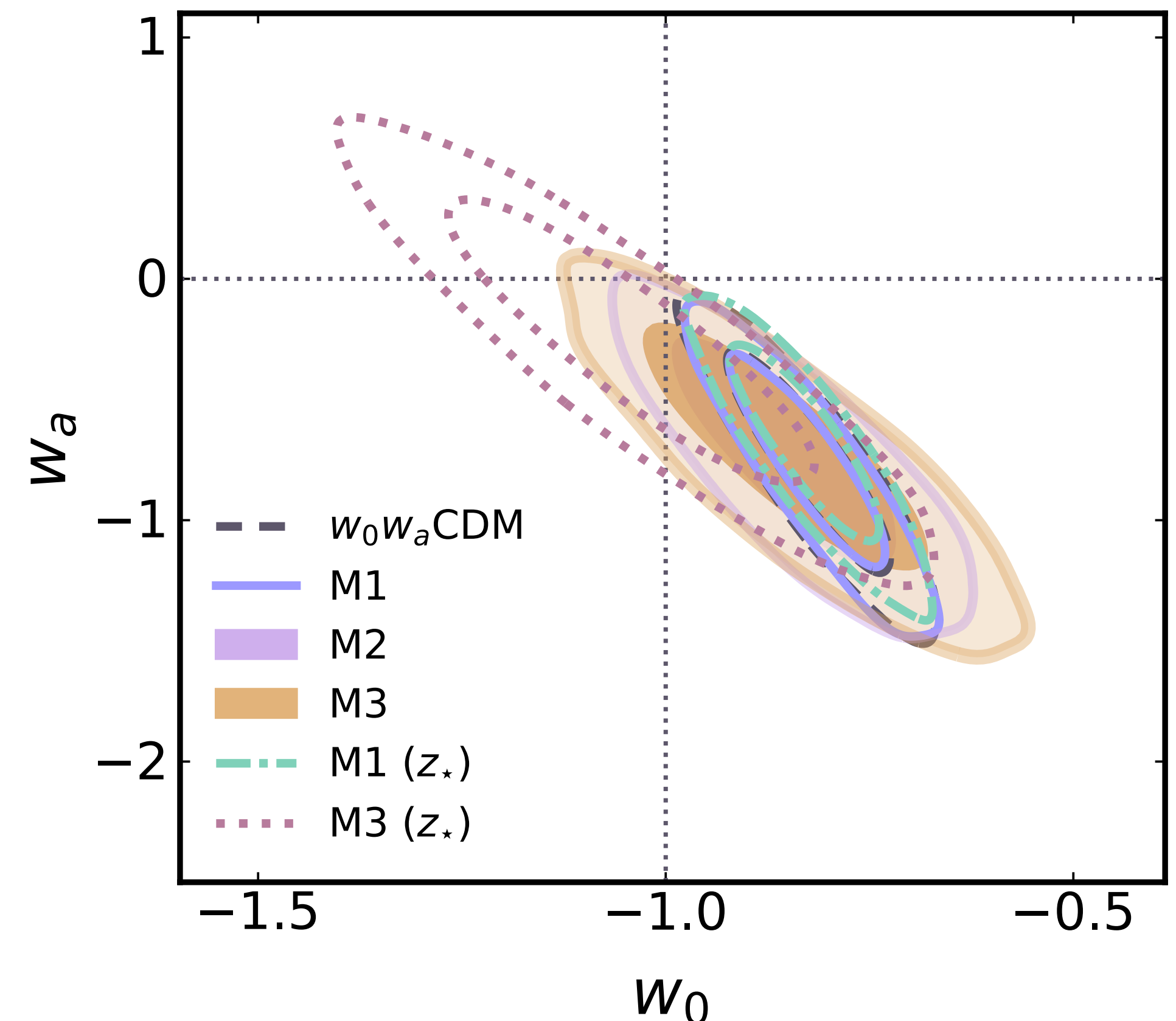


Evidence for dynamical DE

$$\dot{\rho}_{\text{DE}} + 3H\rho_{\text{DE}}(1 + w) = 0, \quad w = w_0 + w_a(1 - a)$$

- Degeneracy between a violation of the DDR and a change in the background cosmology that affects the expansion history $H(z)/H_0$
- In the double exponent DDR (M3(z_*)), the preference for DDE vanishes without SH0ES (Λ CDM at $\sim 1\sigma$) and $H_0 = 72.77 \pm 0.91$ km/s/Mpc
- The preference for DDE can be interpreted as a break in the DDR, with a preference for a deviation occurring at $z < z_*$ (i.e., $\alpha_0 \neq 0, \alpha_1 \sim 0$)
- However: DESI DR2 finds evidence for DDE just from DESI+CMB

Planck18 + DESI + SN



The DDR and the Temperature-Redshift relation

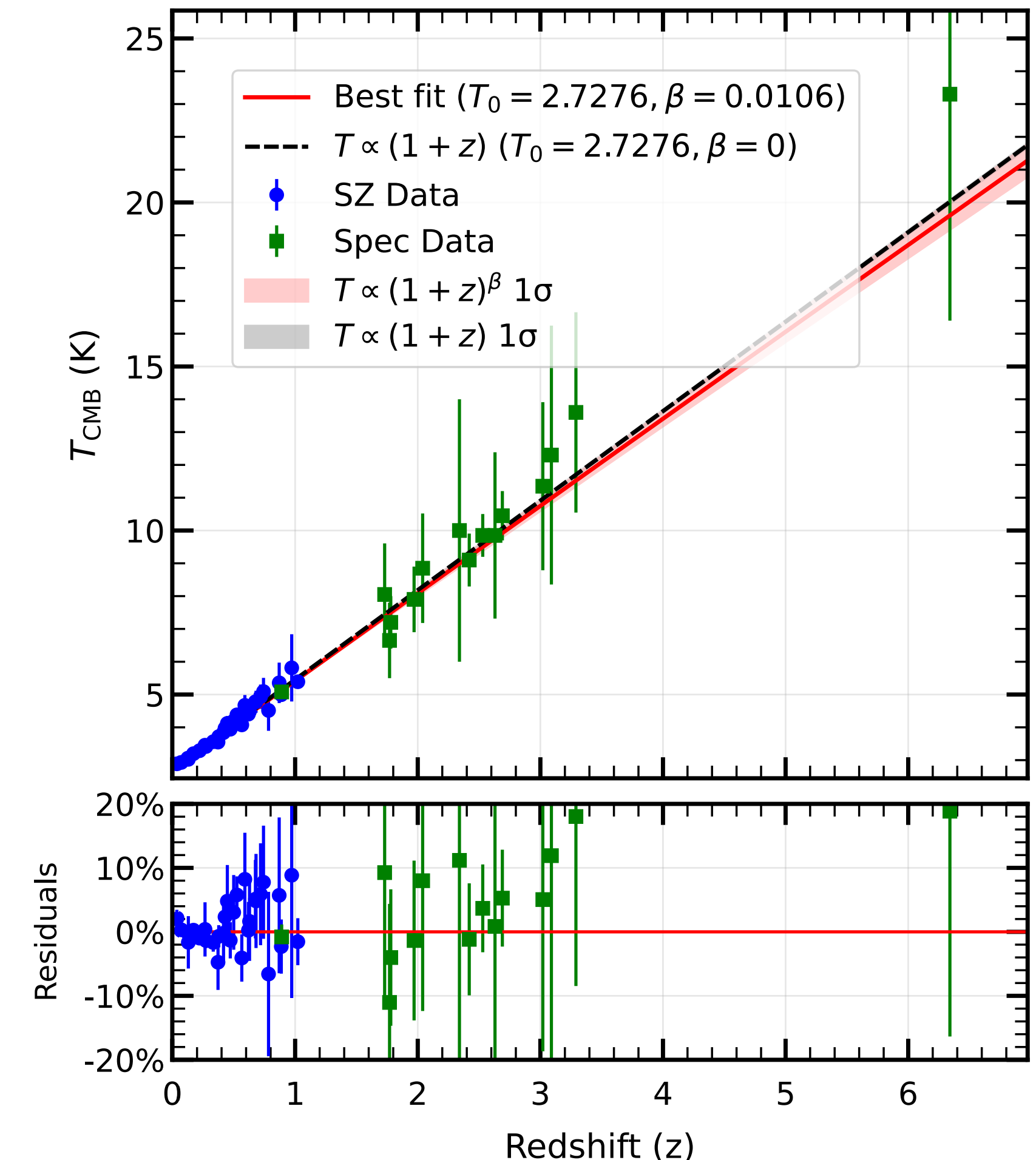
Based on: [Ruchika, W. Giarè, E. M. Teixeira, and A. Melchiorri: [arxiv:2505.02909](https://arxiv.org/abs/2505.02909)]



The DDR and the $T_{\text{CMB}}(z)$ relation

$$T_{\text{CMB}}(z) = T_0(1+z)^{1-\beta} \Rightarrow D_L(z) = D_A(z)(1+z)^{2-\frac{3}{2}\beta}$$

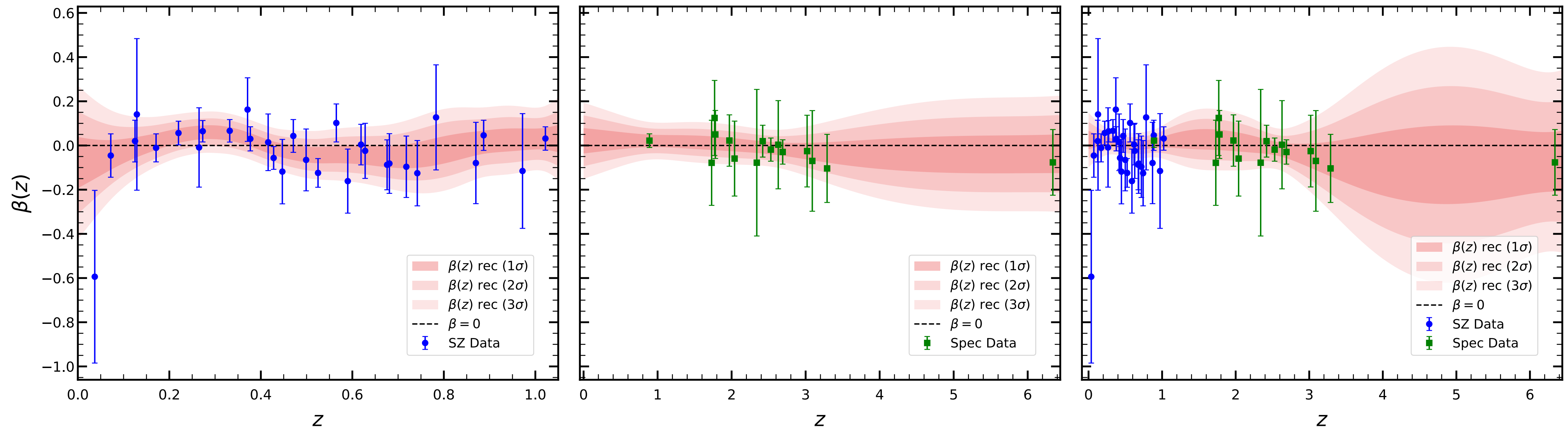
- Violations of the DDR associated with modifications to **redshift evolution of the temperature of CMB photons**
- Gaussian Process reconstruction and χ^2 minimisation of the parameter β using latest SZ effect measurements and molecular line excitation data
- Agreement with $\beta = 0$**
- With $T(z)$ data we find $D_L(z)/D_A(z) \sim (1+z)^{2.0159 \pm 0.0186}$
- In DDR violation we found $D_L(z)/D_A(z) \sim (1+z)^{1.866 \pm 0.021}$
- Hint for **SN/low- z systematics or modified background** (e.g. DDE) expressed as DDR violation

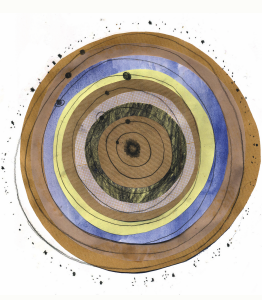




The DDR and the $T_{\text{CMB}}(z)$ relation

$$T_{\text{CMB}}(z) = T_0(1+z)^{1-\beta} \Rightarrow \beta(z) = 1 - \frac{\ln(T_{\text{CMB}}(z)/T_0)}{\ln(1+z)}$$





Conclusions

- The H_0 tension can be recast as a **tension in distances**
- Resolution of the tension with a **preference for a constant shift** in the calibration of the SN and BAO distances
- All parametrisations are **preferred over Λ CDM**, although not all can resolve the tension with SH0ES
- The data currently favours two possibilities:
 1. **Constant violation of the DDR** (equivalent to a calibration shift), $D_L(z)/D_A(z) \simeq 0.925(1+z)^2$
 2. **Change in the power-law redshift-dependence of the DDR**, restricted to $z \lesssim 1$, $D_L(z)/D_A(z) \simeq (1+z)^{1.866}$, together with a **phantom dark energy equation of state** $w \sim -1.155$
- Disentangle DDR-violation models and ‘early-universe’ models with **future independent and precise measurements** of H_0

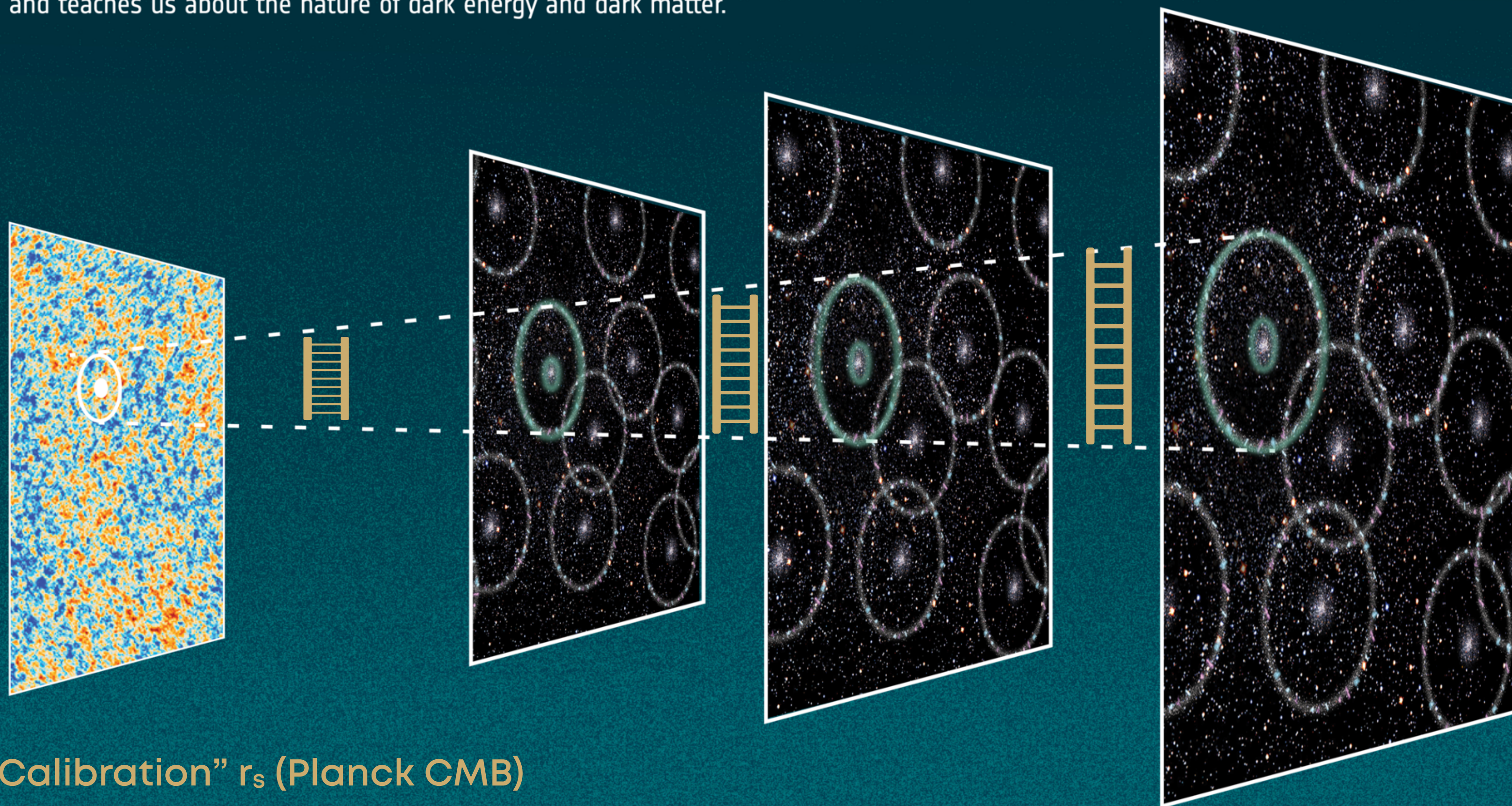


Thank you for your attention!

Illustration Credits: Inês Viegas Oliveira (ivoliveira.com)

WHAT EUCLID WILL MEASURE: BARYONIC ACOUSTIC OSCILLATIONS

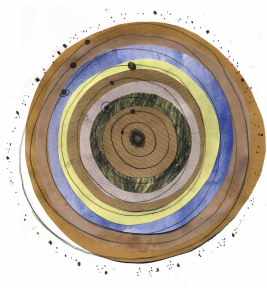
When the early Universe first expanded, the formation of protons and neutrons created sound waves (bubbles) that rippled through the hot particle-radiation soup. About 300 000 years after the Big Bang, when the Universe had cooled down enough for atoms to form and light to travel freely, these waves froze in place. Over time, slightly more galaxies formed in clusters along the frozen ripples. The ripples stretched as the Universe expanded, increasing the distance between galaxies. Euclid will study the distribution of galaxies over immense distances, teasing out these ripple patterns and determining their size. This enables us to measure accurately the accelerated expansion of the Universe and teaches us about the nature of dark energy and dark matter.



Artist's impression of the pattern of baryonic acoustic oscillations imprinted on the large-scale distribution of galaxies (exaggerated)

“Calibration” r_s (Planck CMB)

Distances (DESI BAO)



Distance Duality Relation (DDR)

D_A from BAO

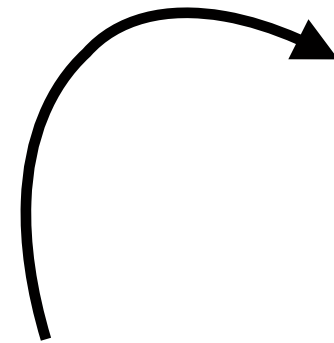
D_L from SN1a

[I. M. H. Etherington (1933)]

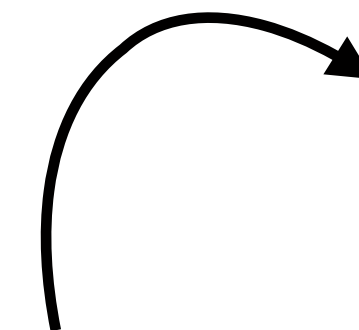
Photons on null geodesics

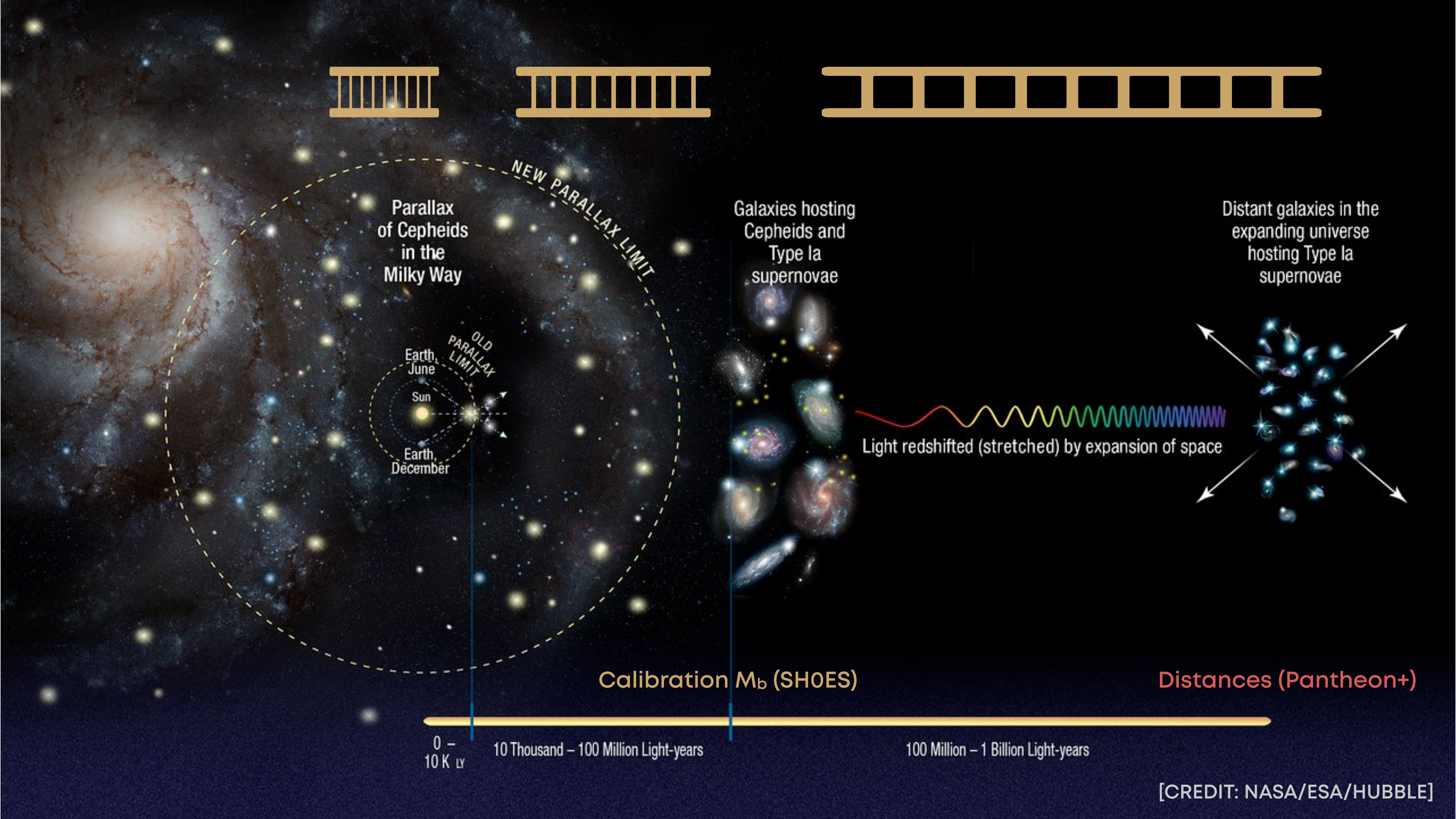


$$\Delta x = a(t)D_M \Delta\theta \text{ with } \Delta\theta \ll 1$$



$$S = 4\pi a_0^2 D_M^2, \quad \frac{L_i}{L_f} = \frac{\Delta E_i}{\Delta E_F} \frac{\Delta t_f}{\Delta t_i} = (1+z)^2$$







Breaking the DDR

Assuming Λ CDM the SH0ES calibration introduces several inconsistencies:

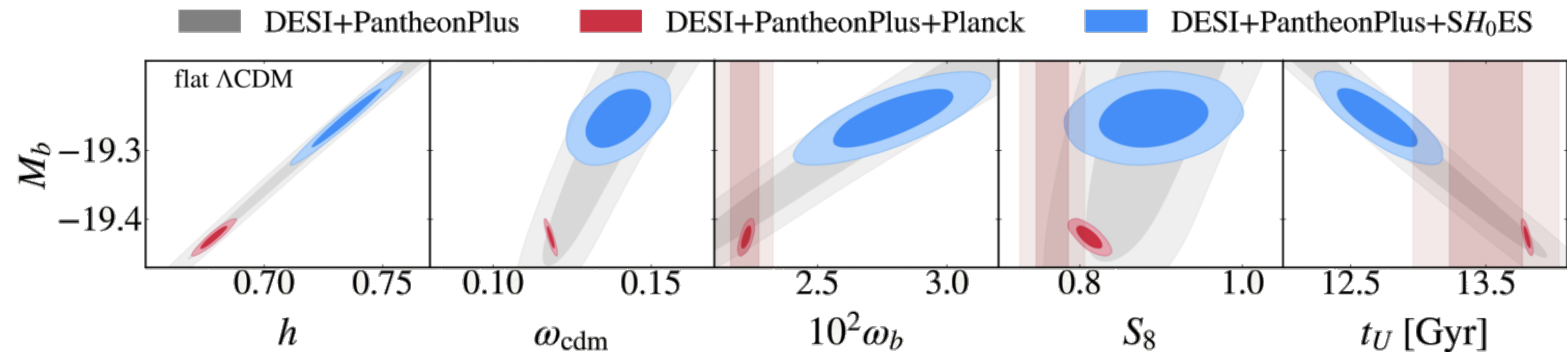
1. H_0 tension: unreconcilable with CMB

2. BBN: larger ω_b to adjust lower r_s

Becomes a challenge to put out all the fires

3. S_8 : increases because of larger w_m

4. Age of the Universe: younger by about 1 Gyr





Evidence for constant DDR

Λ CDM + $\eta(z)$ for <i>Planck</i> 2018 + DESI + PantheonPlus + SH_0 ES prior						
Parameter	Λ CDM	M1	M2	M3	M1 ($z_* = 0.9$)	M3 ($z_* = 0.9$)
α_0	--	-0.075 ± 0.012	-0.070 ± 0.013	-0.049 ± 0.015	-0.076 ± 0.012	-0.066 ± 0.017
α_1	--	--	-0.014 ± 0.010	--	-0.039 ± 0.024	0.010 ± 0.026
M_B	-19.395 ± 0.011	-19.254 ± 0.026	-19.253 ± 0.026	-19.370 ± 0.013	-19.253 ± 0.027	-19.367 ± 0.013
r_s	147.62 ± 0.22	147.35 ± 0.23	147.42 ± 0.23	147.73 ± 0.22	147.32 ± 0.23	147.72 ± 0.22
H_0	68.89 ± 0.38	68.00 ± 0.41	68.18 ± 0.42	69.10 ± 0.38	67.93 ± 0.41	69.08 ± 0.38
Ω_m	0.2953 ± 0.0048	0.3066 ± 0.0054	0.3042 ± 0.0055	0.2925 ± 0.0047	0.3076 ± 0.0055	0.2928 ± 0.0047
$\Delta\chi^2_{\min}$	--	-32.84	-34.73	-10.27	-35.40	-17.25
$\log \mathcal{Z}_M / \mathcal{Z}_{\Lambda\text{CDM}}$	0	13.6	10.9	2.3	12.3	3.3

TABLE II: Observational constraints at a 68% confidence level on the cosmological parameters for a Λ CDM cosmology with different models of DDR violation, inferred from analyses of the combination of *Planck* 2018 data, DESI BAO and PantheonPlus SNIa calibrated with a SH_0 ES prior.



Evidence for constant DDR

Λ CDM + $\eta(z)$ for <i>Planck</i> 2018 + DESI + PantheonPlus						
Parameter	Λ CDM	M1	M2	M3	M1 ($z_* = 0.9$)	M3 ($z_* = 0.9$)
α_0	—	$-0.088^{+0.096}_{-0.11}$	$-0.062^{+0.12}_{-0.090}$	-0.024 ± 0.016	$-0.066^{+0.12}_{-0.086}$	-0.040 ± 0.017
α_1	—	—	-0.014 ± 0.010	—	$-0.027^{+0.12}_{-0.094}$	0.026 ± 0.027
M_B	-19.422 ± 0.012	-19.21 ± 0.23	$-19.26^{+0.19}_{-0.29}$	-19.407 ± 0.015	$-19.27^{+0.19}_{-0.29}$	-19.402 ± 0.015
r_s	147.36 ± 0.23	147.35 ± 0.22	147.43 ± 0.23	147.44 ± 0.23	147.32 ± 0.23	147.44 ± 0.23
H_0	68.00 ± 0.40	68.00 ± 0.39	68.17 ± 0.41	68.20 ± 0.42	67.91 ± 0.41	68.22 ± 0.42
Ω_m	0.3065 ± 0.0054	0.3065 ± 0.0052	0.3043 ± 0.0054	0.3039 ± 0.0055	0.3078 ± 0.0054	0.3036 ± 0.0055
$\Delta\chi^2_{\min}$	—	-0.33	-2.44	-2.51	-3.34	-7.36
$\log \mathcal{Z}_M / \mathcal{Z}_{\Lambda\text{CDM}}$	0	-0.6	-3.1	-2.1	-2	-2.1
GT w/ $SH_0\text{ES } M_B$	5.7σ	0.2σ	0.03σ	5.0σ	0.08σ	4.8σ

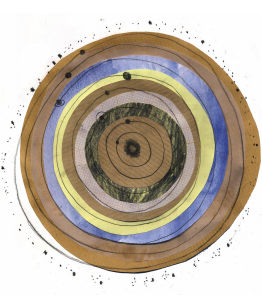
TABLE III: Same as Table II without the $SH_0\text{ES } M_B$ prior. We also report the Gaussian tension (GT) with the $SH_0\text{ES } M_B$ measurement defined in (18).



Evidence for dynamical DE

$w_0 w_a + \eta(z)$ for <i>Planck</i> 2018 + DESI + PantheonPlus + SH_0ES prior						
Parameter	$w_0 w_a$	M1	M2	M3	M1 ($z_* = 0.9$)	M3 ($z_* = 0.9$)
α_0	--	-0.066 ± 0.014	-0.060 ± 0.021	-0.090 ± 0.023	-0.071 ± 0.017	-0.134 ± 0.028 [-0.128 ± 0.021]
α_1	--	--	-0.008 ± 0.021	--	-0.035 ± 0.029	-0.040 ± 0.029 [--]
w_0	-0.784 ± 0.067	-0.820 ± 0.065	-0.846 ± 0.091	-1.029 ± 0.089	-0.821 ± 0.075	-1.155 ± 0.095 [-1.158 ± 0.033]
w_a	$-1.20^{+0.34}_{-0.30}$	$-0.78^{+0.32}_{-0.27}$	$-0.74^{+0.33}_{-0.29}$	$-0.52^{+0.36}_{-0.32}$	$-0.72^{+0.36}_{-0.30}$	$-0.09^{+0.37}_{-0.32}$ [--]
M_B	-19.350 ± 0.016	-19.254 ± 0.027	-19.254 ± 0.028	-19.283 ± 0.024	-19.253 ± 0.032	-19.264 ± 0.024
r_s	147.06 ± 0.25	147.13 ± 0.26	147.12 ± 0.26	147.06 ± 0.26	147.15 ± 0.31	147.11 ± 0.26
H_0	69.77 ± 0.60	68.00 ± 0.71	68.5 ± 1.3	72.08 ± 0.88	67.70 ± 0.89	72.77 ± 0.91
Ω_m	0.2939 ± 0.0055	0.3089 ± 0.0067	$0.305^{+0.011}_{-0.013}$	0.2756 ± 0.0070	0.3116 ± 0.0085	0.2700 ± 0.0070
$\Delta\chi^2_{\min}$	-20.37	-40.89	-40.16	-34.82	-42.75	-43.20 [-41.86]
$\log \mathcal{Z}_M / \mathcal{Z}_{\Lambda\text{CDM}}$	5.6	12.7	10.4	10.5	11.5	12.4 [15.4]

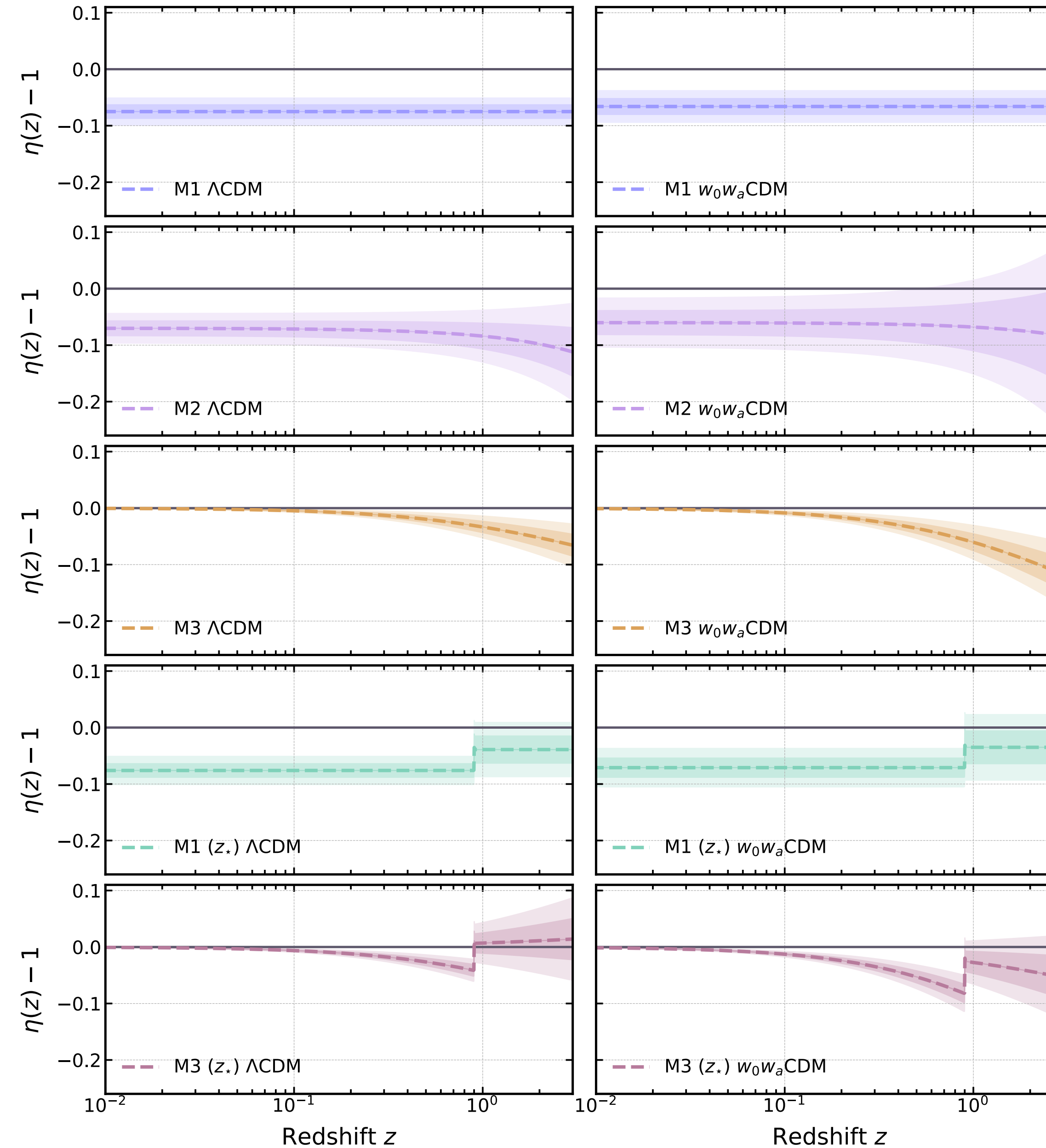
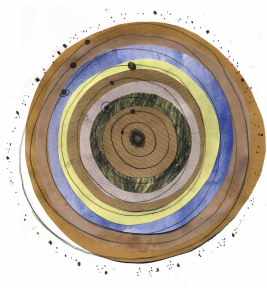
TABLE IV: Same as Table II, now in the $w_0 w_a$ CDM cosmology and with the SH_0ES M_B prior. In square brackets, we list the values for M3 ($z_* = 0.9$) with $\alpha_1 = w_a = 0$ (the corresponding cosmological parameters are consistent with the full case).

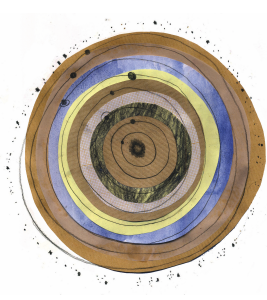


Evidence for dynamical DE

$w_0 w_a + \eta(z)$ for <i>Planck</i> 2018 + DESI + PantheonPlus						
Parameter	$w_0 w_a$	M1	M2	M3	M1 ($z_* = 0.9$)	M3 ($z_* = 0.9$)
α_0	--	-0.072 ± 0.096	$-0.057^{+0.12}_{-0.096}$	-0.012 ± 0.040	$-0.065^{+0.12}_{-0.093}$	-0.085 ± 0.054
α_1	--	--	-0.008 ± 0.021	--	$-0.03^{+0.12}_{-0.10}$	-0.011 ± 0.040
w_0	-0.822 ± 0.065	-0.821 ± 0.063	-0.846 ± 0.092	-0.85 ± 0.12	-0.825 ± 0.061	-1.04 ± 0.15
w_a	-0.77 ± 0.30	$-0.77^{+0.31}_{-0.27}$	$-0.74^{+0.33}_{-0.29}$	$-0.72^{+0.35}_{-0.32}$	$-0.69^{+0.29}_{-0.25}$	-0.28 ± 0.39
M_B	-19.404 ± 0.020	$-19.23^{+0.21}_{-0.27}$	$-19.25^{+0.20}_{-0.29}$	-19.390 ± 0.049	$-19.26^{+0.20}_{-0.29}$	-19.323 ± 0.060
r_s	147.13 ± 0.26	147.13 ± 0.26	147.12 ± 0.26	147.12 ± 0.26	147.16 ± 0.26	147.13 ± 0.26
H_0	67.99 ± 0.71	68.00 ± 0.70	68.4 ± 1.3	68.5 ± 1.7	67.65 ± 0.74	70.8 ± 2.1
Ω_m	0.3091 ± 0.0068	0.3090 ± 0.0067	$0.305^{+0.011}_{-0.013}$	0.305 ± 0.015	0.3118 ± 0.0071	0.286 ± 0.017
$\Delta\chi^2_{\min}$	-8.05	-7.95	-6.66	-8.16	-10.50	-11.06
$\log \mathcal{Z}_M / \mathcal{Z}_{\Lambda\text{CDM}}$	-1.2	-1.4	-6.1	-2.7	-3	-2.8
GT w/ $SH_0\text{ES } M_B$	4.4σ	0.08σ	0.01σ	2.4σ	0.03σ	1.1σ

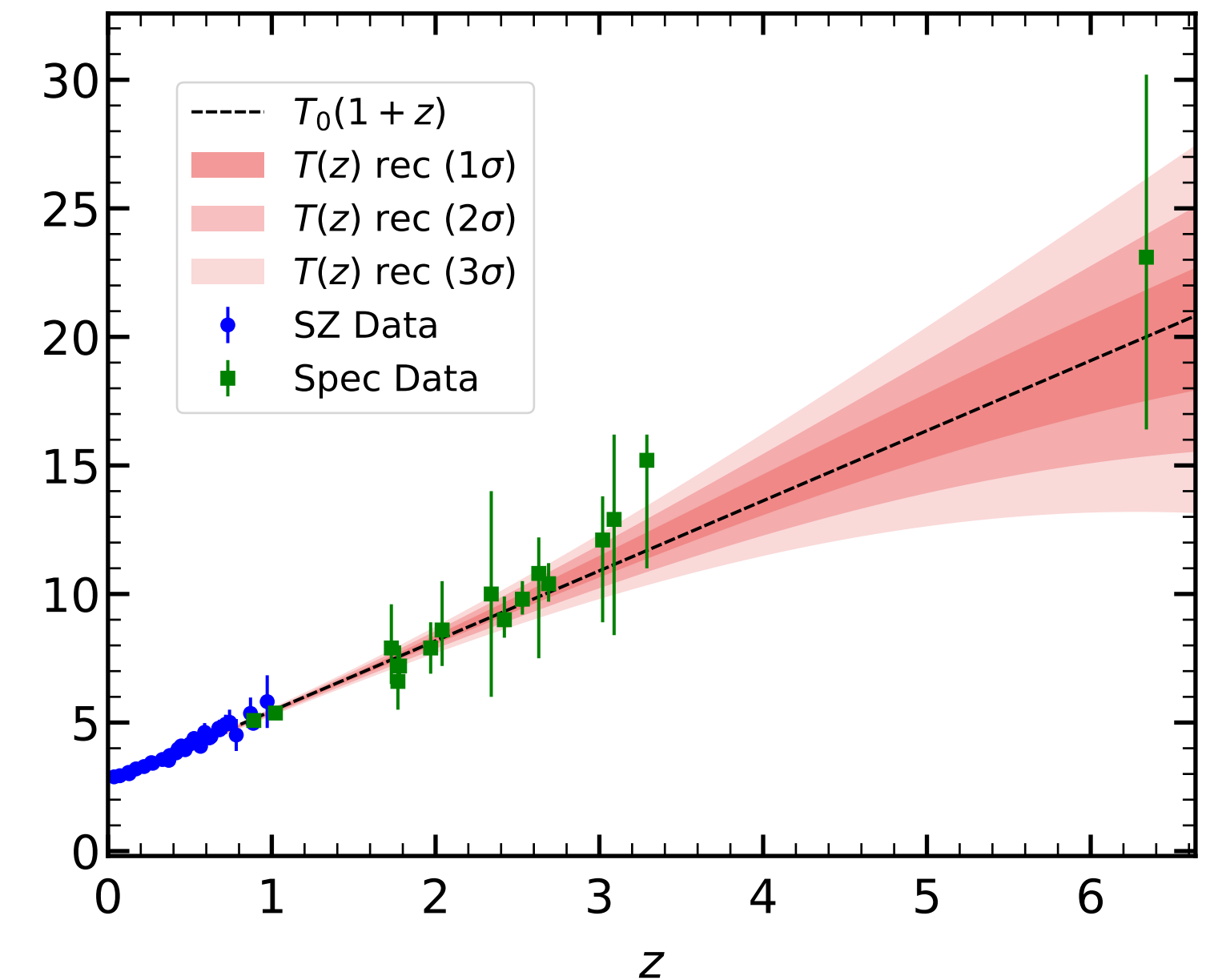
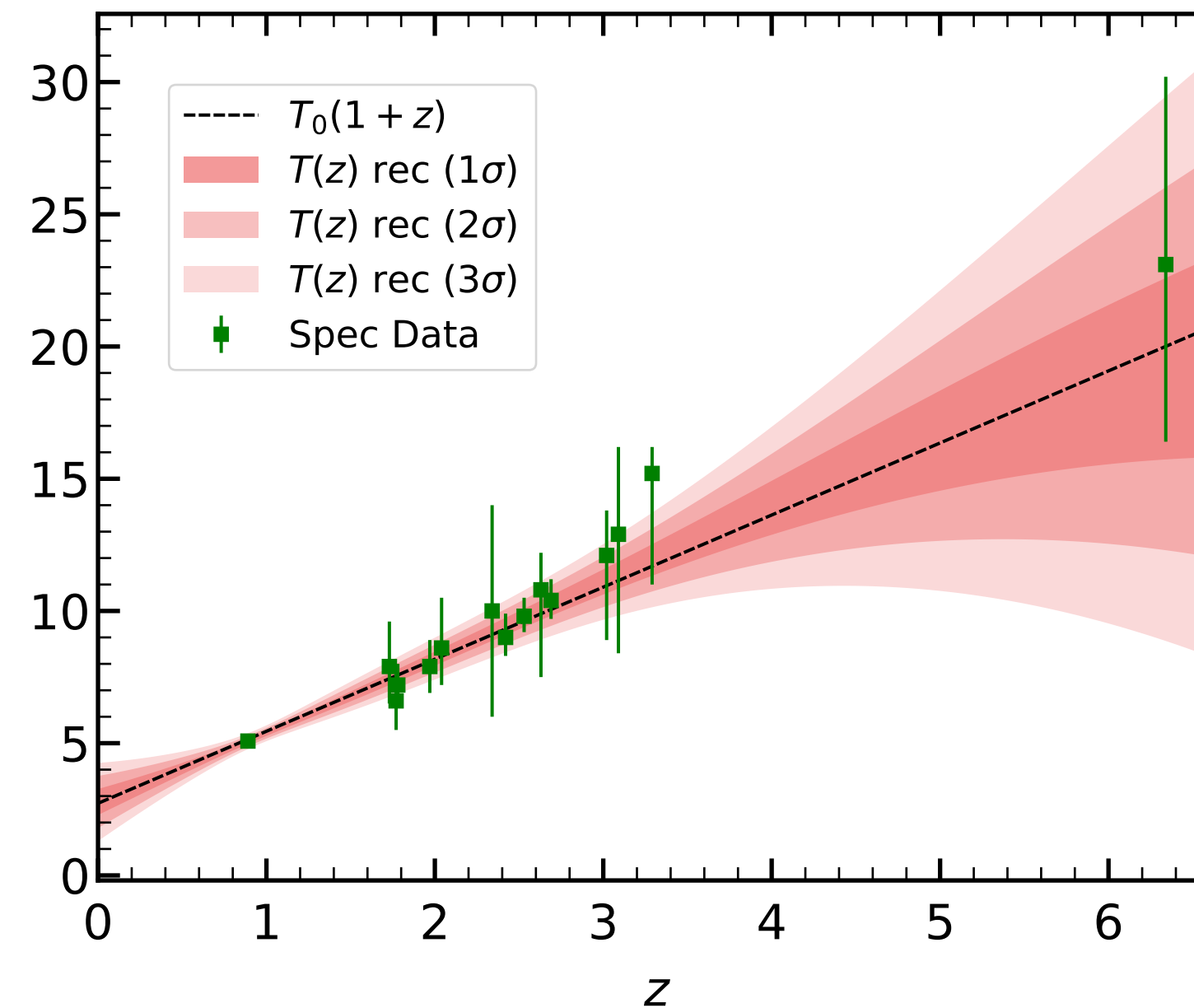
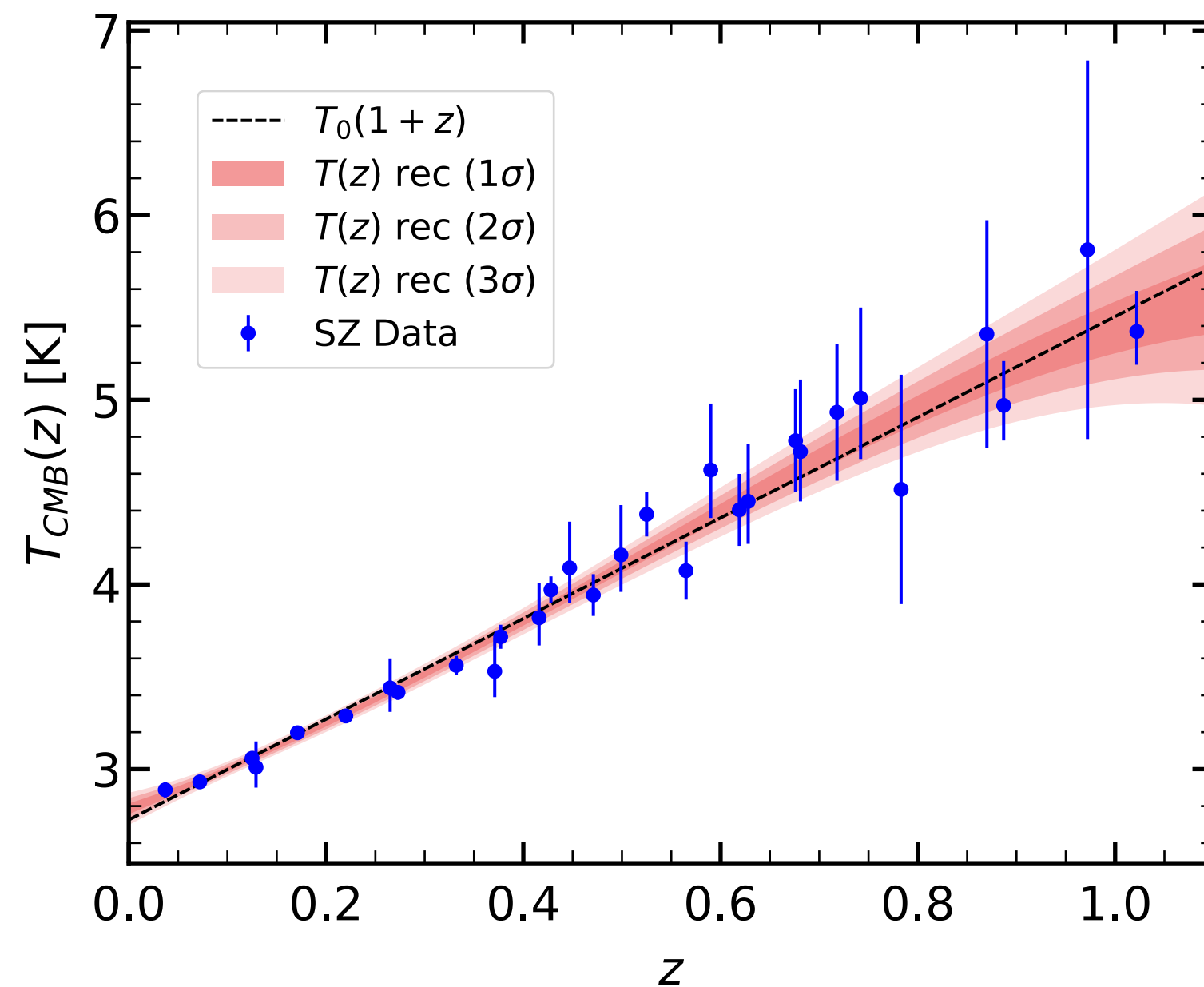
TABLE V: Same as Table IV without the $SH_0\text{ES } M_B$ prior. We also report the Gaussian tension (GT) with the $SH_0\text{ES } M_B$ measurement defined in (18).

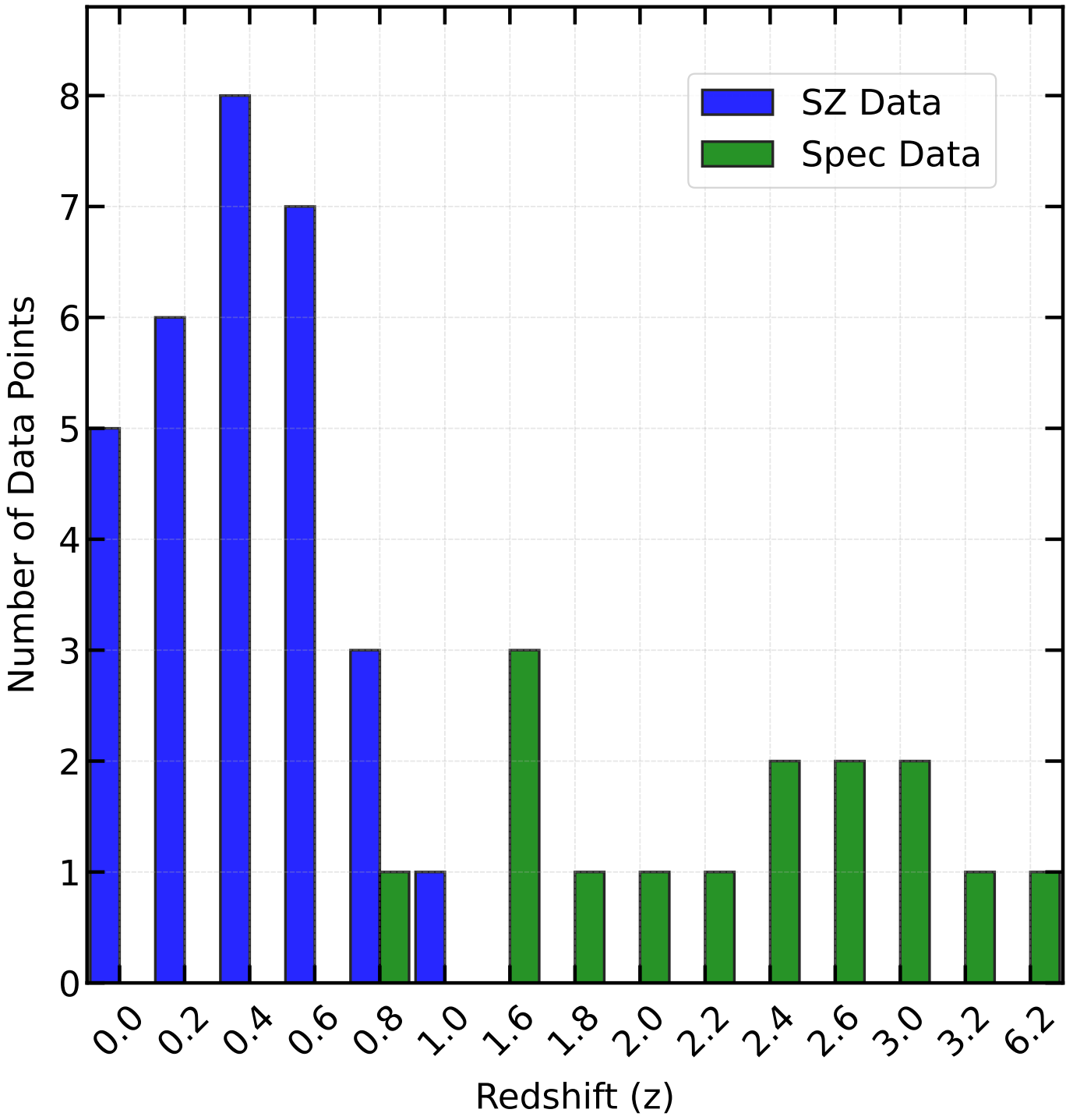
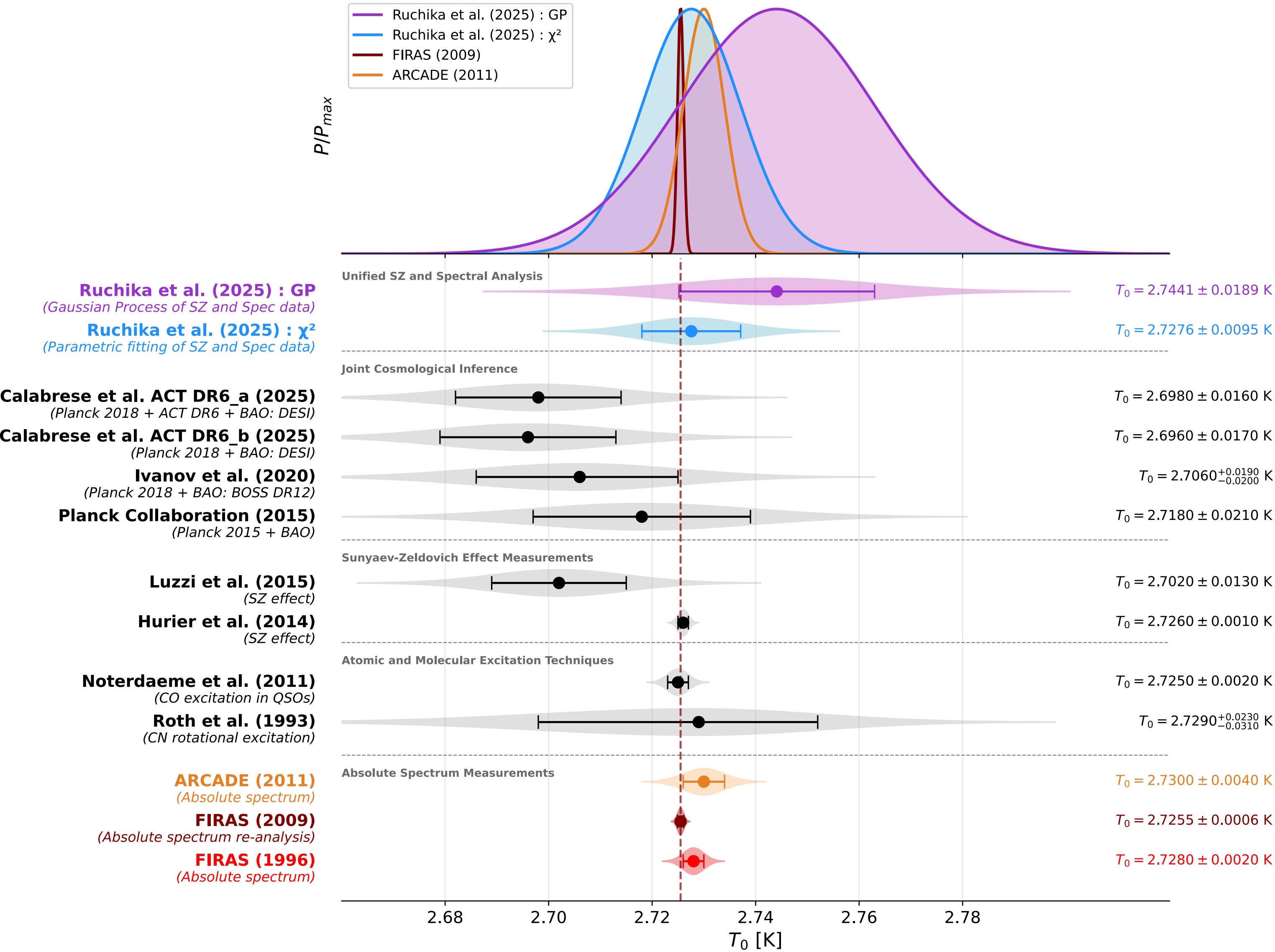
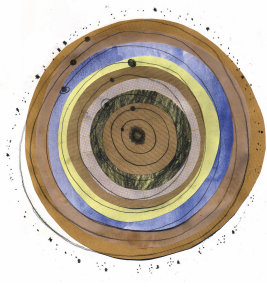




The DDR and the $T_{\text{CMB}}(z)$ relation

$$T_{\text{CMB}}(z) = T_0(1 + z) \Rightarrow T_0 = 2.744 \pm 0.019 \text{ K}$$





[Ruchika et al.: arXiv: [2505.02909](https://arxiv.org/abs/2505.02909)]