Quantifying BAO tension and its implications for late-time solutions to the Hubble crisis

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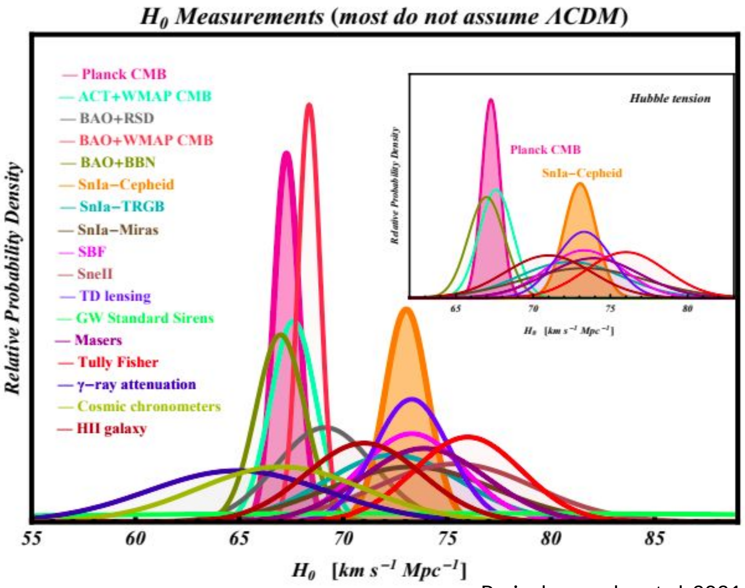
Tensions in the standard Cosmology

With the advent of high-precision cosmology, mismatches (>2σ) between the best-fit ΛCDM model preferred by *Planck* and constraints on cosmological parameters derived from different data sets have arisen in the last decade



Perivolaropoulos et al. 2021

Measure H_0 model-independently

Local distance ladder: standard candles such as SNIa

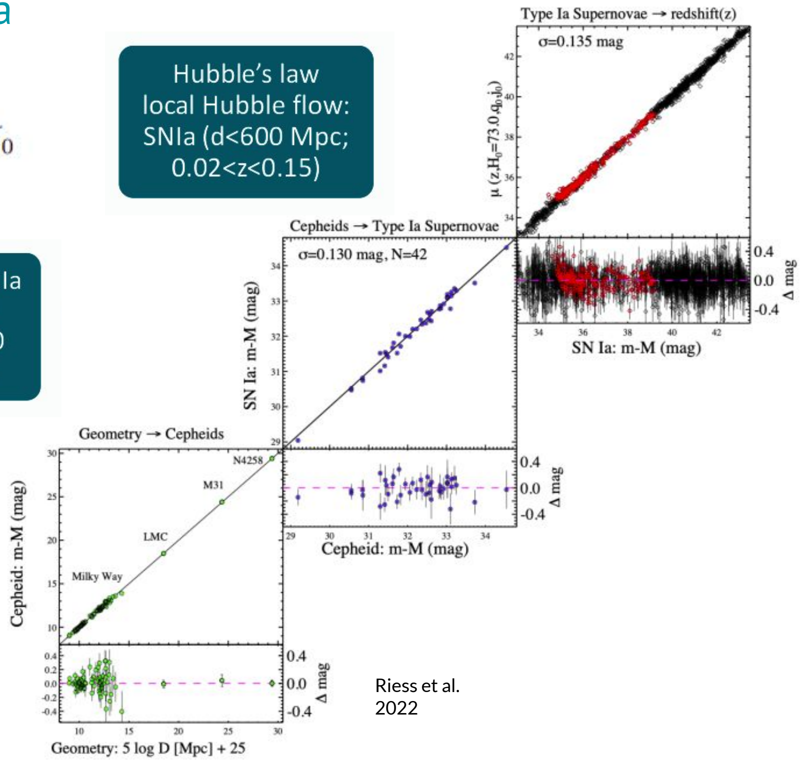
$$\log D_L(z) \approx \log z \left[1 + \frac{1}{2}(1-q_0)z - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0)z^2\right] - \log H_0$$

$$M = m(z) - 25 - 5\log\left(\frac{D_L(z)}{1\ Mpc}\right)$$

Hubble's law
local Hubble flow:
SNIa (d<600 Mpc;
0.02<z<0.15)

Calibration of SNIa
Host galaxies:
Cepheids (d<40
Mpc; z<0.01)

Calibration of
Cepheids
Anchors: Milky way
(30 kpc) parallaxes



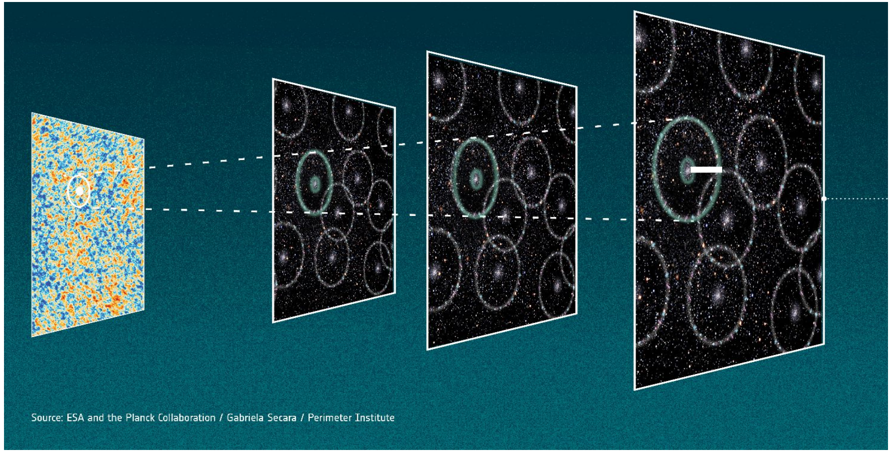
Riess et al.
2022

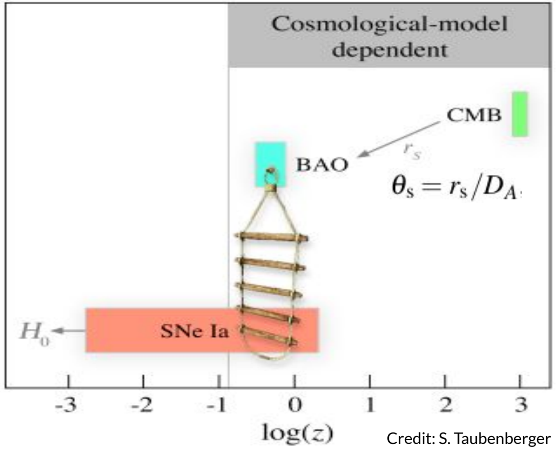
Measure H_0 model-dependently

Inverse distance ladder (Aubourg et al. 2015, Cuesta et al. 2015)
The sound horizon at the baryon-drag epoch is used as a standard ruler to calibrate cosmic distances

$$r_d H(z), \quad D_V(z)/r_d, \quad D_M(z)/r_d$$

$$r_s = \int_{z_{ls}}^{\infty} \frac{c_s(z)\,dz}{H(z)} = \frac{c}{\sqrt{3}H_{ls}} \int_{z_{ls}}^{\infty} \frac{dz}{[\rho(z)/\rho(z_{ls})]^{1/2}(1+R)^{1/2}}$$

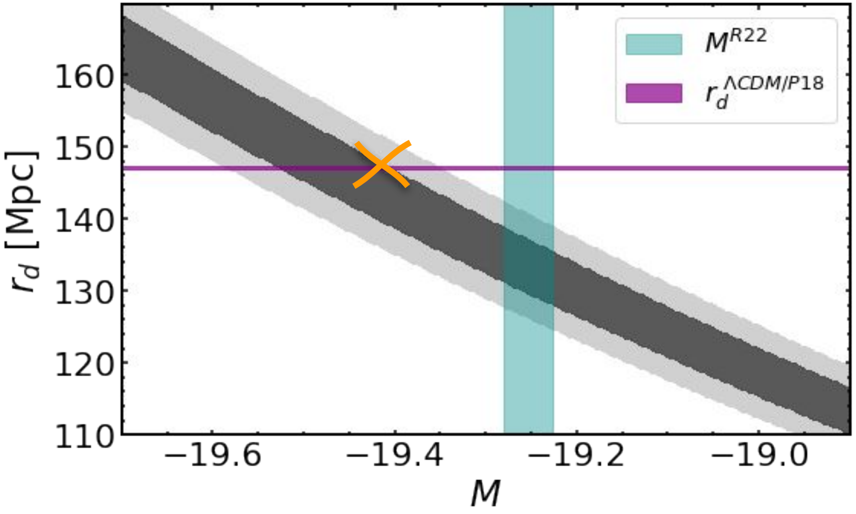




Credit: S. Taubenberger

The calibrators plane

The Hubble tension can be translated in a tension in the calibrators *M* and/or *rd*



$$M^{R22} = (-19.253 \pm 0.027)\ \mathrm{mag}$$
$$r_d^{P18} = (147.09 \pm 0.26)\ \mathrm{Mpc}$$
$$H_0^{R22} = (73.04 \pm 1.04)\ \mathrm{km/s/Mpc}$$

Late-time phenomenology to solve the H_0 tension

*Gómez-Valent, **AF**, Migliaccio, A. Sen, Phys.Rev.D 109 (2024) 2, 023525* [arXiv:2309.07795]

By considering standard pre-recombination physics, one can define a flexible parametrization of *H(z)* (in a FLRW universe)

$$H(z) = \begin{cases} \bar{H}(z) + \delta H_1(z) & \text{if } 0 < z \leq z_p \\ \bar{H}(z) + \delta H_2(z) & \text{if } z_p < z < z_{max} \\ H_\Lambda(z) & \text{if } z \geq z_{max} \end{cases}$$

$$\bar{H}(z) \equiv \bar{H}_0\sqrt{1 + \bar{\Omega}_m[(1+z)^3 - 1]},$$

$$H_\Lambda(z) \equiv \tilde{H}_0\sqrt{1 + \tilde{\Omega}_m[(1+z)^3 - 1] + \tilde{\Omega}_r[(1+z)^4 - 1]}$$

$$\delta H_1(z) \equiv a + bz + cz^2; \qquad \delta H_2(z) \equiv d + ez + fz^2.$$

$$\{\tilde{\Omega}_m, \tilde{H}_0, H_0, z_p, \delta H_p\}$$

and fit it to a combination of the following data:

BAO + (CMB+SH0ES priors)

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and fit it to a combination of the following data:

BAO + (CMB+SH0ES priors) + SNIa ⇨ *M(z)* ⇔ *Gaussian Processes (GP)*

Late-time phenomenology to solve the H_0 tension

Answer the following questions:

➔ Is the inferred absolute magnitude of SNIa compatible with the constant value measured by SH0ES or do low-*z* solutions to the H0 tension require an evolution of it?

➔ Does the background expansion history required to solve the H0 tension depend on the BAO data set employed?

Late-time phenomenology to solve the H_0 tension

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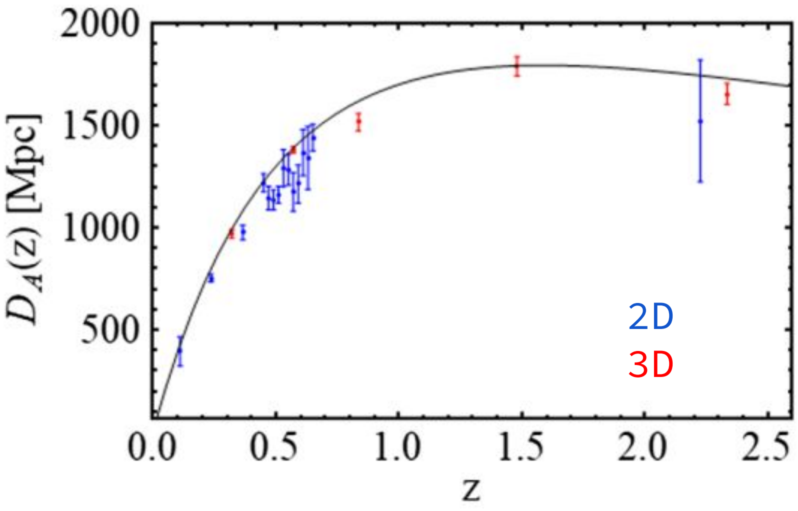
BAO: 2D vs 3D measurements

3D (anisotropic) analysis needs to employ a fiducial cosmology to convert the measured redshifts and angles into comoving distances to build the 3D tracer map (e.g. Carvalho et al. 2016)

2D (angular) analysis is performed in the angular space since it relies on the measurement of the angular BAO peak position (Sánchez et al. 2011).

➔ Weakly dependent on a cosmological model (Sánchez et al 2011, de Carvalho et al. 2021)



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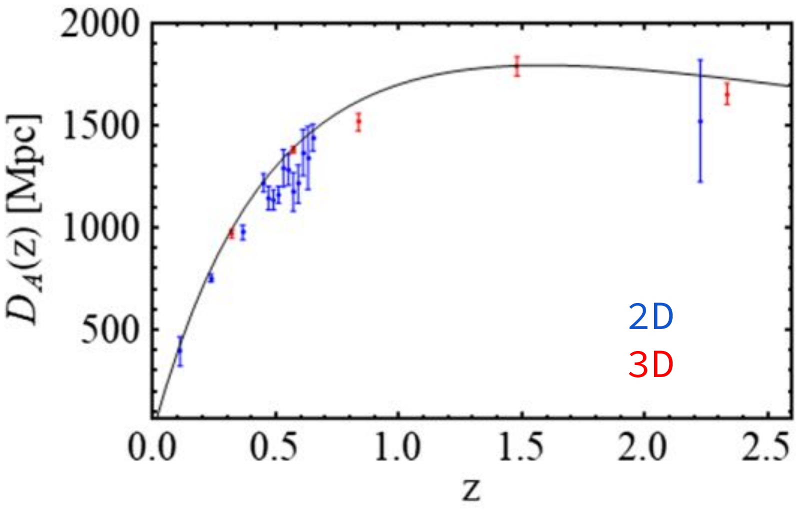
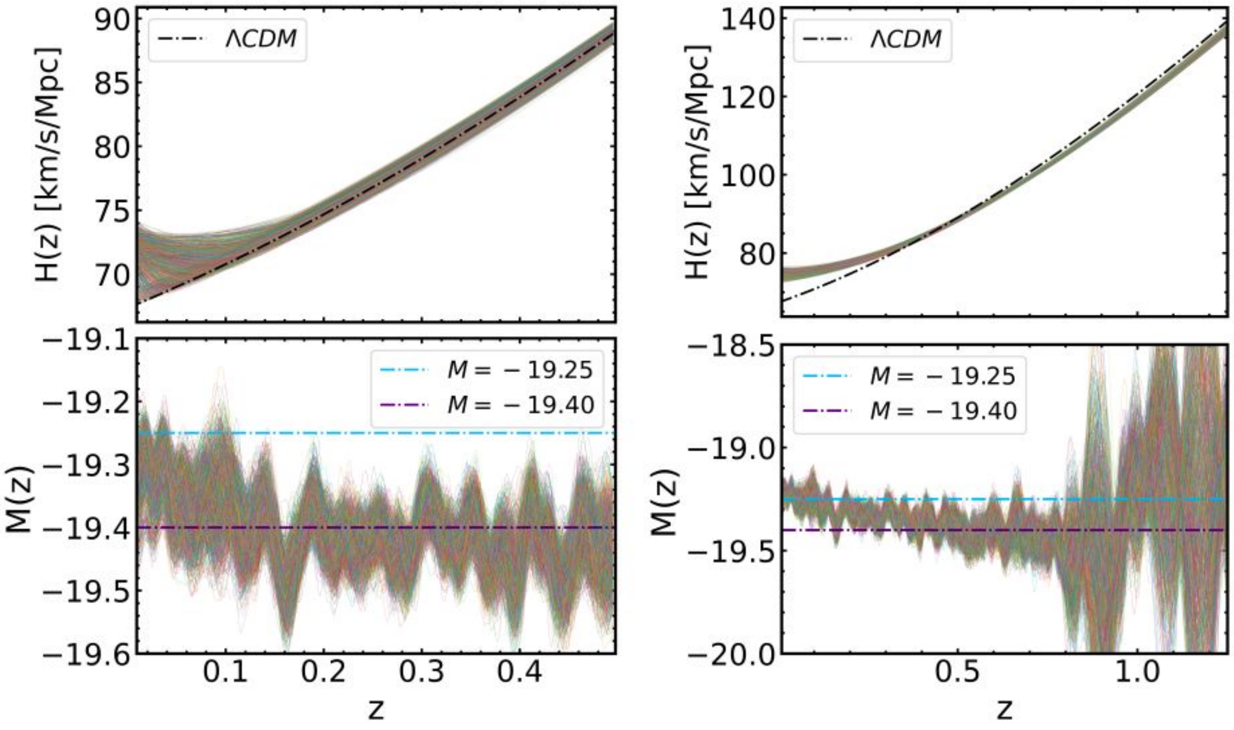


Underestimation of the 3D BAO uncertainties by a factor 2 (Anselmi et al. 2019)

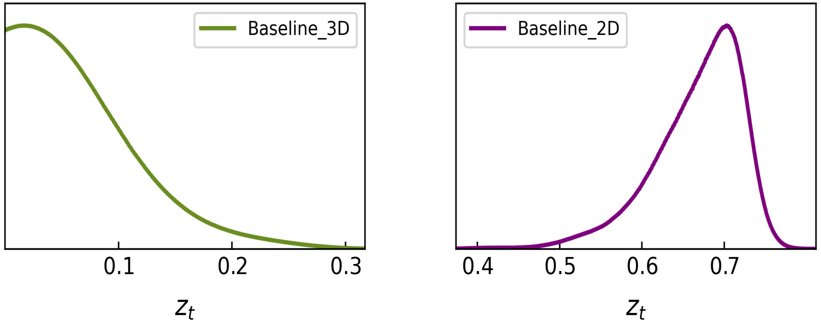
Impact of the BAO data sets

3D BAO **2D BAO**



Impact of the BAO data sets

The two datasets leave an imprint on completely different redshift ranges



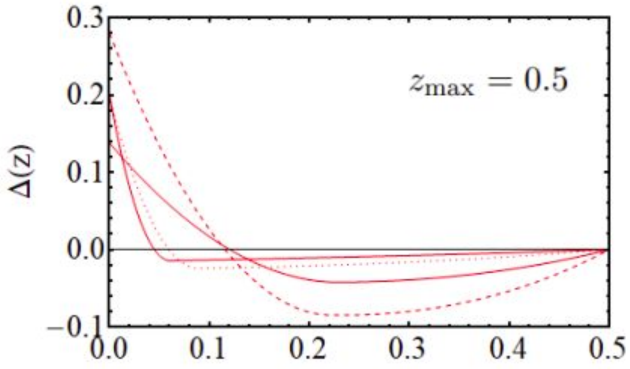
What happens if we interpret these results in terms of an effective self-conserved dark energy (DE) fluid?

Impact of the BAO data sets

$$\Delta(z) \equiv \frac{\rho_{\rm de}(z) - \tilde{\rho}_\Lambda}{\tilde{\rho}_\Lambda} = \frac{H^2(z) - H^2_\Lambda(z)}{H^2_\Lambda(z) - \tilde{\Omega}_m \tilde{H}_0^2 (1+z)^3}$$

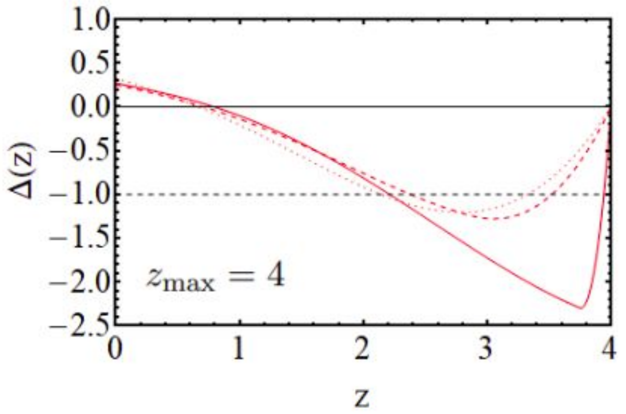
3D BAO: fast phantom evolution of DE at $z<0.2$

e.g. *M* phantom transition (Alestas et al. 2021); crossing of the phantom divide (Heisenberg et al. 2023)



2D BAO: effective DE density must be negative at $z>2$ and DE fraction values larger than in ΛCDM at $z<1$

e.g. ΛsCDM (Akarsu et al. 2024); *w*XCDM model (Gómez-Valent & Solà Peracaula 2024)



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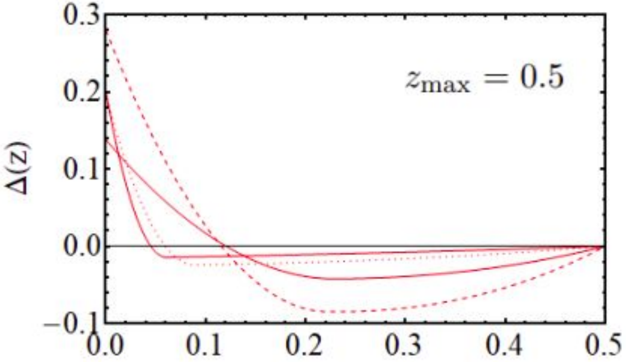

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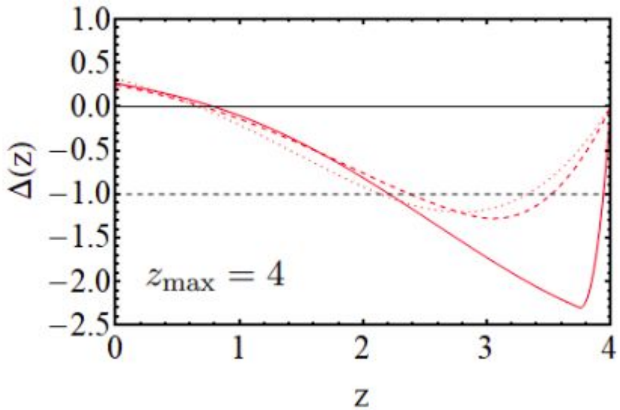
Tension between 2D and 3D BAO?

(Camarena & Marra 2020)



2D & 3D BAO: quantification of the tension

AF, *Gómez-Valent, Migliaccio, Phys.Lett.B 858 (2024) 139027* [arXiv:2405.12142]

Model- and calibrator- independent method to quantify the tension between state-of-art data on 2D and 3D BAO by making use of SNIa data from Pantheon+ and DESY5

$$D_L(z) = (1+z)^2 D_A(z)$$

Etherington or *cosmic distance duality relation (CDDR)* (see Elsa's talk)

$$\eta(z) \equiv \frac{D_L(z)}{(1+z)^2 D_A(z)} = 1$$

$$D_L(z) = 10^{m(z)-M-25}$$

$$D_M(z) = (1+z)D_A(z)$$

$$\theta(z) = \frac{r_d}{D_M(z)}$$

GP reconstruction (SNIa)

$$\eta(z) = \frac{10^{m(z)/5}\theta(z)}{(1+z)} \frac{10^{-5-M/5}\mathrm{Mpc}}{r_d}$$

$\eta(z) = 1$ ⟹

$$\bar{r}_d 10^{M/5} = \frac{10^{m(z)/5}\theta(z)}{10^5(1+z)}$$

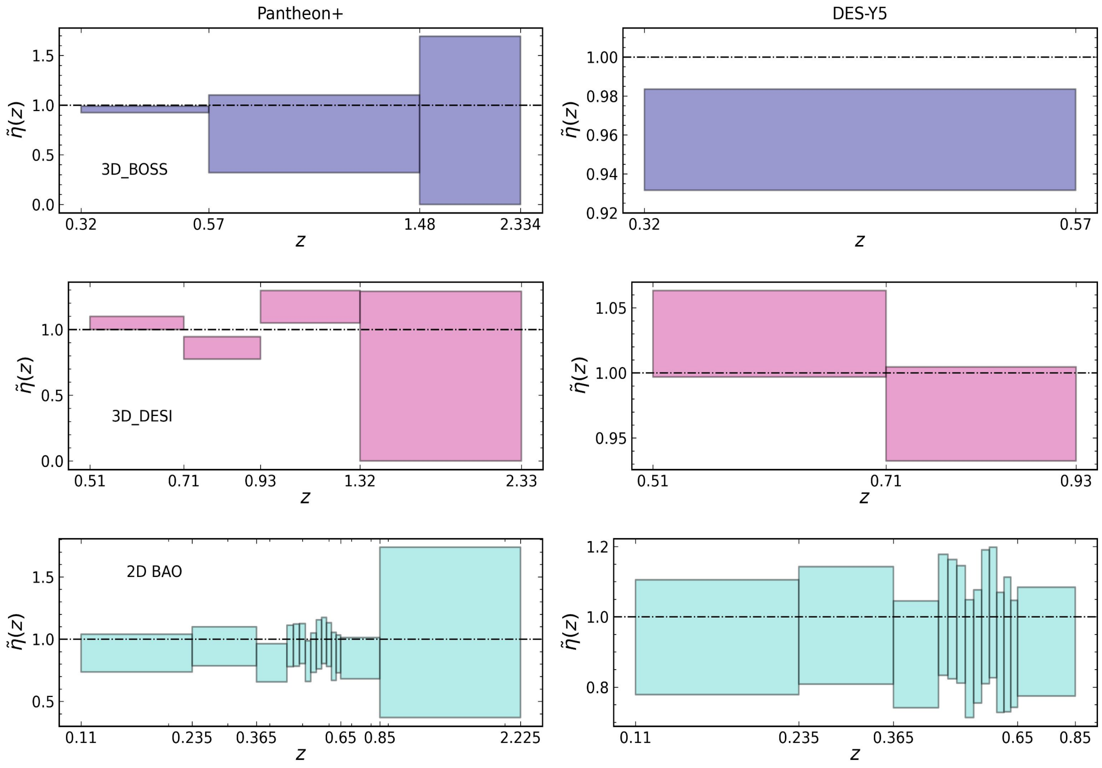
2D & 3D BAO: test of the CDDR

The validity of the CDDR is taken for granted in the quantification of the BAO tension

➔ cross-check any hint for a departure from it with both 2D and 3D BAO

$$\eta_{i,j} \equiv \frac{\eta(z_i)}{\eta(z_j)} = \frac{10^{m(z_i)/5}\theta(z_i)}{10^{m(z_j)/5}\theta(z_j)}\left(\frac{1+z_j}{1+z_i}\right)$$

(Tonghua et al., 2023)



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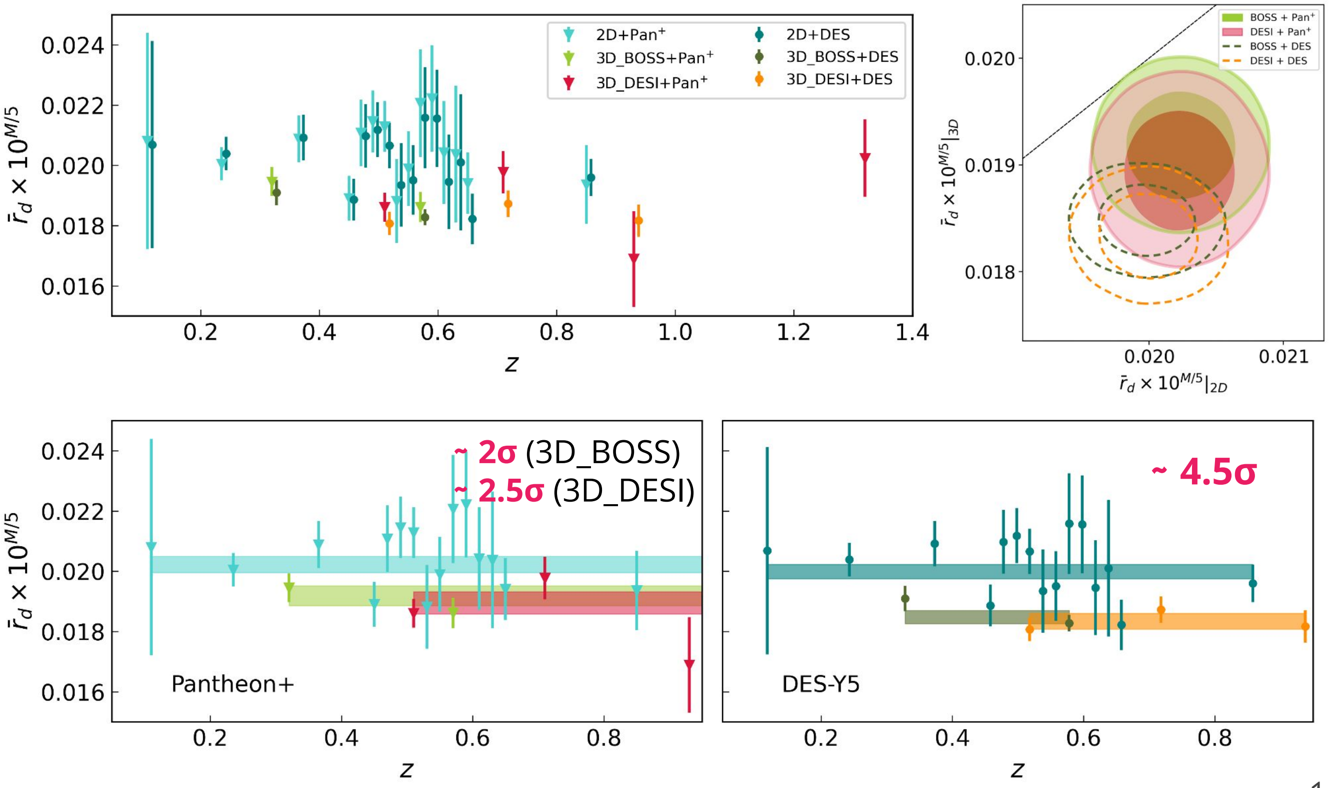

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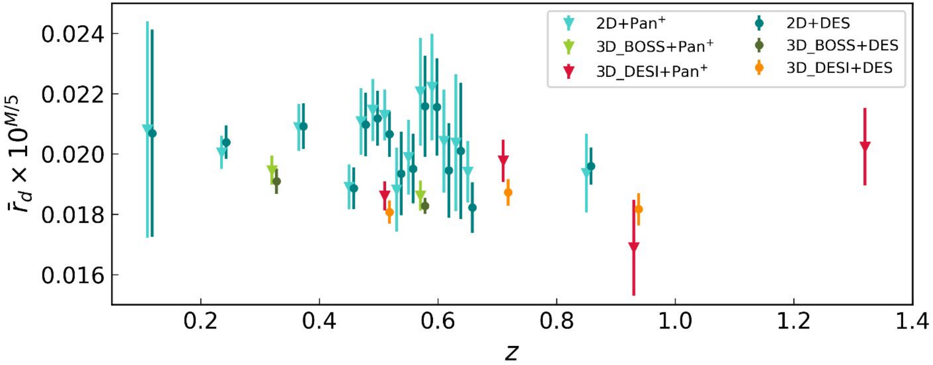
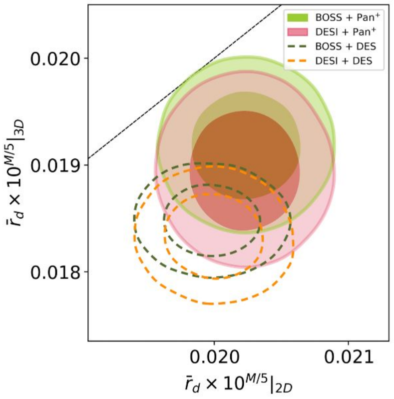
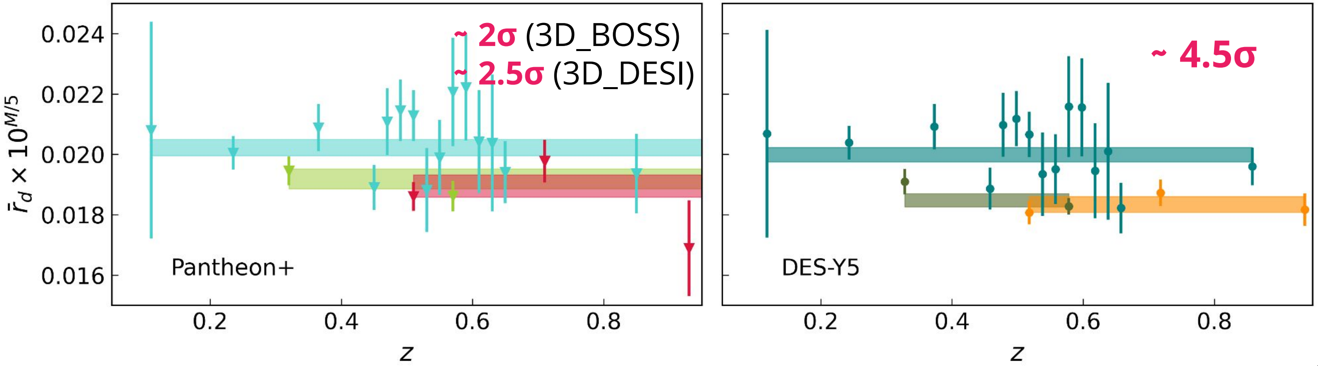




Underestimation of the 3D BAO uncertainties by a factor 2 (Anselmi et al. 2019) decrease the tension below 2σ and ~3.5σ



Conclusions

- The door for a late- and an ultra-late-time solution to the Hubble tension is still open and its concrete form seems to depend crucially on the BAO data set considered.

- 2D and 3D BAO are found to be in tension with a lower bound currently at ~4.5σ C.L. when DESY5 SNIa are employed.

- In the era of precision cosmology and the existing tensions afflicting the ΛCDM model, it is crucial to elucidate what is causing these discrepancies.

- Future background and BAO data such as those from Euclid are meant to be pivotal on the discussion and eventual solutions to the cosmic tensions.

Thank you for your attention!

BACKUP SLIDES

BAO: 2D vs 3D measurements

3D BAO

Survey	z	Observable	Measurement	References
6dFGS + SDSS MGS	0.122	$D_V(r_d^{\rm fid}/r_d)$	539 ± 17 (Mpc)	[97]
WiggleZ	0.44	$D_V(r_d^{\rm fid}/r_d)$	1716.4 ± 83.1 (Mpc)	[98]
	0.60	$D_V(r_d^{\rm fid}/r_d)$	2220.8 ± 100.6 (Mpc)	
	0.73	$D_V(r_d^{\rm fid}/r_d)$	2516.1 ± 86.1 (Mpc)	
BOSS DR12	0.32	$r_d H/(10^3\ \mathrm{km/s})$	11.549 ± 0.385	[99]
		D_A/r_d	6.5986 ± 0.1337	
	0.57	$r_d H/(10^3\ \mathrm{km/s})$	14.021 ± 0.225	
		D_A/r_d	9.389 ± 0.103	
DES Y3	0.835	D_M/r_d	18.92 ± 0.51	[100]
Quasars eBOSS DR16	1.48	D_M/r_d	30.21 ± 0.79	[101]
		$c/(H r_d)$	13.23 ± 0.47	
Lyα-Forests eBOSS DR16	2.334	D_M/r_d	$37.5^{+1.2}_{-1.1}$	[102]
		$c/(H r_d)$	$8.99^{+0.20}_{-0.19}$	

2D BAO

z	$\theta_{\rm BAO}$ (deg)	$\sigma_{\rm BAO}$ (deg)	References
0.11	19.8	3.26	[51]
0.235	9.06	0.23	[103]
0.365	6.33	0.22	
0.45	4.77	0.17	[104]
0.47	5.02	0.25	
0.49	4.99	0.21	
0.51	4.81	0.17	
0.53	4.29	0.30	
0.55	4.25	0.25	
0.57	4.59	0.36	[105]
0.59	4.39	0.33	
0.61	3.85	0.31	
0.63	3.90	0.43	
0.65	3.55	0.16	
2.225	1.77	0.31	[106]

BAO: 2D vs 3D measurements

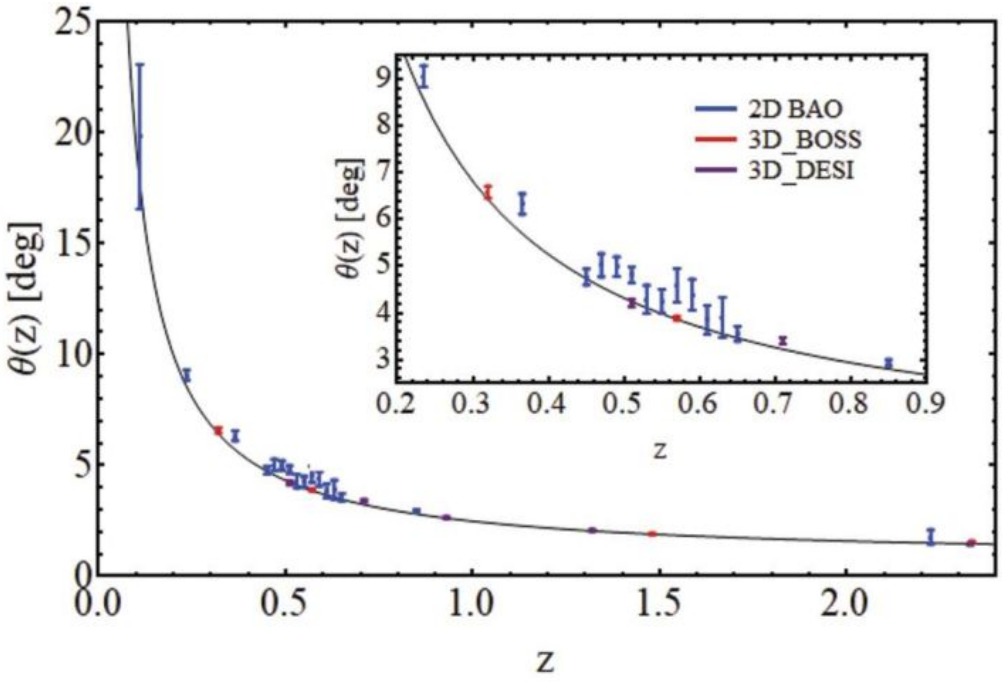
3D_BOSS / 3D_DESI

Survey	z	D_A/r_d	References
BOSS DR12	0.32	6.5986 ± 0.1337	Gil-Marín et al. [51]
	0.57	9.389 ± 0.103	
eBOSS DR16Q	1.48	12.18 ± 0.32	Hou et al. [58]
Lyα-F eBOSS DR16	2.334	$11.25^{+0.36}_{-0.33}$	du Mas des Bourboux et al. [25]
LRG1 DESI Y1	0.51	9.02 ± 0.17	Adame et al. [4]
LRG2 DESI Y1	0.71	9.85 ± 0.19	Adame et al. [4]
LRG3+ELG1 DESI Y1	0.93	11.25 ± 0.16	Adame et al. [4]
ELG2 DESI Y1	1.32	11.98 ± 0.30	Adame et al. [4]
Lyα-F DESI Y1	2.33	11.92 ± 0.29	Adame et al. [5]

2D BAO

Survey	z	θ_{BAO} [deg]	References
SSDS DR12	0.11	19.8 ± 3.26	de Carvalho et al. [34]
SDSS DR7	0.235	9.06 ± 0.23	Alcaniz et al. [12]
	0.365	6.33 ± 0.22	
SDSS DR10	0.45	4.77 ± 0.17	Carvalho et al. [36]
	0.47	5.02 ± 0.25	
	0.49	4.99 ± 0.21	
	0.51	4.81 ± 0.17	
	0.53	4.29 ± 0.30	
	0.55	4.25 ± 0.25	
SDSS DR11	0.57	4.59 ± 0.36	Carvalho et al. [37]
	0.59	4.39 ± 0.33	
	0.61	3.85 ± 0.31	
	0.63	3.90 ± 0.43	
	0.65	3.55 ± 0.16	
DES Y6	0.85	2.932 ± 0.068	Abbott et al. [2]
BOSS DR12Q	2.225	1.77 ± 0.31	de Carvalho et al. [35]

Anisotropic and angular BAO



Anisotropic and angular BAO

$$\chi^2 = -2 \ln f(x_{2D}, x_{3D})$$

BAO data set	$\bar{r}_d 10^{M/5}$	p-value	$\bar{r}_d 10^{M/5}$	p-value
	Pantheon+		DES Y5	
2D	$(20.23 \pm 0.27) \cdot 10^{-3}$	–	$(19.98 \pm 0.24) \cdot 10^{-3}$	-
3D_BOSS	$(19.19 \pm 0.33) \cdot 10^{-3}$	$0.048^{+0.008}_{-0.009}$	$(18.48 \pm 0.22) \cdot 10^{-3}$	$< 10^{-5}$
3D_BOSS*	$(19.10 \pm 0.48) \cdot 10^{-3}$	$0.116^{+0.016}_{-0.008}$	$(18.47 \pm 0.38) \cdot 10^{-3}$	$< 10^{-3}$
3D_DESI	$(18.95 \pm 0.37) \cdot 10^{-3}$	$0.018^{+0.011}_{-0.002}$	$(18.34 \pm 0.26) \cdot 10^{-5}$	$< 10^{-5}$
3D_DESI*	$(18.98 \pm 0.53) \cdot 10^{-3}$	$0.105^{+0.014}_{-0.008}$	$(18.29 \pm 0.43) \cdot 10^{-3}$	$< 10^{-3}$

Late-time phenomenology to solve the H_0 tension

$$\delta H_1(z=0) = H_0 - \bar{H}_0 \equiv \delta H_0,$$

$$\delta H_1(z_p) = \delta H_2(z_p) \equiv \delta H_p,$$

$$\left.\frac{\partial \delta H_1}{\partial z}\right|_{z=z_p} = \left.\frac{\partial \delta H_2}{\partial z}\right|_{z=z_p} = 0,$$

$$\delta H_2(z_{\max}) = H(z_{\max}) - \bar{H}(z_{\max}) \equiv \delta H_{\max}$$

$$\begin{pmatrix} \delta H_0 \\ \delta H_p \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & z_p & z_p^2 \\ 0 & 1 & 2z_p \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

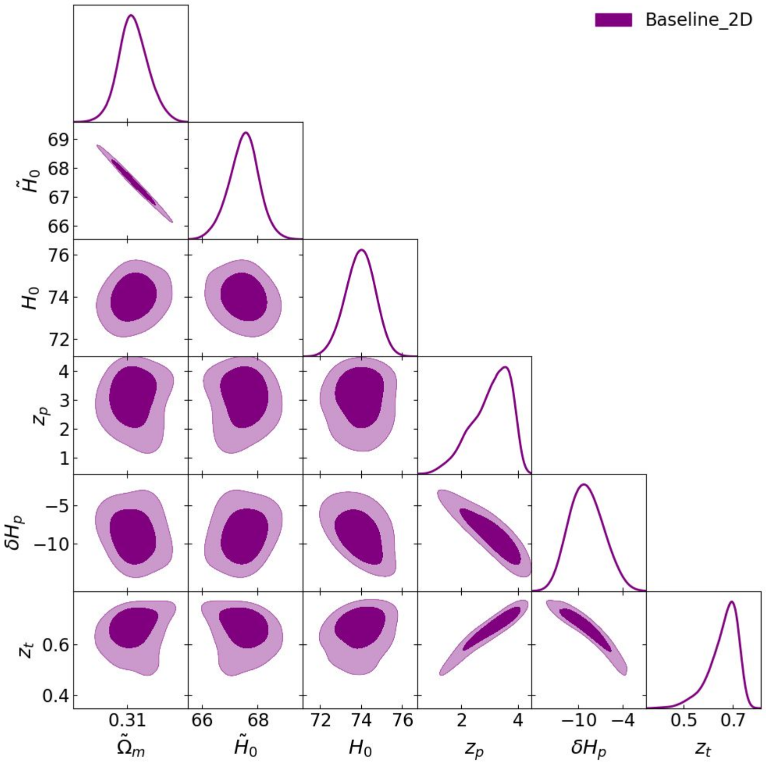
$$\begin{pmatrix} \delta H_{\max} \\ \delta H_p \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & z_{\max} & z_{\max}^2 \\ 1 & z_p & z_p^2 \\ 0 & 1 & 2z_p \end{pmatrix} \begin{pmatrix} d \\ e \\ f \end{pmatrix}$$

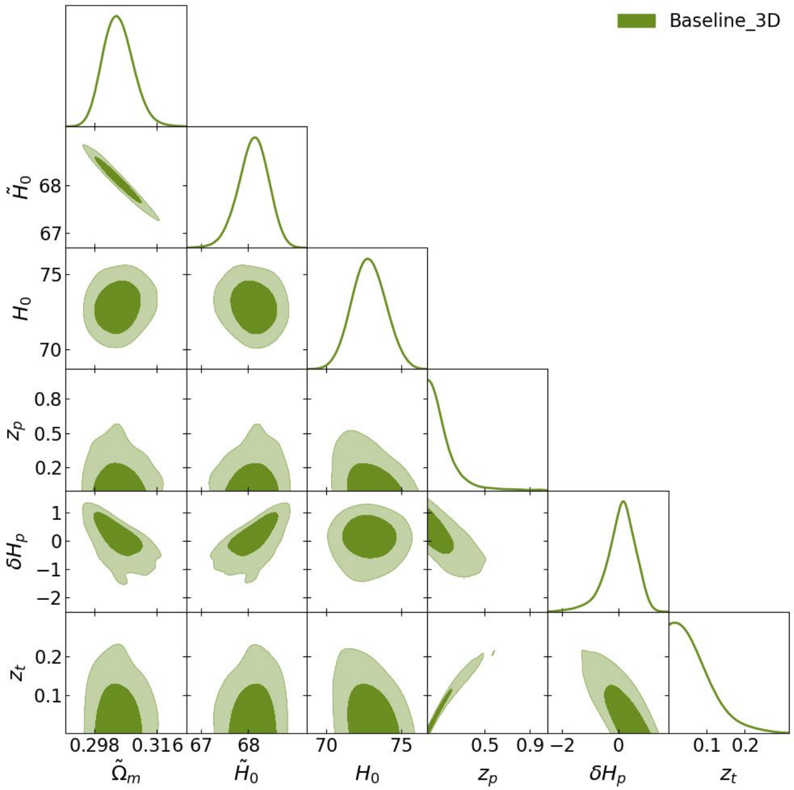
$$\mathrm{IOI}[i,j] = \frac{1}{2}\mu^T (C^{(i)} + C^{(j)})^{-1}\mu$$

$$w = \exp(-\mathrm{IOI}[\mathrm{BAO}, \mathrm{SNIa}])$$

$$D_A(z) = \frac{D_L(z)}{(1+z)^2}$$

Late-time phenomenology to solve the H_0 tension





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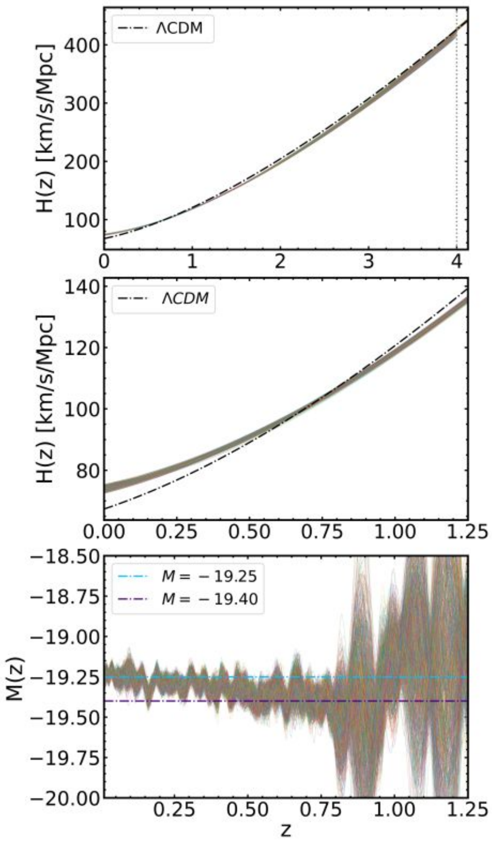

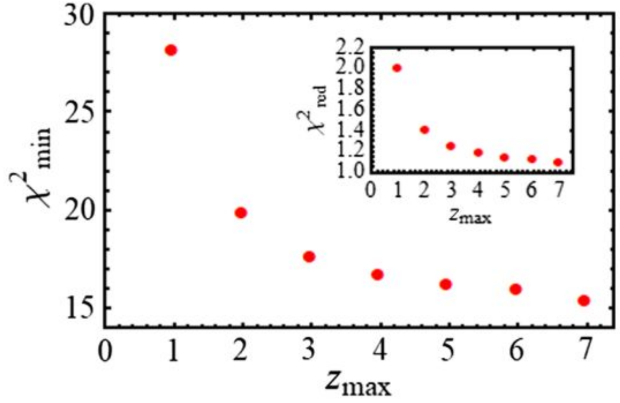

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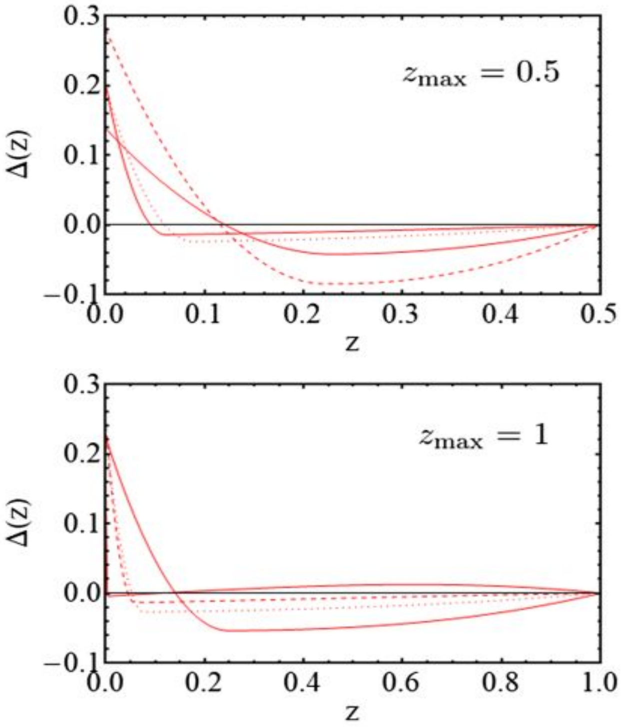

$$H^2(z) \equiv \frac{8\pi G}{3}[\tilde{\rho}_m^0(1+z)^3 + \rho_{\rm de}(z)]$$
$$= \tilde{\Omega}_m \tilde{H}_0^2(1+z)^3 + \frac{8\pi G}{3}\rho_{\rm de}(z), \qquad (27)$$

with $H(z)$ given by Eq. (1) and also use

$$H_\Lambda^2(z) = \frac{8\pi G}{3}[\tilde{\rho}_m^0(1+z)^3 + \tilde{\rho}_\Lambda]$$
$$= \tilde{\Omega}_m \tilde{H}_0^2(1+z)^3 + \frac{8\pi G}{3}\tilde{\rho}_\Lambda, \qquad (28)$$

$$\Delta(z) \equiv \frac{\rho_{\rm de}(z) - \tilde{\rho}_\Lambda}{\tilde{\rho}_\Lambda} = \frac{H^2(z) - H_\Lambda^2(z)}{H_\Lambda^2(z) - \tilde{\Omega}_m \tilde{H}_0^2(1+z)^3}$$