Perspectives of future Cosmology: Gravitational Waves

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The far-reaching scientific potential of GW observations

- GW direct detection from Earth is a great theoretical and experimental achievement, providing observational access to many new physical phenomena

- Astrophysics:
  - Discovery of new astrophysical objects (black hole binaries…)
  - Provide information on their population and characteristics
  - Enlighten astrophysical phenomena (fast gamma-ray bursts, Active Galactic Nuclei, supernovae explosions…)
  - Probe the content of our galaxy and the environment of galactic centres
  - Provide information on the formation and growth of massive black holes and massive black hole binaries
  - All this potential would be enhanced by the detection of electromagnetic counterparts

- Cosmology:
  - Expansion of the universe, dark energy, Hubble tension
  - Nature of Dark Matter (primordial black holes, black holes accretion…)
  - Cosmological structure formation, galaxy mergers, probe of LambdaCDM
  - Early universe before Recombination in general

The far-reaching scientific potential of GW observations

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- Fundamental physics:
  - Test General Relativity in the strong field regime (Post Newtonian terms, tests of the horizon, GW polarisations, space-time around black holes…)
  - Test of General Relativity at cosmological scales (GW propagation, GW lensing…)
  - High energy and beyond the standard model physics (phase transitions: Electroweak scale, QCD scale, cosmic strings; Inflation…)
  - Matter in extreme conditions (neutron stars equation of state, elements synthesis…)

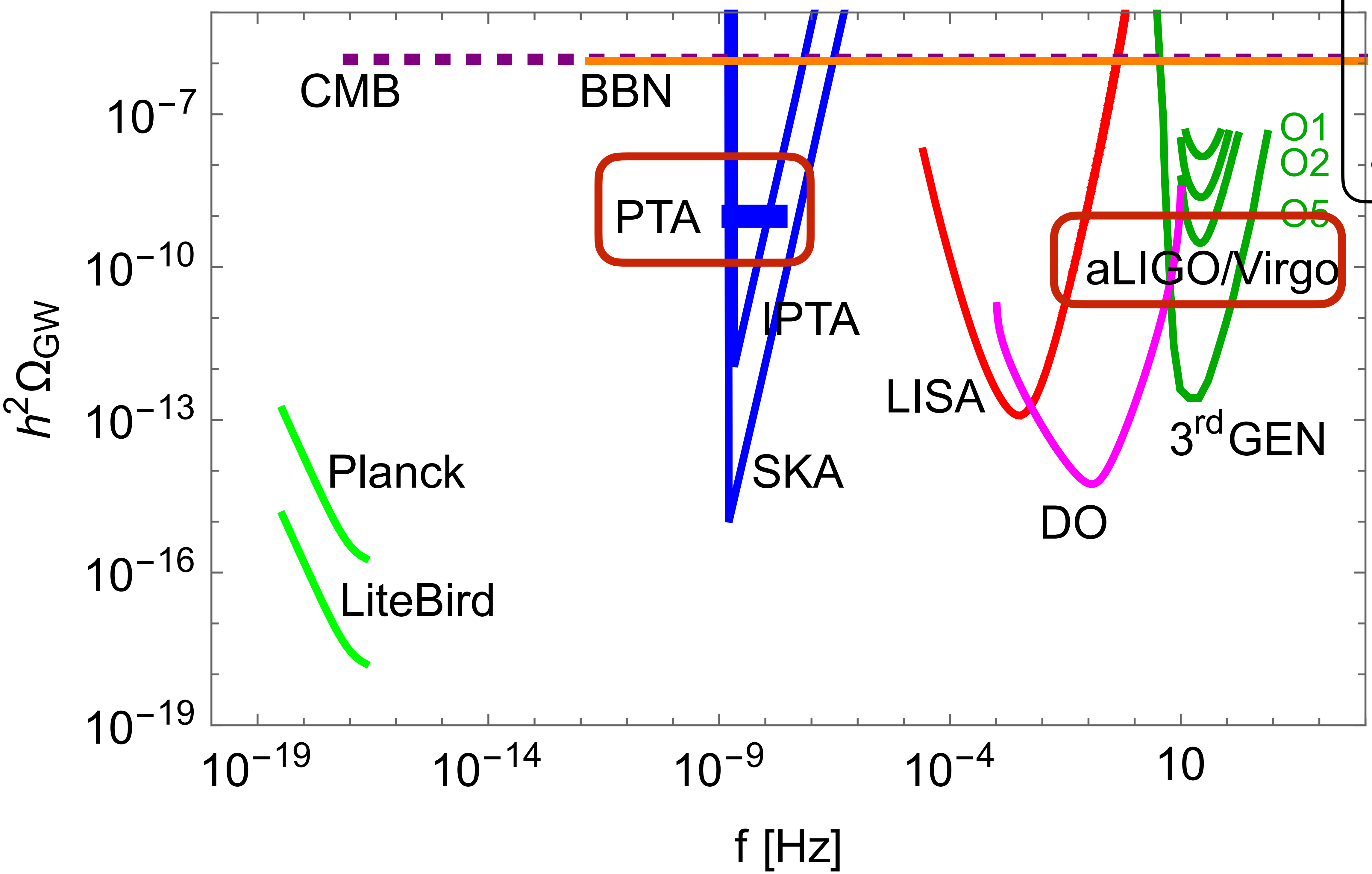
- Data Analysis (Bayesian methods, noise and foreground subtraction, machine learning…)

- Detectors (stabilisation, cryogeny, quantum limits, free fall, atom interferometry…)

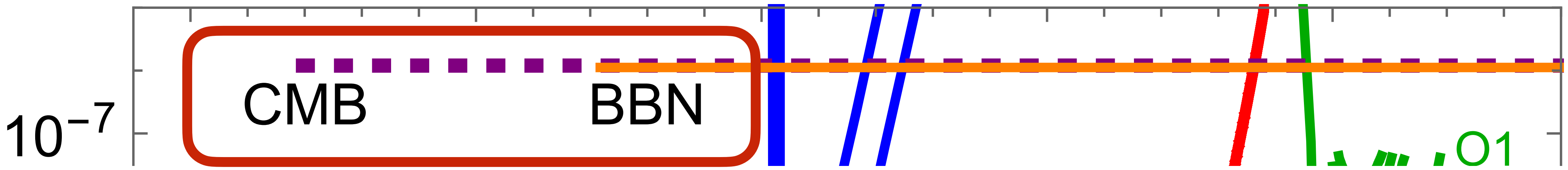
Non exhaustive list and non exhaustive review

GW observational landscape

high frequency detectors up to 10^{18} GeV, sensitivity still above BBN and CMB bounds

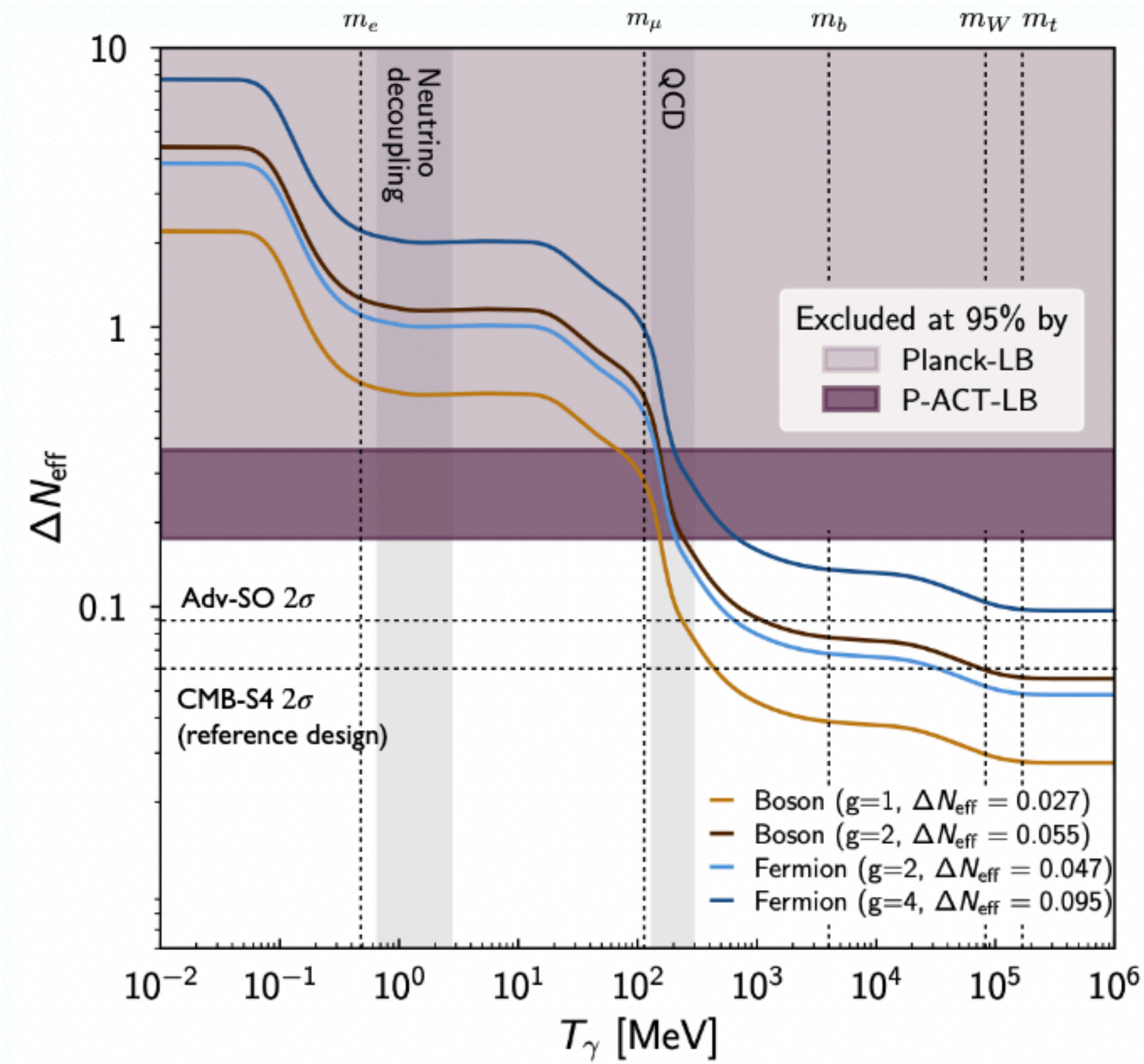


CMB and BBN bounds

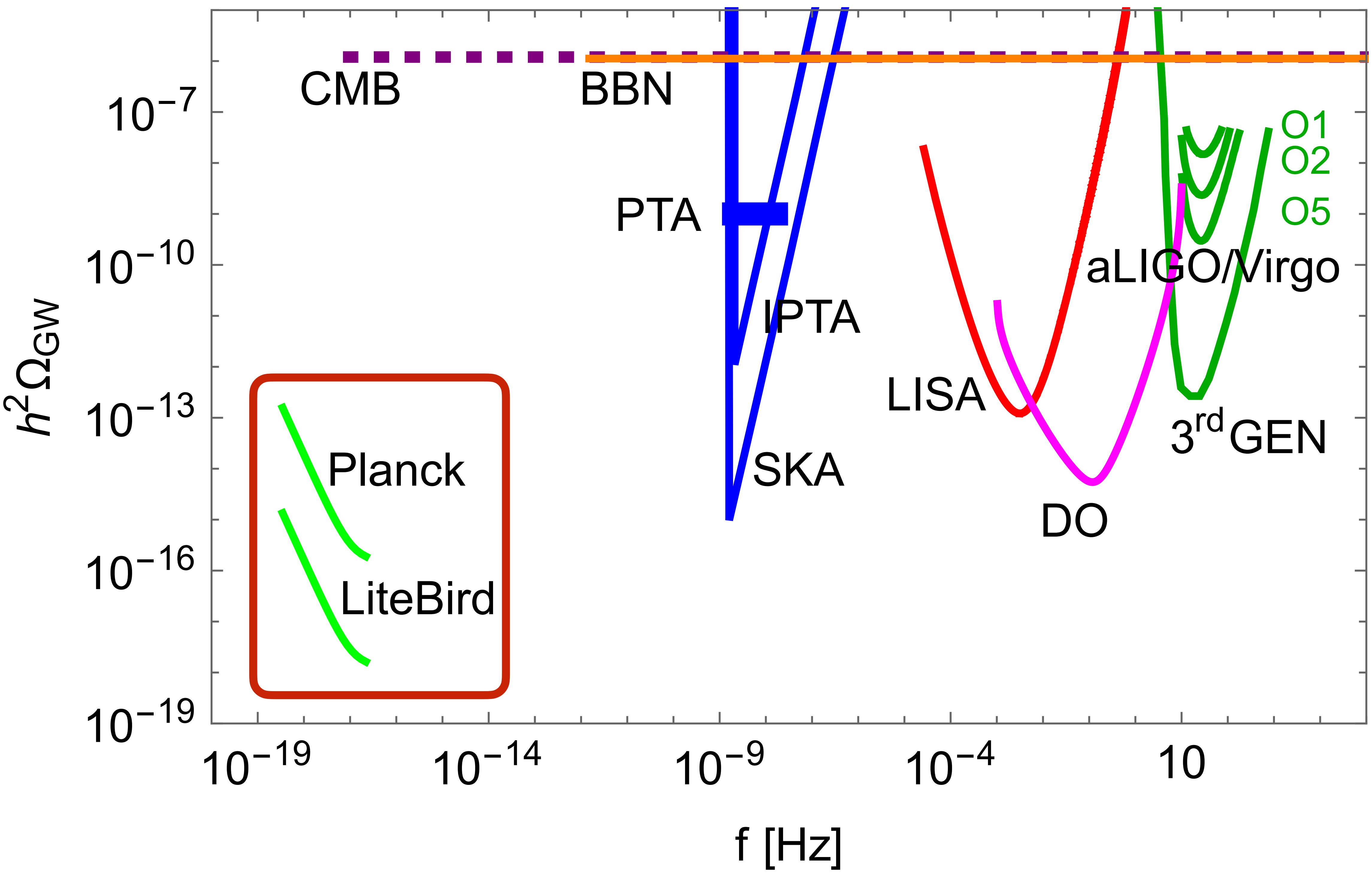


Talks by A. Challinor, Greg Jackson, J. Ghiglieri

- Constraints from CMB will be improved
- There are still theoretical uncertainties on Neff
- It can be a probe of new physics
- Future progress not very relevant in the context of BBN and CMB bounds on primordial GWs (?)



CMB anisotropies and polarisation



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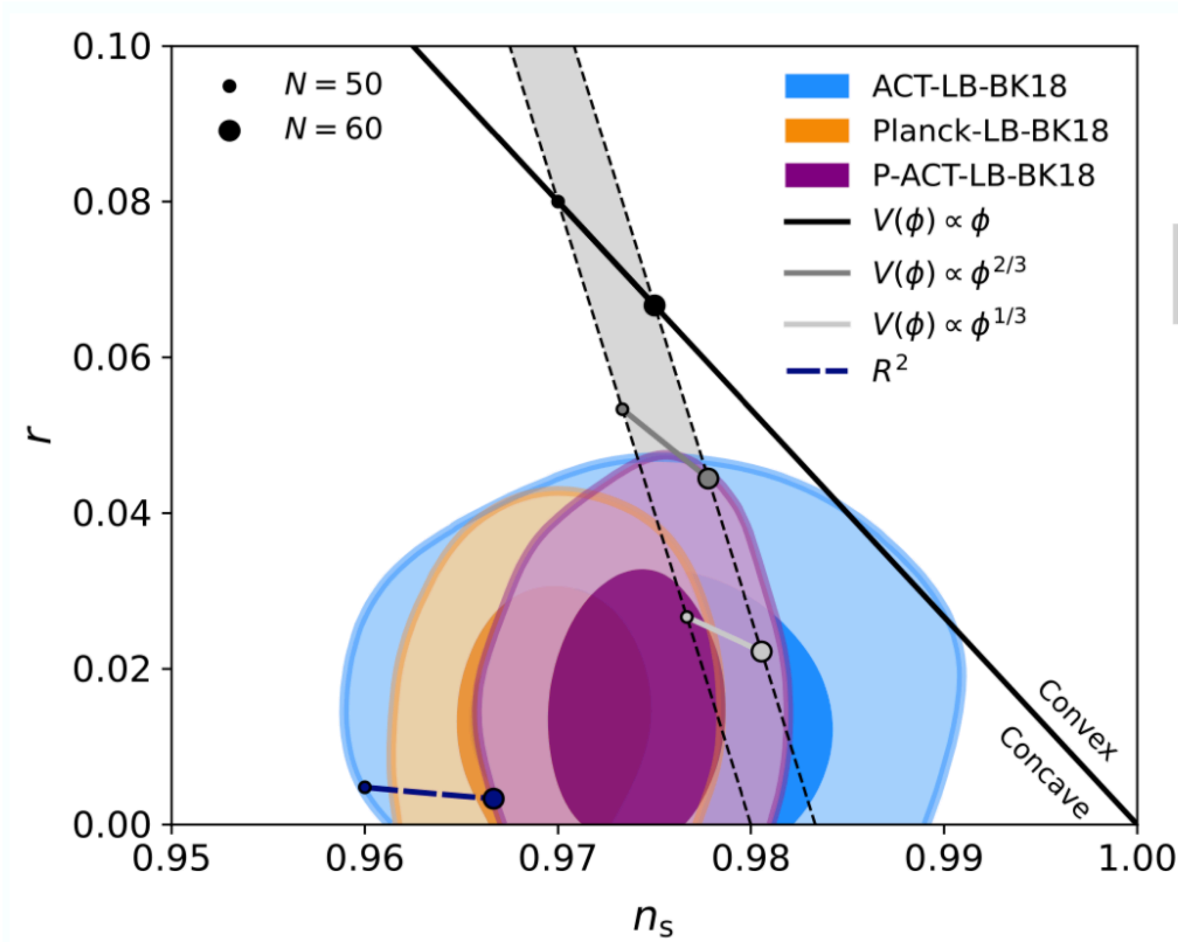


Talk by A. Challinor

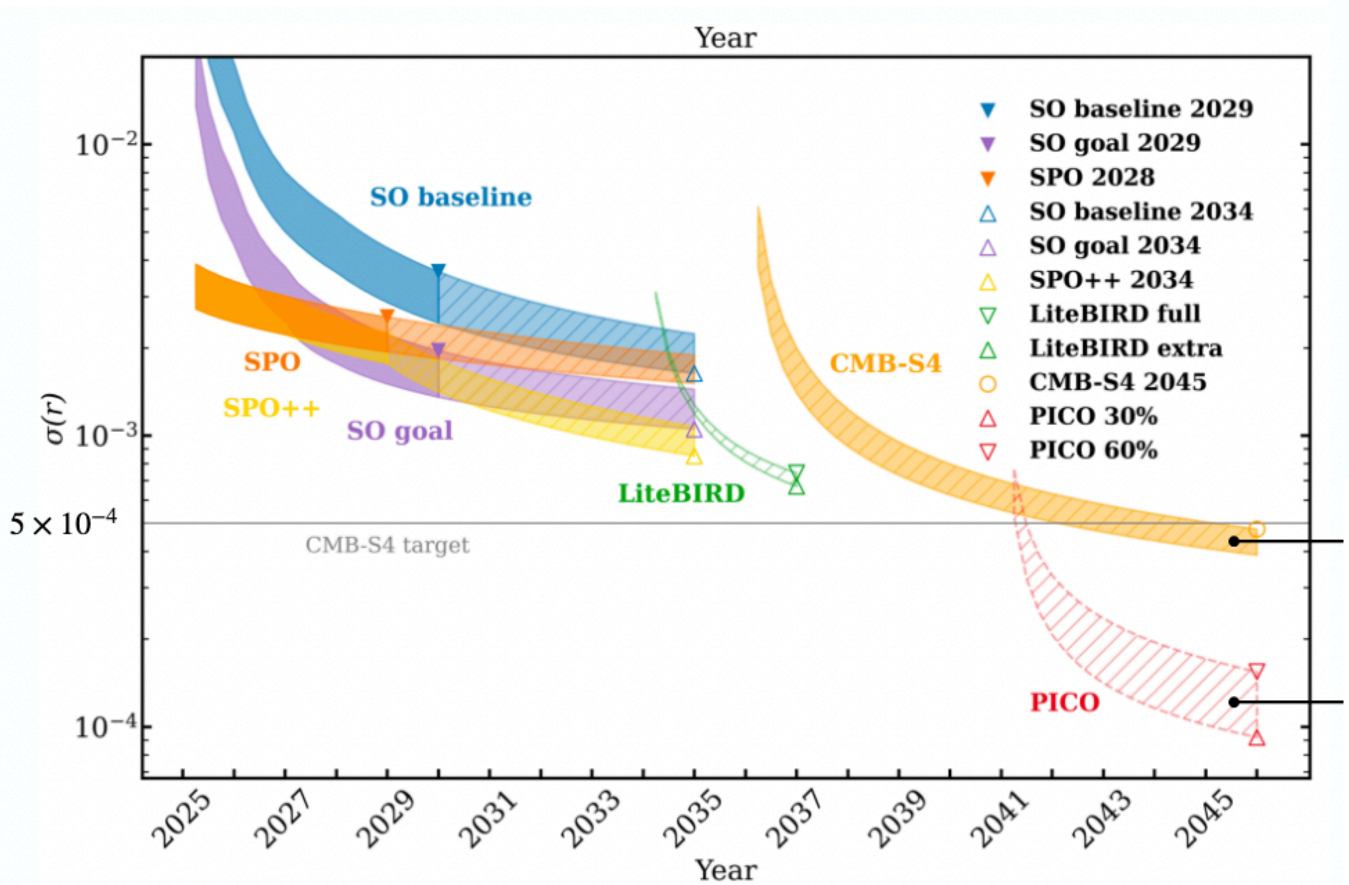
- Constraint on r driven by BICEP/Keck

$$r_{0.05} < 0.038 \quad (95\%; \text{Planck+ACT+BK+lensing+BAO})$$

- n_s pushed up 1σ by lower $\Omega_m h^2$ from DESI BAO
  - Implications for natural targets such as R^2 inflation with $r = O(1/N^2)$?
- Goals for future surveys:

SO	$\sigma(r) \leq 0.003$ for $r = 0$
CMB-S4	$\sigma(r) = 5 \times 10^{-4}$ for $r = 0$

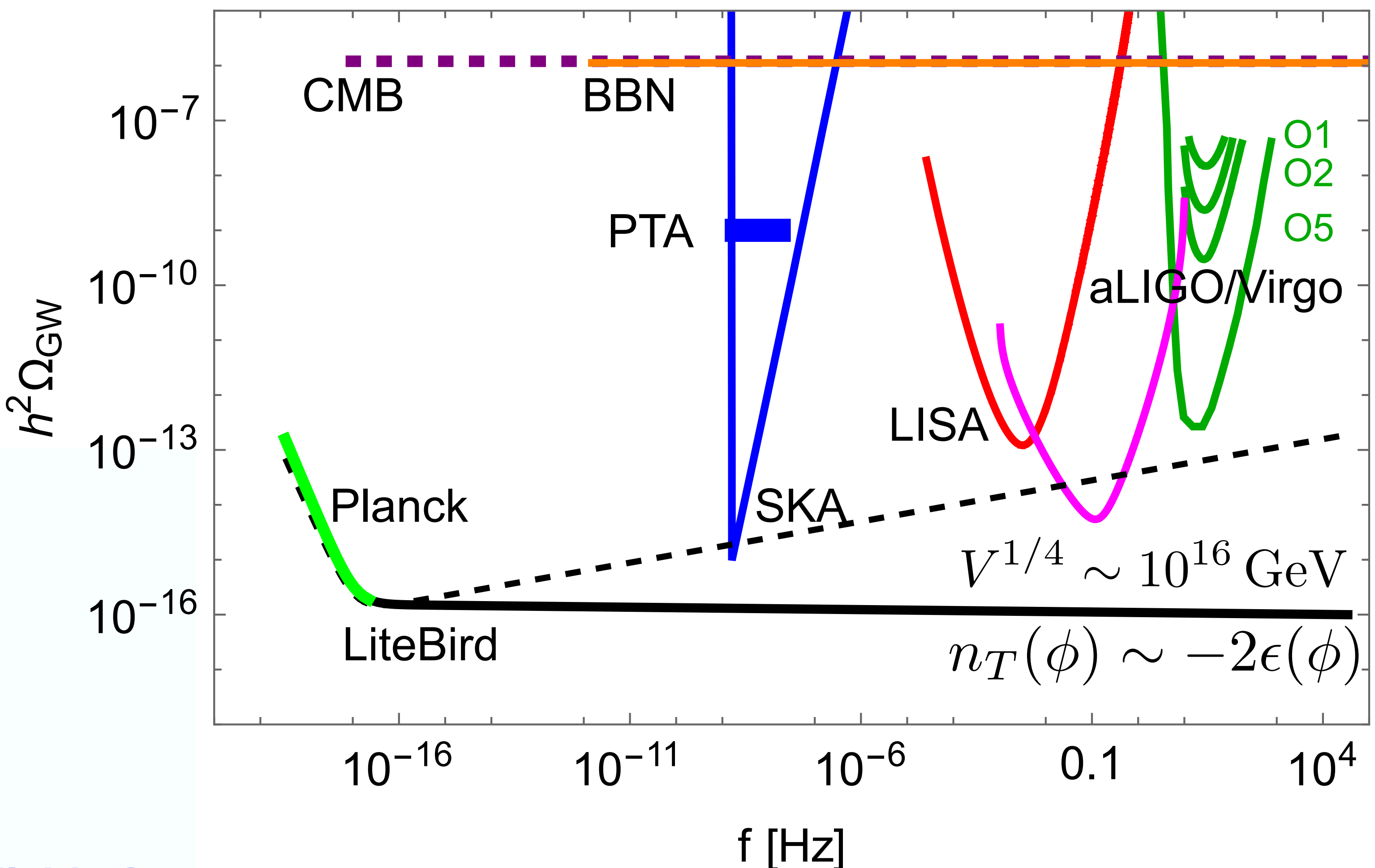
Requires aggressive delensing (see Julien Carron's talk)



CMB anisotropies and polarisation

- GW observatories probe at higher frequencies than CMB -> smaller scales -> closer to the end of inflation
- This should be used as an opportunity to test inflation far from CMB scales
- However, the standard expectation is that the power spectrum gets smaller at smaller scales! Is it instead possible to enhance it?

Talk by D. Figueroa:
a motivated example is
axion inflation



$$f \simeq 15\,\mathrm{mHz}\left(\frac{k}{10^{13}\,\mathrm{Mpc}^{-1}}\right)$$

PROs
- Rooted in HEP
- Rich Pheno (GW, PBH, BAU, …)
- Potentially Observable

CONs
- Current Predictions reliable ?
- Non-Linear (local) dynamics
- Lattice Sims extremely difficult

Here where progress is needed

CMB anisotropies and polarisation

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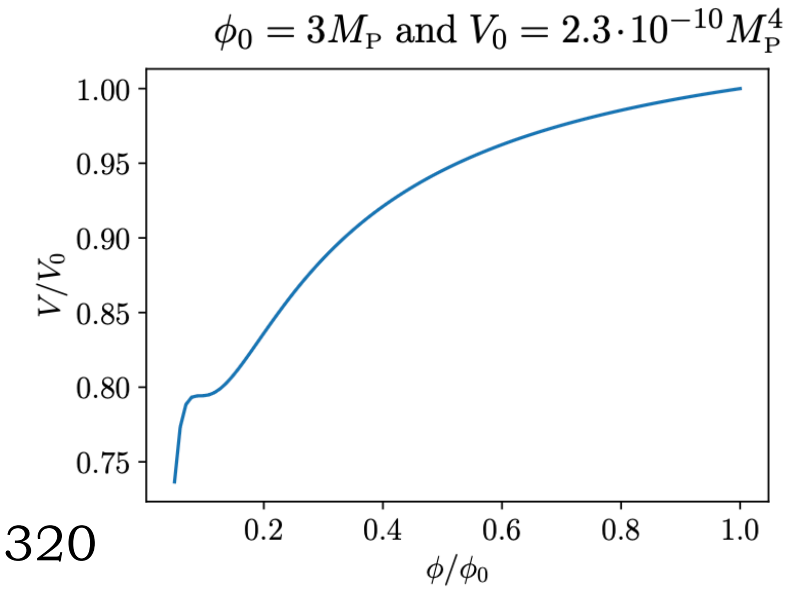


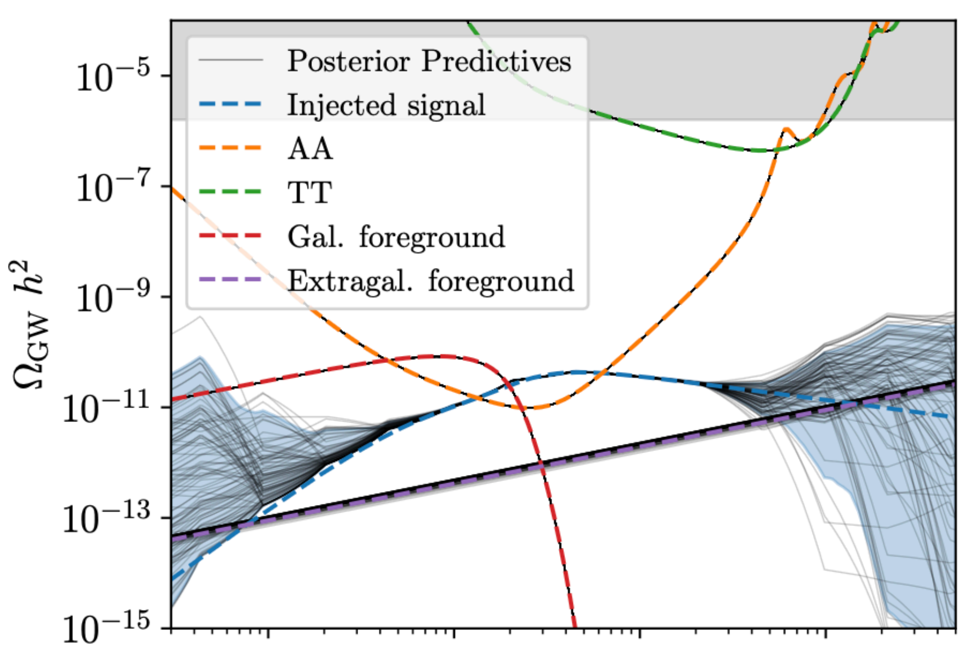
Another motivated example:
tensors at second order from enhanced scalars, in connection with PBHs

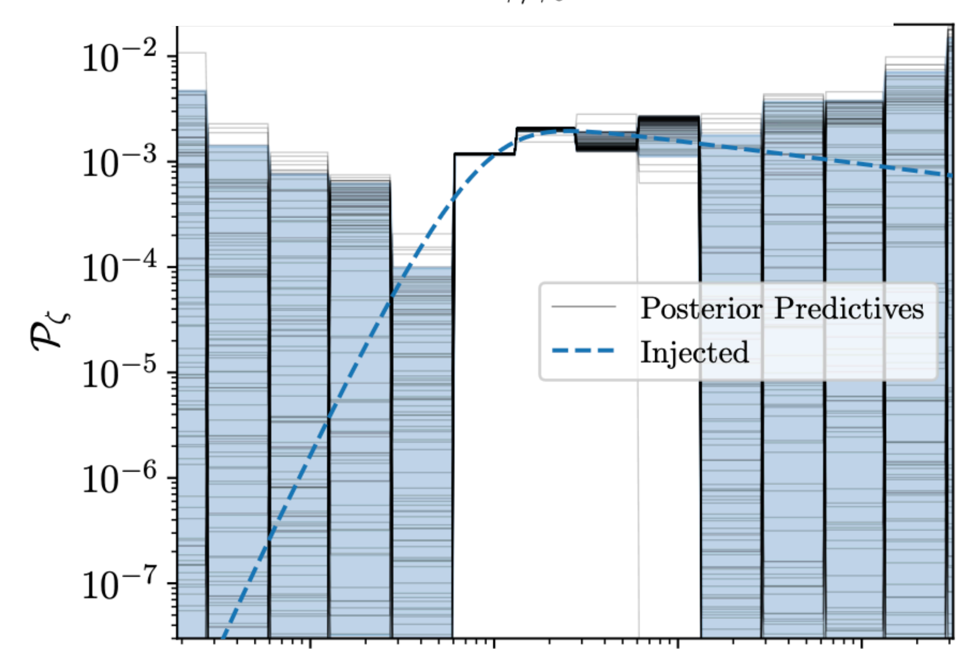
LISA CosWG, arXiv:2501.11320

Progress is needed:

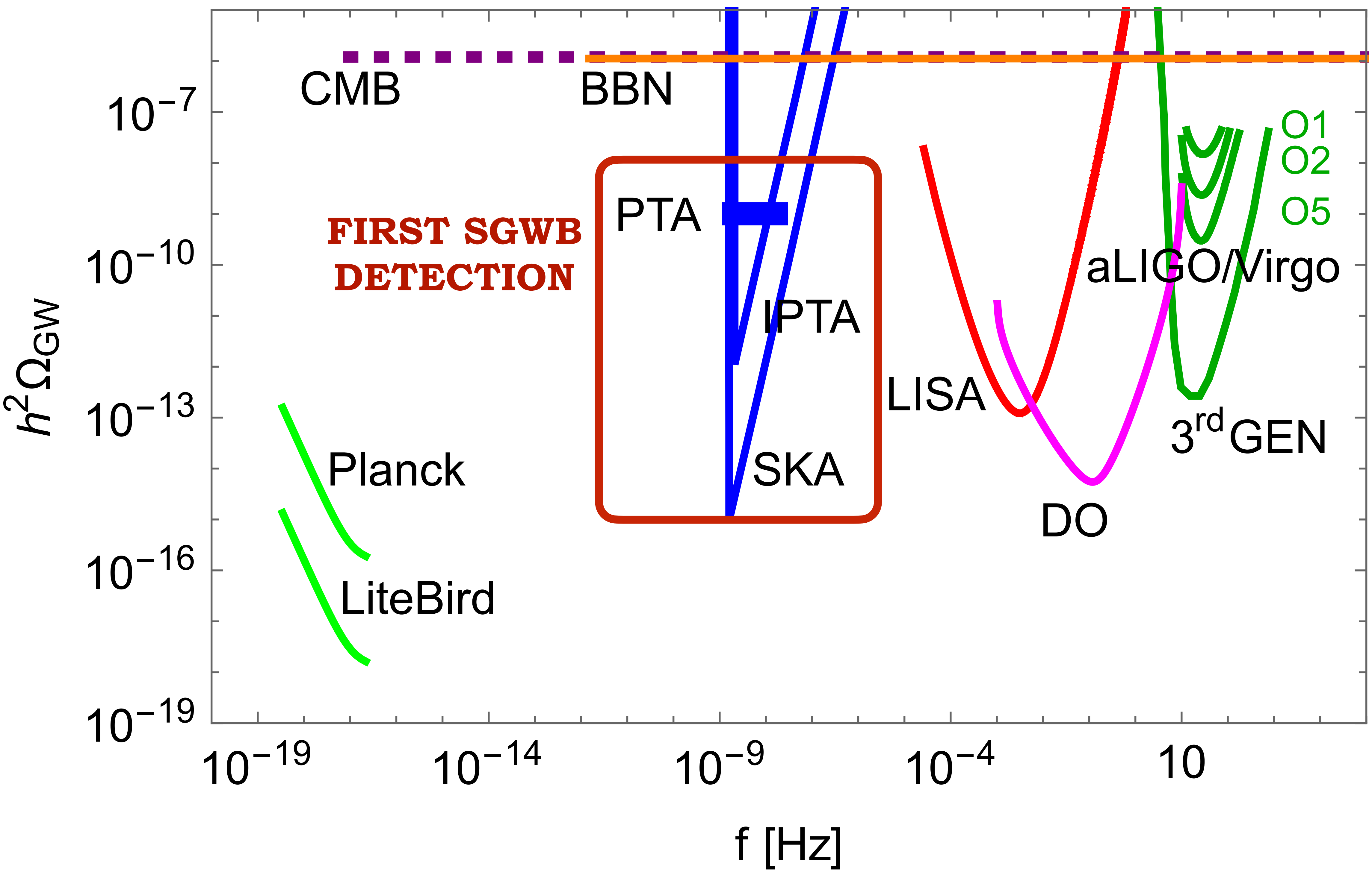
- Non-gaussianity & collapse
- Quantum regime

Poster by C. Joana
Talk by B. Blachier





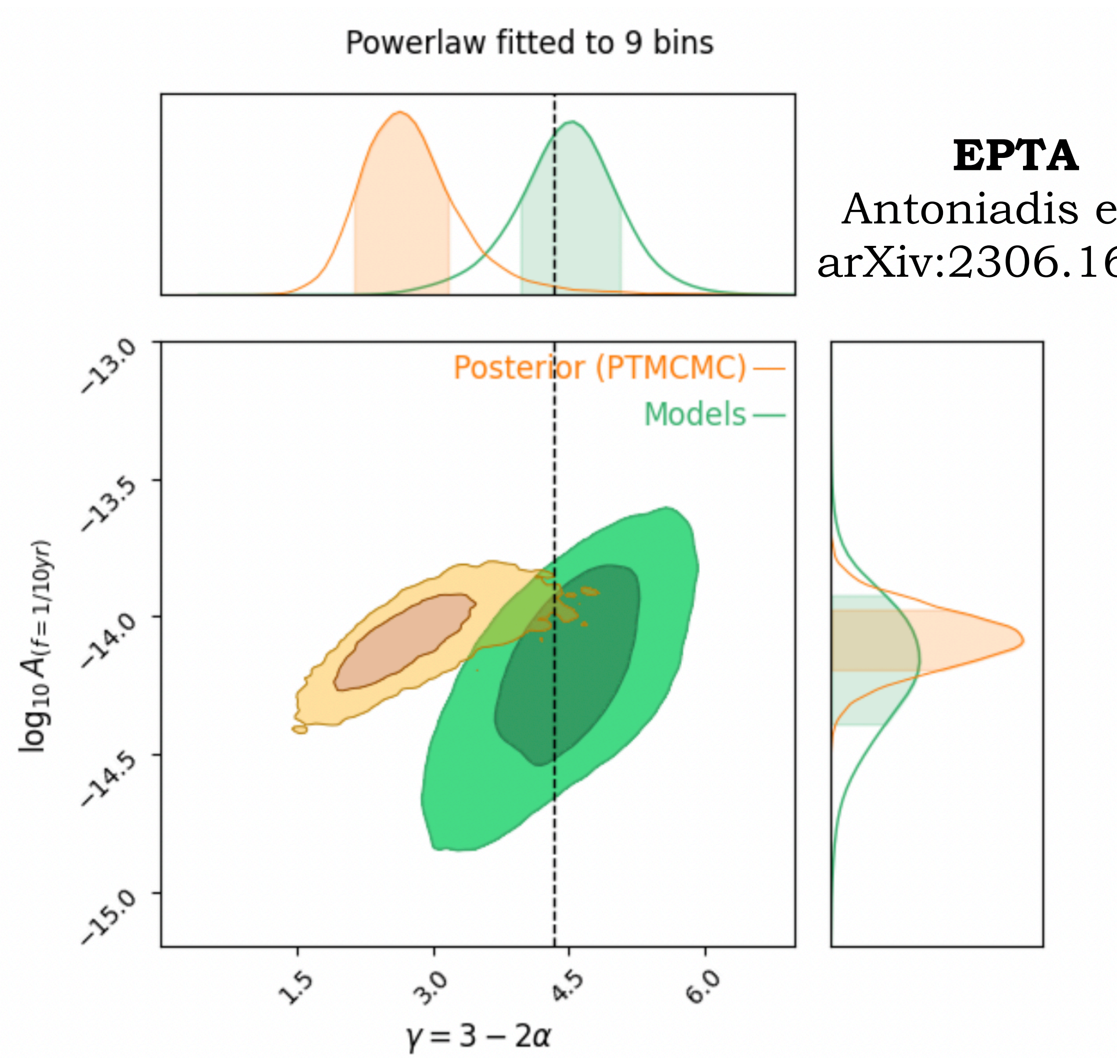
Pulsar timing arrays



Pulsar timing arrays

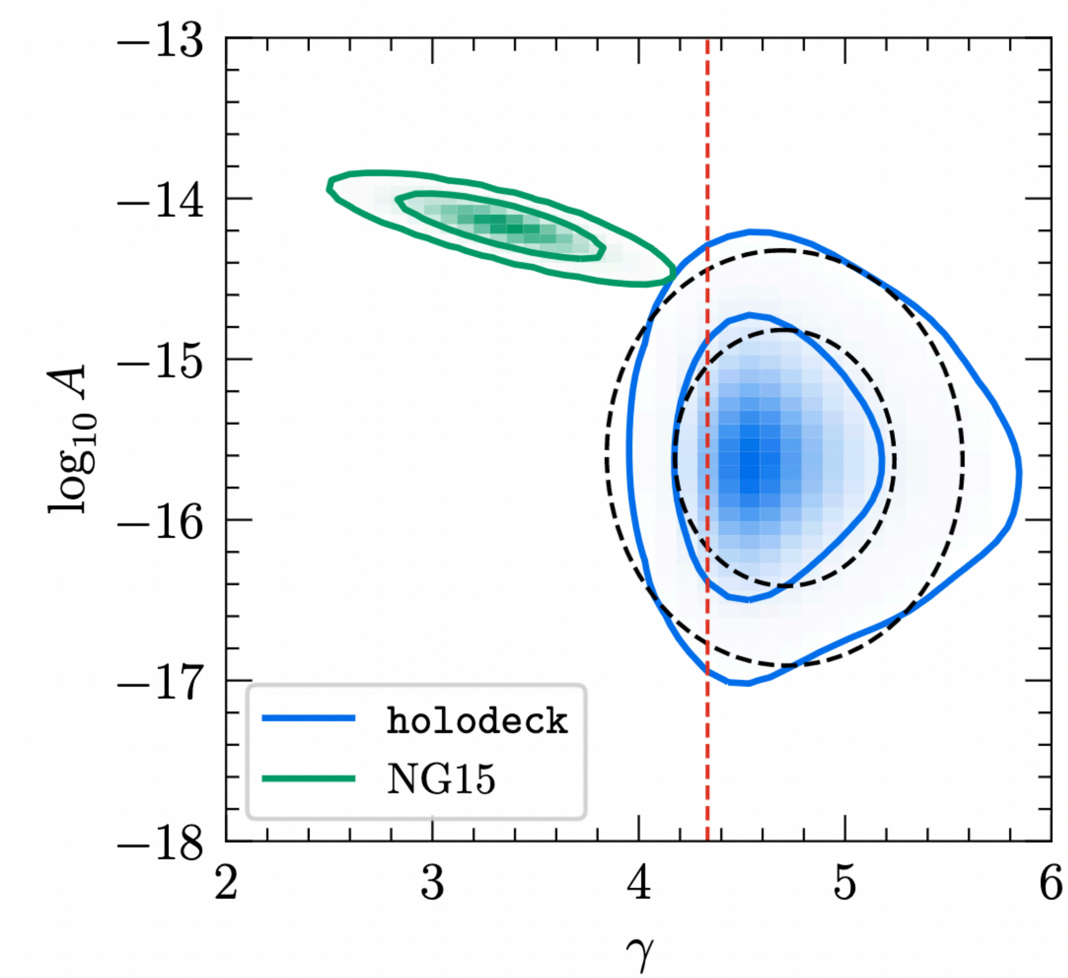
The SGWB from super-massive black hole binaries at the centre of galaxies is the best candidate source for this signal, but there is room for a primordial SGWB

- Signal in *mild tension* with the astrophysical predictions: **higher amplitude and shallower spectral slope**



EPTA
Antoniadis et al
arXiv:2306.16227

NANOGrav
Afzal et al arXiv:2306.16219



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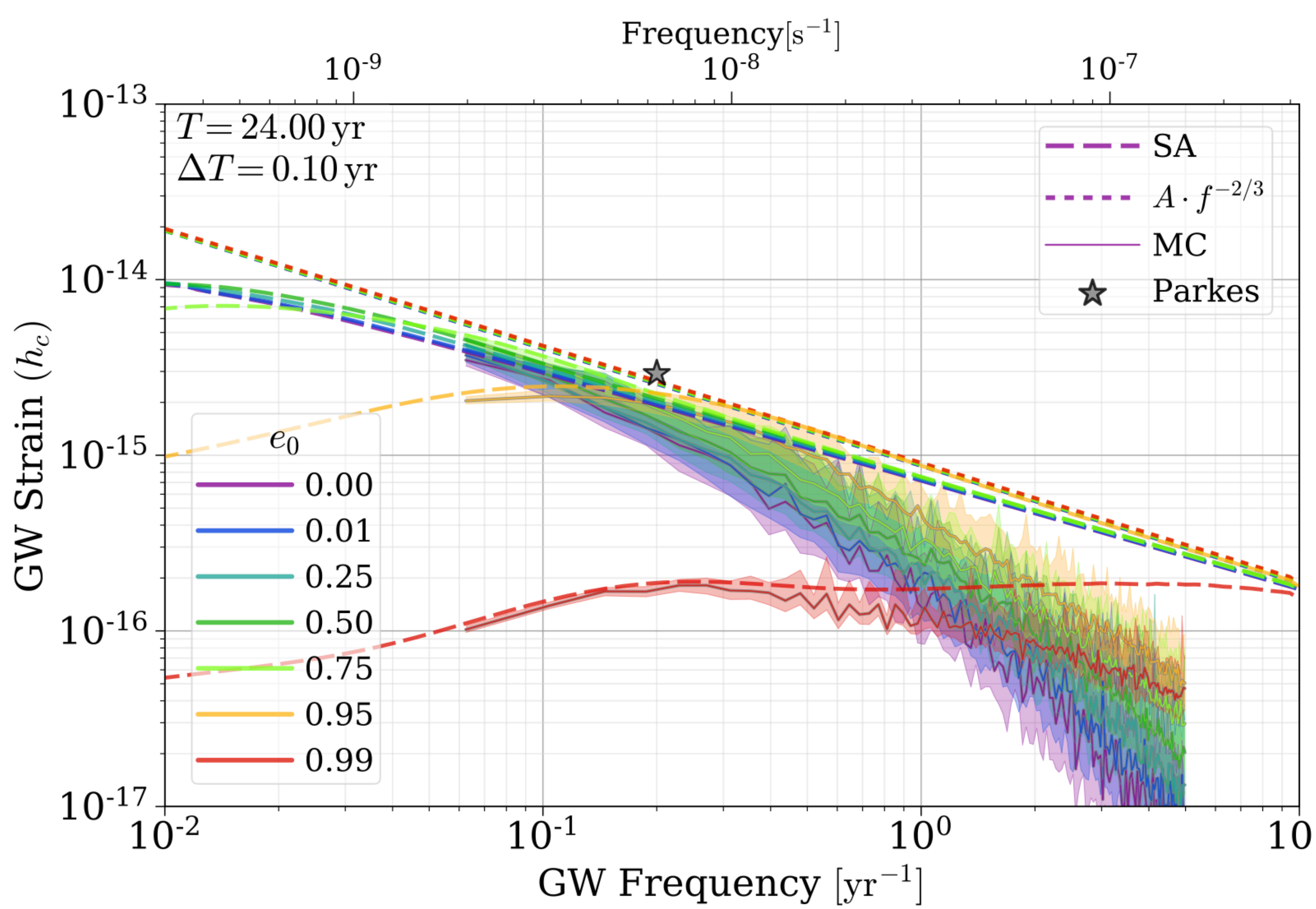
- The pure SMBHB interpretation points towards high merger rate, efficient accretion, possibly strong environmental interaction and eccentricity…

L.Z. Kelley et al, arXiv:1702.02180

- No homogeneous and isotropic SGWB at high frequency: less SMBHBs, steeper slope, discreteness with spikes from loudest SMBHBs
- Interaction with the binary environment makes hardening stronger and suppresses SGWB power at low frequency
- Eccentricity enhances GW emission at higher frequencies



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- However, *one can also interpret this tension as another origin for the signal* and/or a contribution from other SGWB generation processes

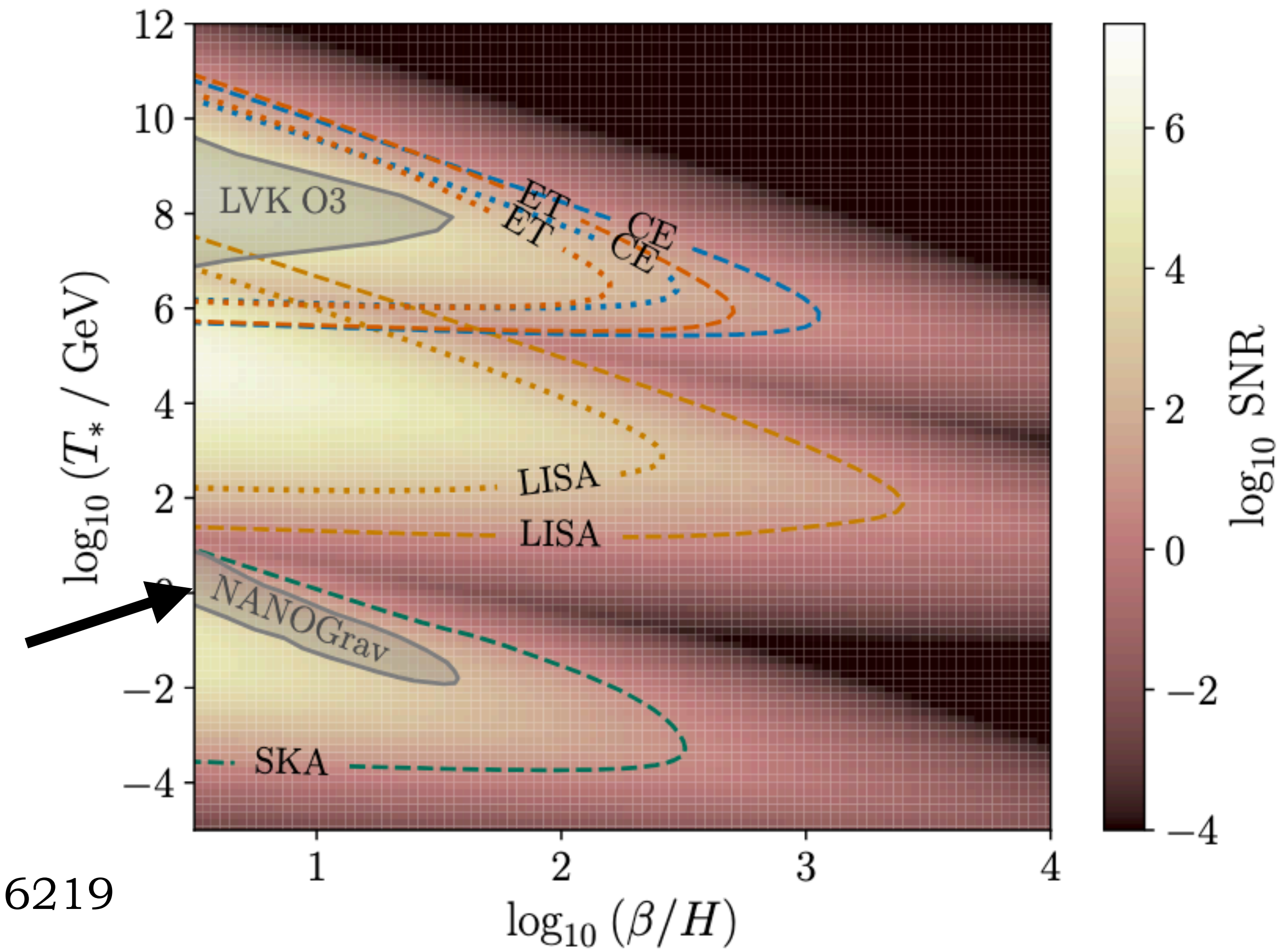
CC et al, ArXiv:2406.02359

Many primordial SGWBs possible, but important to point out that PTAs offer the possibility to probe the **QCD energy scale**

$$10^{-9}\,\mathrm{Hz} < f < 10^{-7}\,\mathrm{Hz} \rightarrow 1\,\mathrm{MeV} \lesssim T_* \lesssim 1\,\mathrm{GeV}$$

Talk by A. Rajantie

Parameter space region that could explain the measurement



Afzal et al arXiv:2306.16219

Parameter to which the signal amplitude is *inversely* proportional

Pulsar timing arrays

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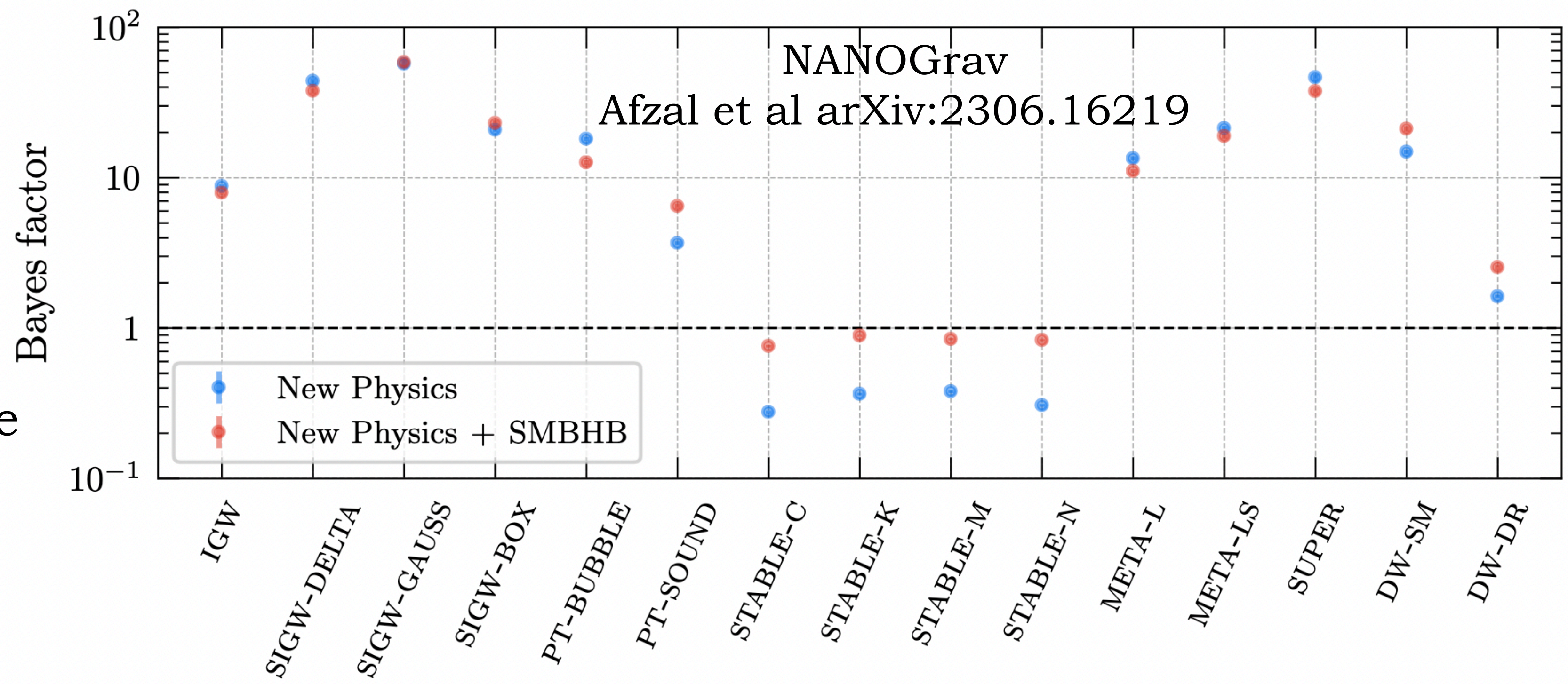
Assessing the signal origin using spectral shape only has its limits



- All new physics models except stable cosmic strings fit the signal *better than the baseline SMBHB model*
- Models in which the *two signals are present do not improve the fit quality consistently* a part in a few cases
- **HOWEVER!** Bayes factors do not account for the *huge theoretical uncertainties on the models and are prior dependent*

Talk by Atkins and Di Ferrante

Pulsar timing arrays

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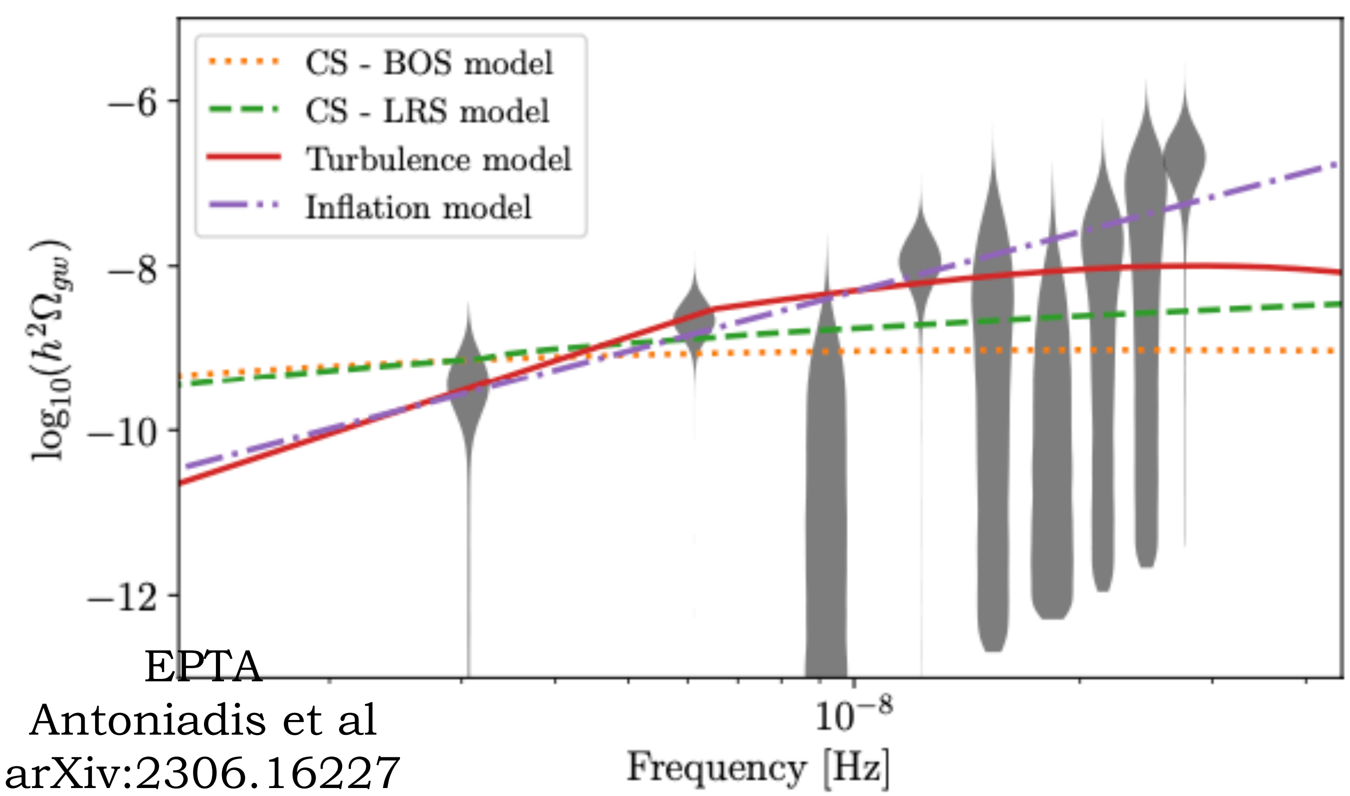
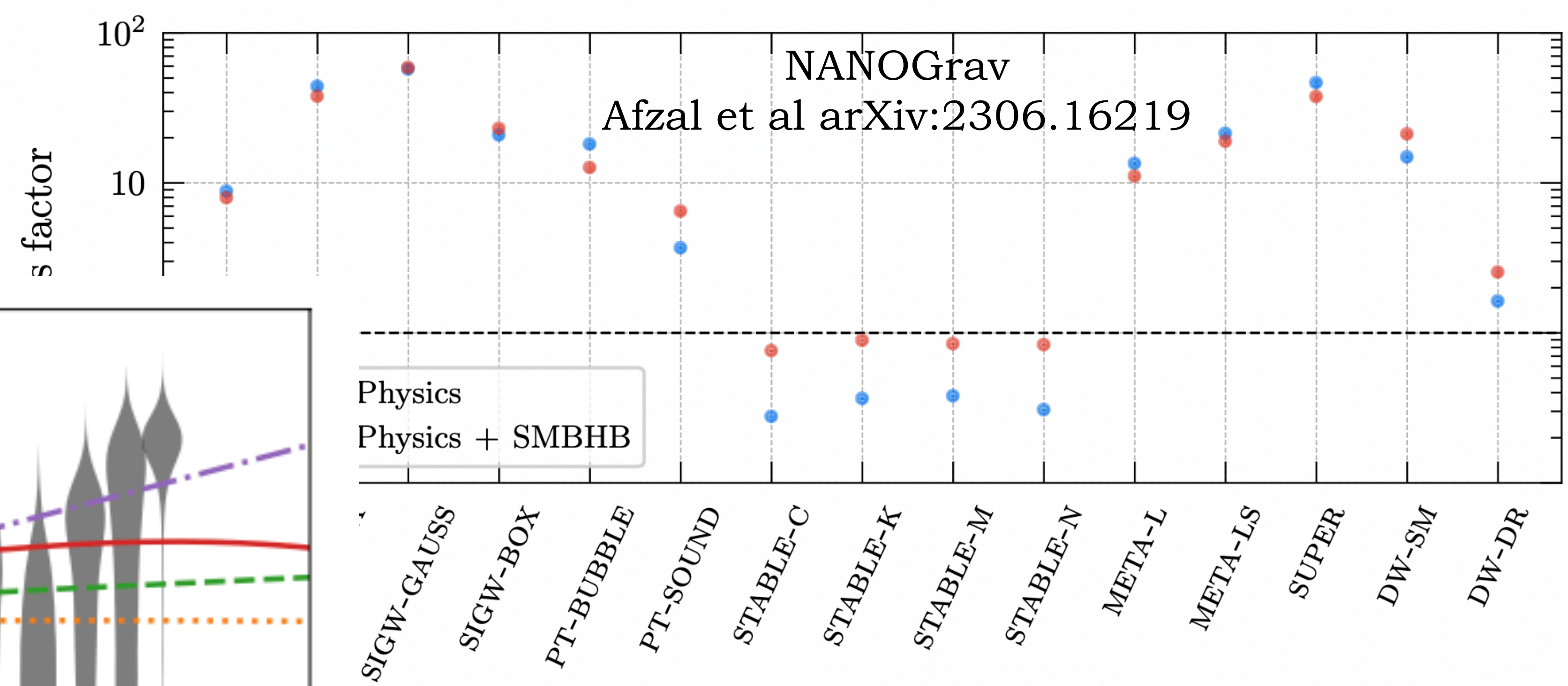


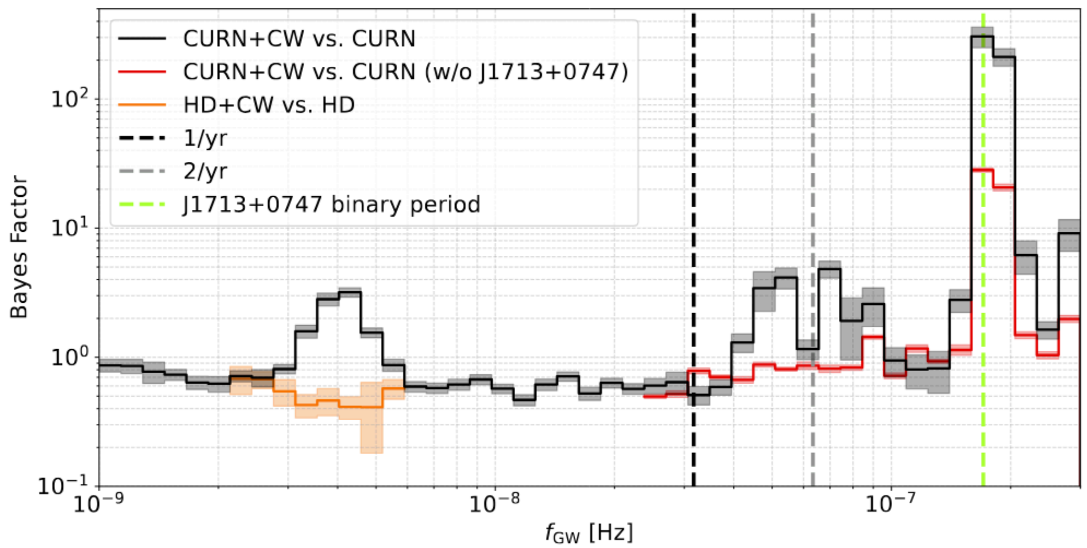
Antoniadis et al arXiv:2306.16227



Anisotropy is the key observable

Pulsar timing arrays

Maybe already some evidence for a continuous wave emission around 4-5 nanoHz in both EPTA and NanoGrav



NanoGrav, arXiv:2306.16222

Figure 1. Savage-Dickey Bayes factors for the CW+CURN model versus the CURN model as a function of frequency (black). Also shown are Bayes factors when excluding PSR J1713+0747 (red, only computed for $f_{GW} > 24$ nHz) and Bayes factors based on a resampled posterior that takes into account the presence of HD correlations in the common red noise process, i.e., CW+HD versus HD (orange, only computed for 2.1 nHz $< f_{GW} <$ 5.9 nHz). Shaded regions show the 1-σ uncertainties.

EPTA, arXiv:2306.16226

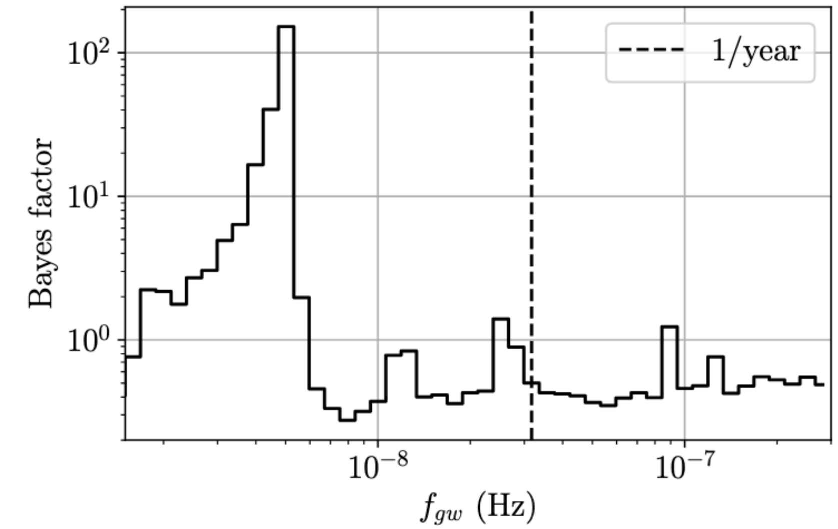


Fig. 4: Bayes factor for the model comparison PSRN+CURN+CGW (Earth term) vs PSRN+CURN for 50 logarithmically spaced frequency sub-bands in the region $f_{gw} \in [1.5, 320]$ nHz.

Constraints on anisotropy will improve with more pulsars and more sensitive instruments

Depta et al, arXiv:2407.14460

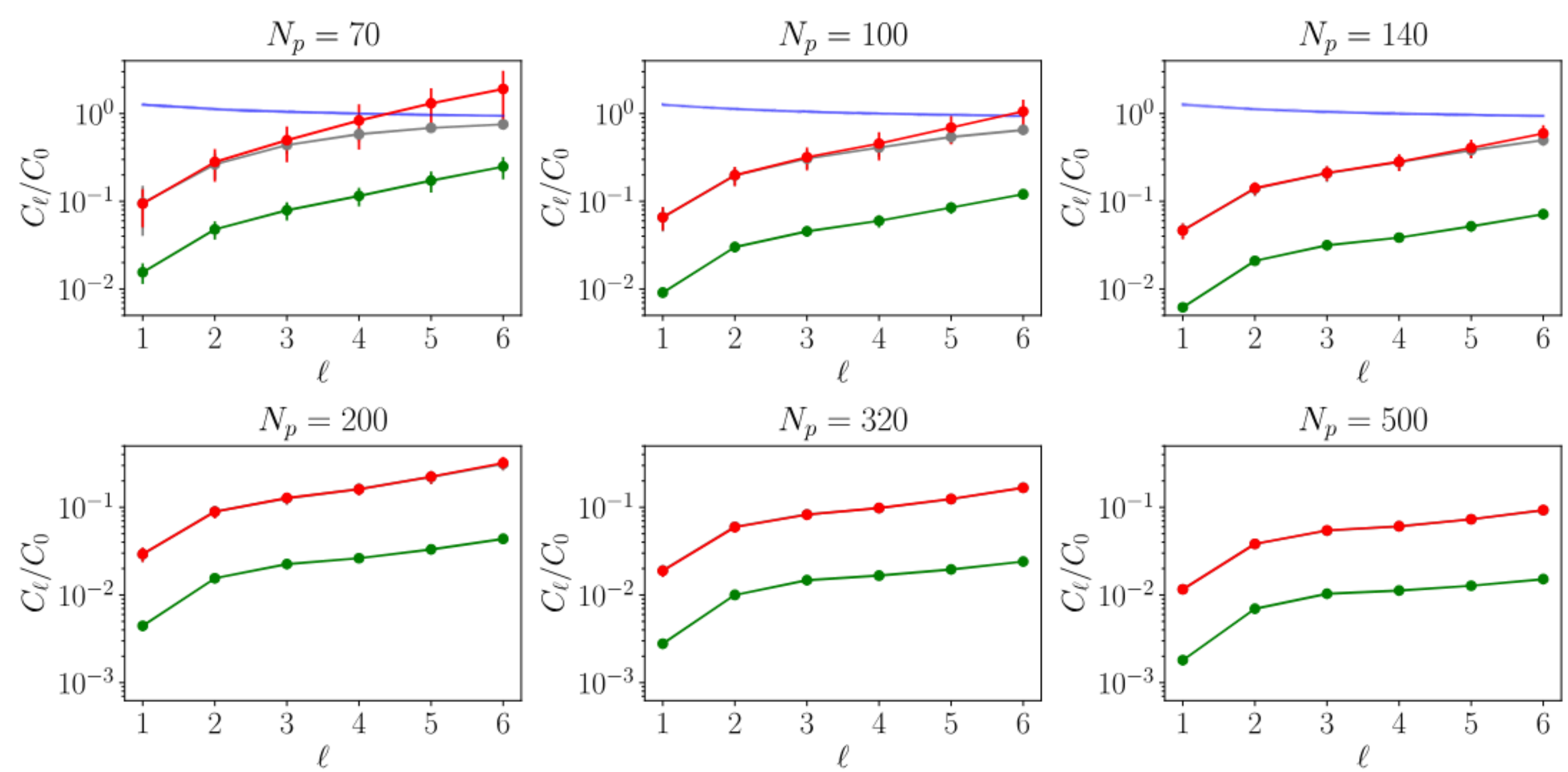


FIG. 4. 95% C.L. upper bounds on C_ℓ assuming EPTA-like noise (red) and SKA-like noise (green) for different numbers of pulsars and $T_{obs} = 15$ yr. We sample 10 realizations of noise and pulsar locations in the sky (with uniform distribution on the sphere). The gray lines indicate the result adopting the prior $|c_{\ell m}| < 5/(4\pi)$, while the blue reports the upper bound just drawing from the same uniform prior. In the bottom row, due to the increased sensitivity, such a prior plays no role.

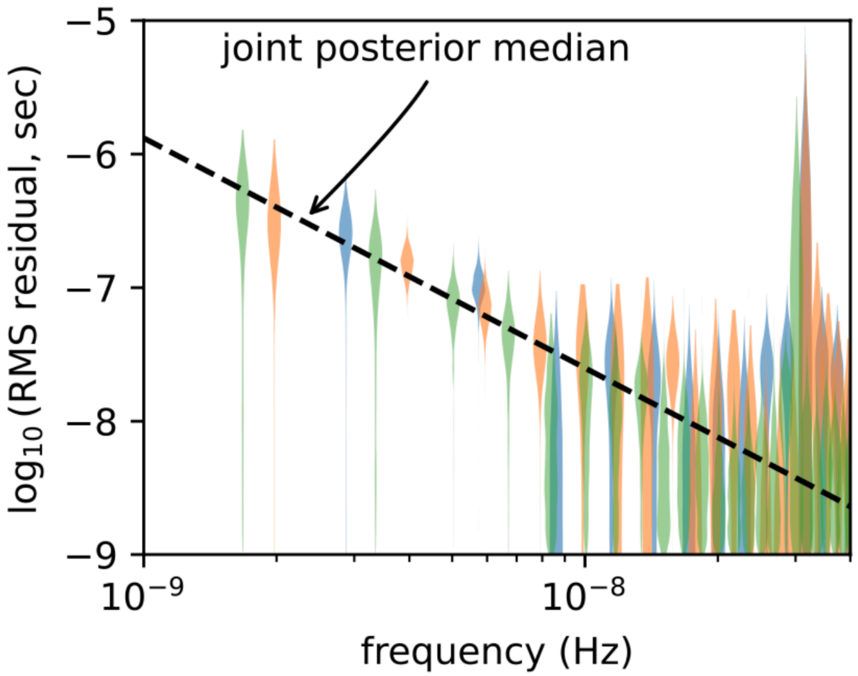
Talk by Jimenez Cruz on the measurement of the kinematic dipole

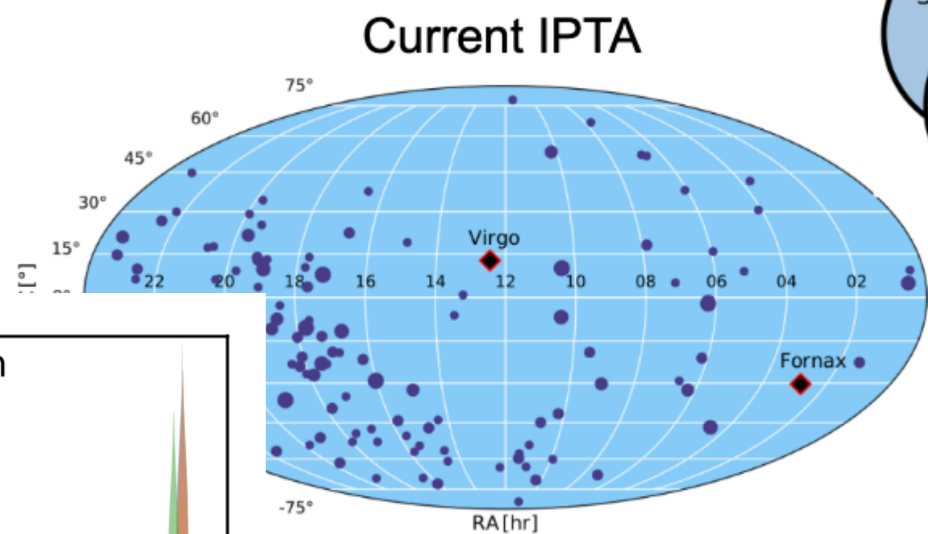
Pulsar timing arrays

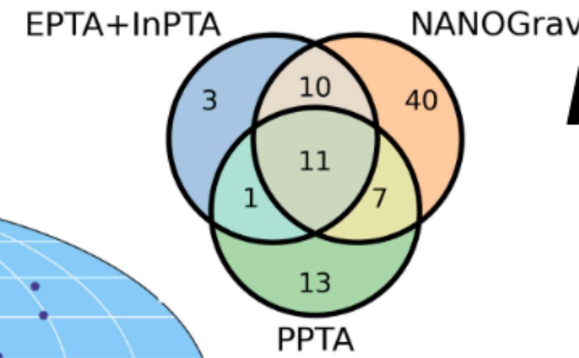
Slide from Gilles Theureau
https://indico.cern.ch/event/1267450/contributions/5887065/attachments/2872403/5029414/PTA_Noirmoutier2024.pdf

Full IPTA DR3 = 125 pulsars

IPTA Collaboration, arXiv:2309.00693





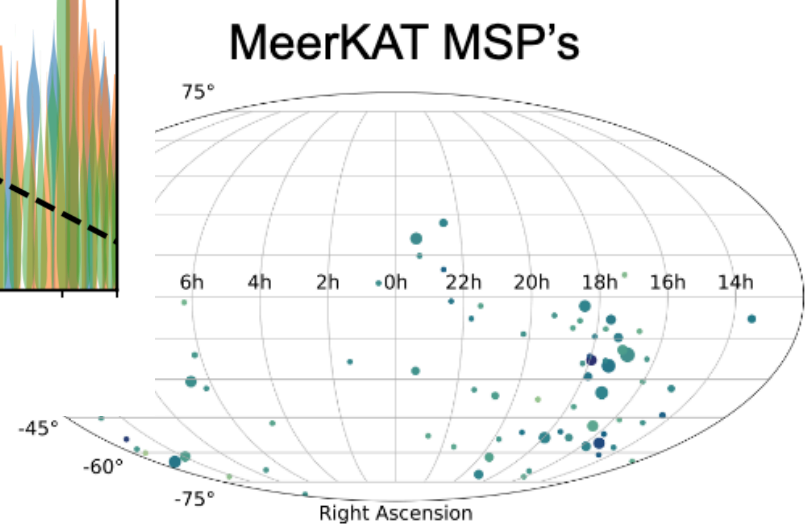


From PTA's to IPTA

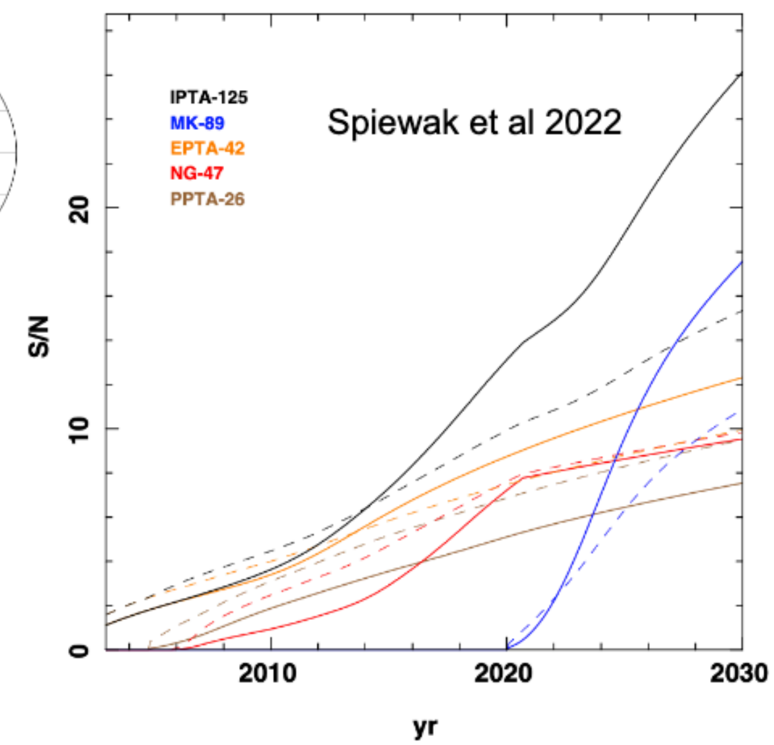
(Verbiest et al 2024)

$$S/N \propto N_{psr} \sqrt{T} \left(\frac{A\sqrt{C}}{\sigma} \right)^{3/13}$$

GW signal amplitude; number of pulsars; time span; timing precision; cadence

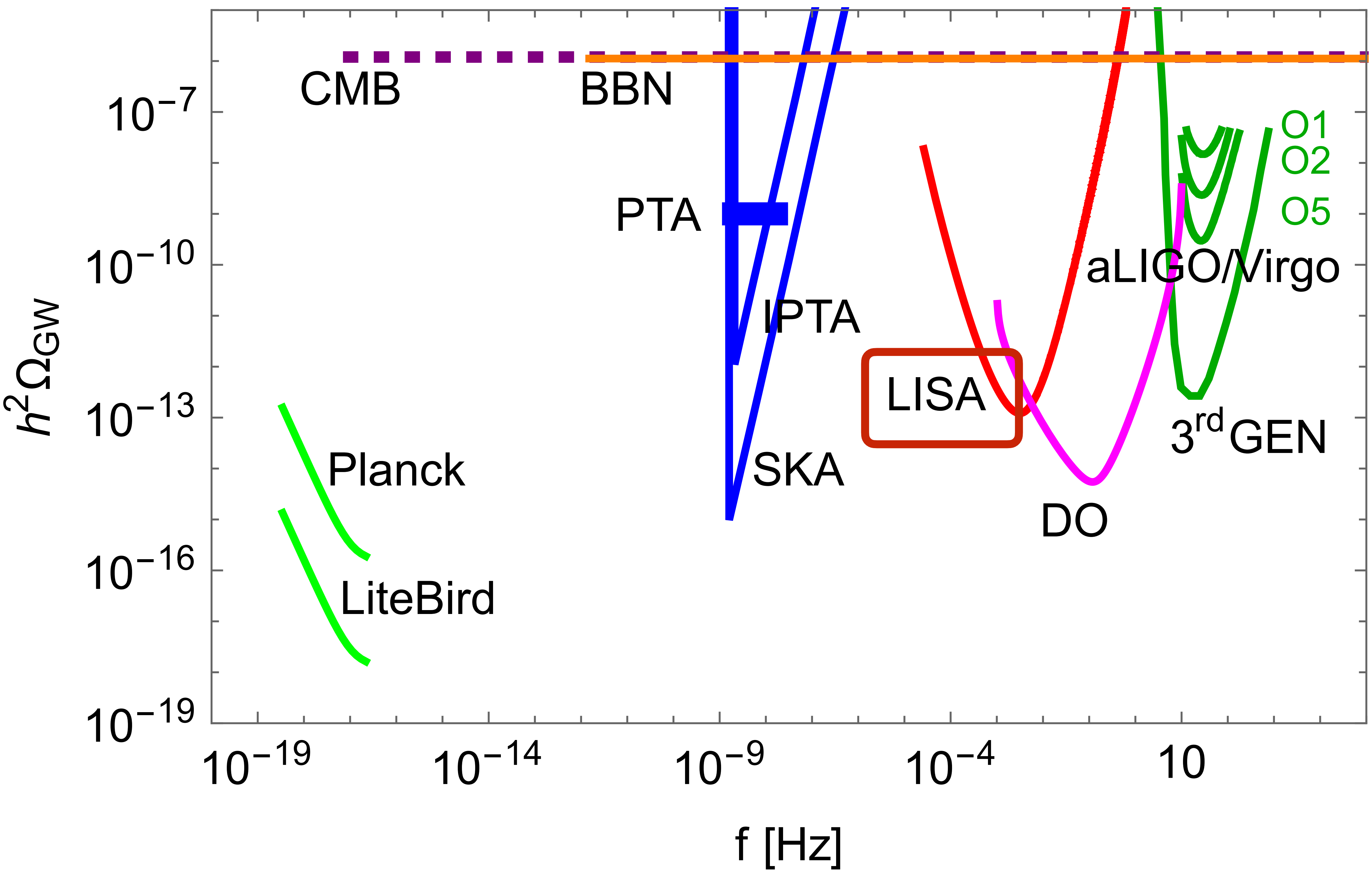


Miles et al 2023



reach the 5-σ level by summer 2025?

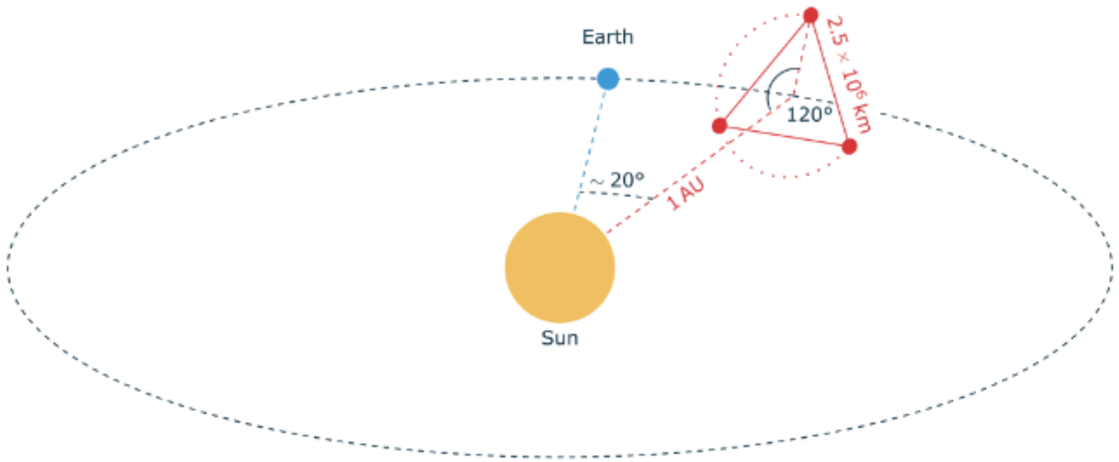
LISA

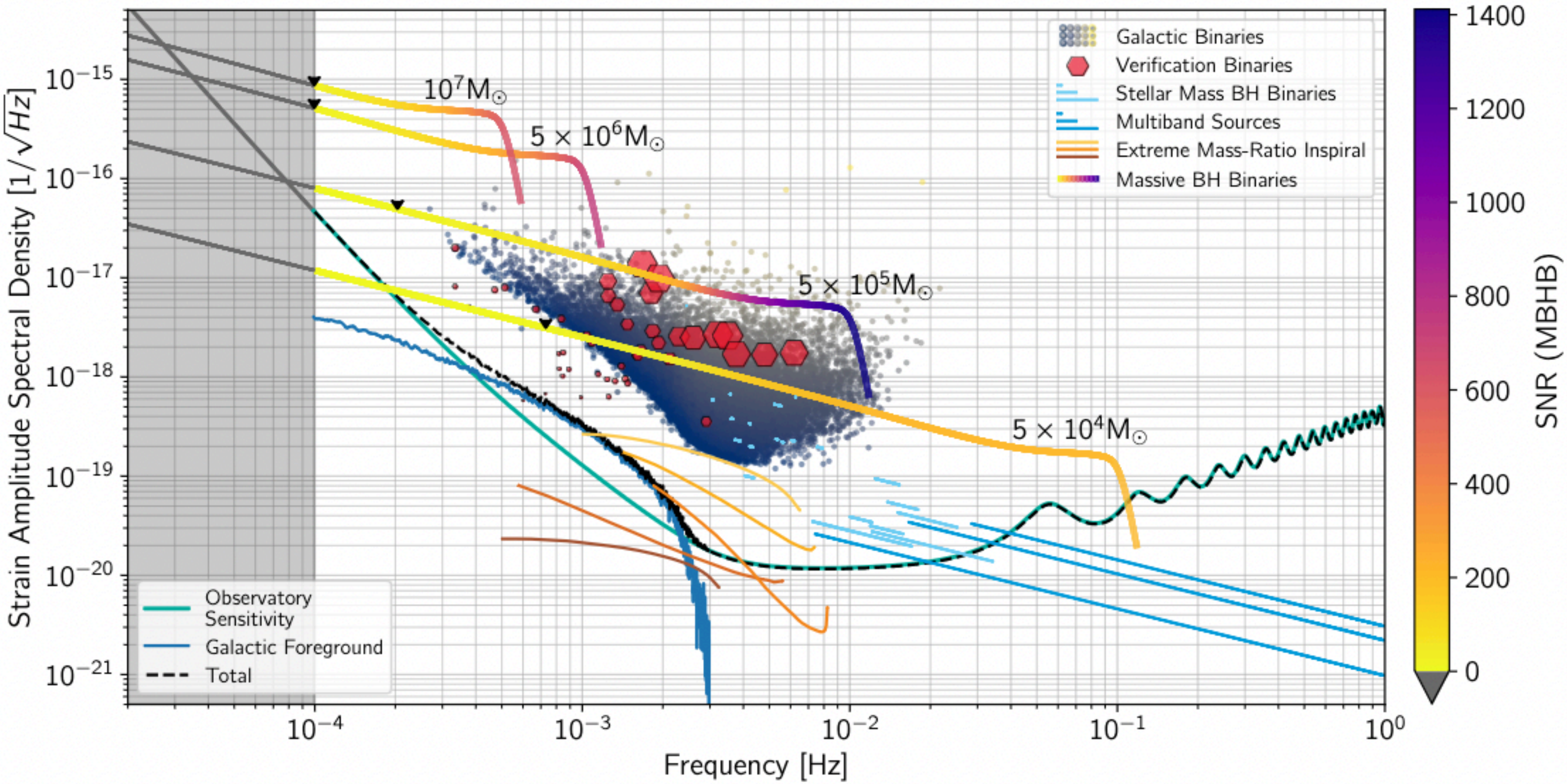


CMB
BBN
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
$h^2\Omega_{GW}$
PTA
IPTA
SKA
LISA
DO
aLIGO/Virgo
O1
O2
O5
3rd GEN
Planck
LiteBird
10^{-19}
10^{-14}
10^{-9}
10^{-4}
10
f [Hz]

LISA

A completely new frequency window with an amazing richness of expected sources and far-reaching scientific potential, but also new challenges for data analysis





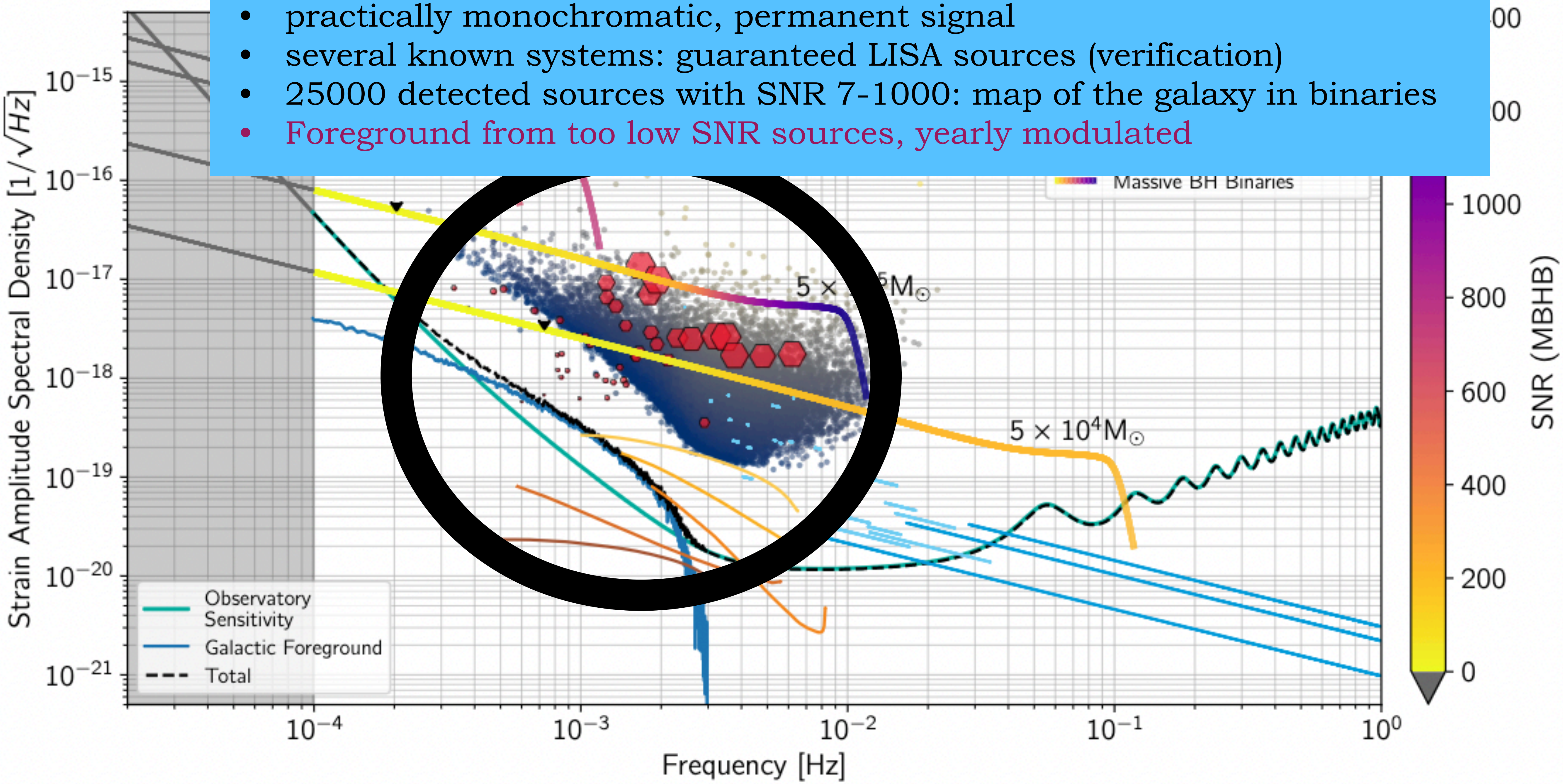
LISA Red Book arXiv:2402.07571

LISA

Compact binaries in the galaxy

$$M_c = 1\,\mathrm{M}_\odot \qquad \tau = 10^5\,\text{years} \qquad \longrightarrow \qquad f = 3\,\mathrm{mHz}$$

- practically monochromatic, permanent signal
- several known systems: guaranteed LISA sources (verification)
- 25000 detected sources with SNR 7-1000: map of the galaxy in binaries
- Foreground from too low SNR sources, yearly modulated



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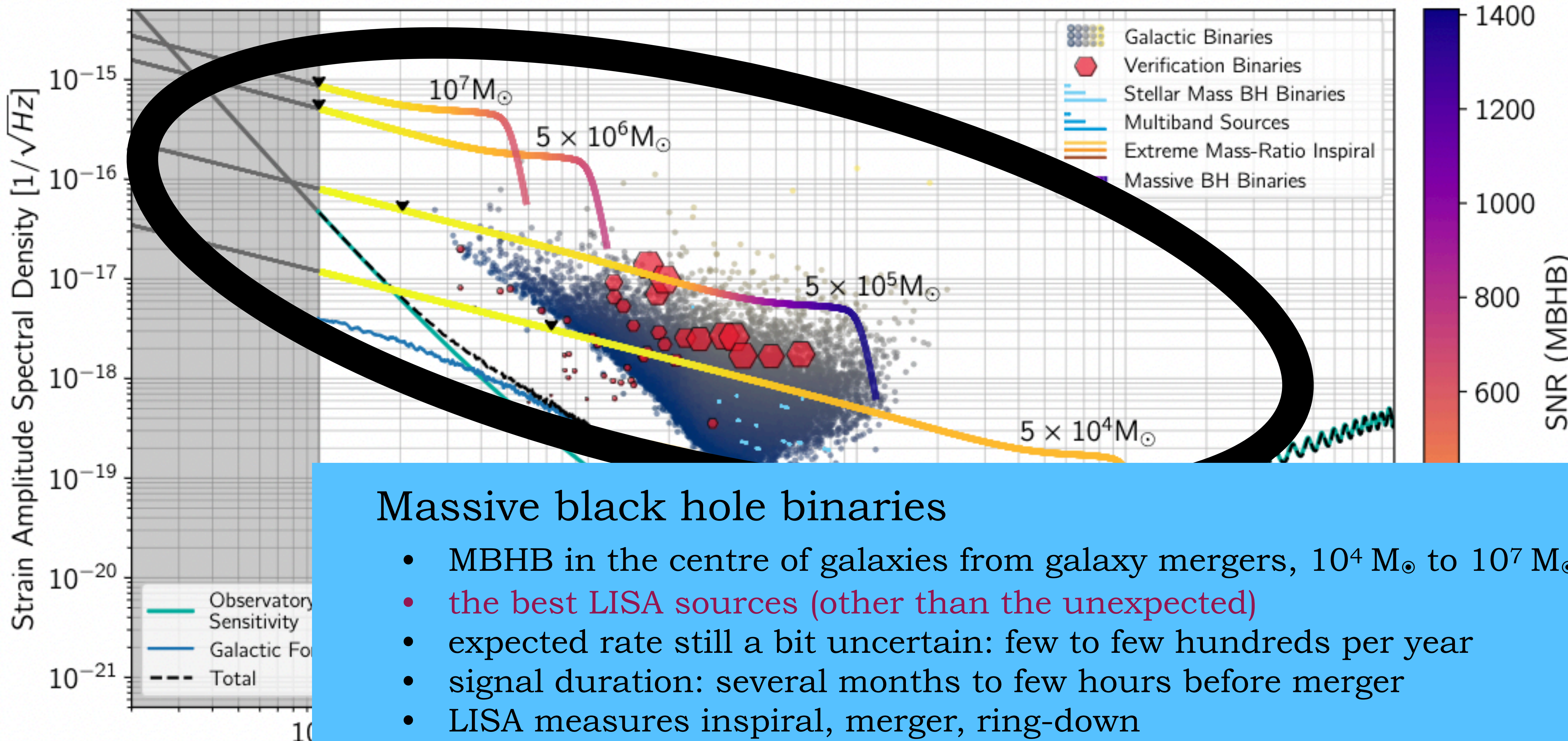


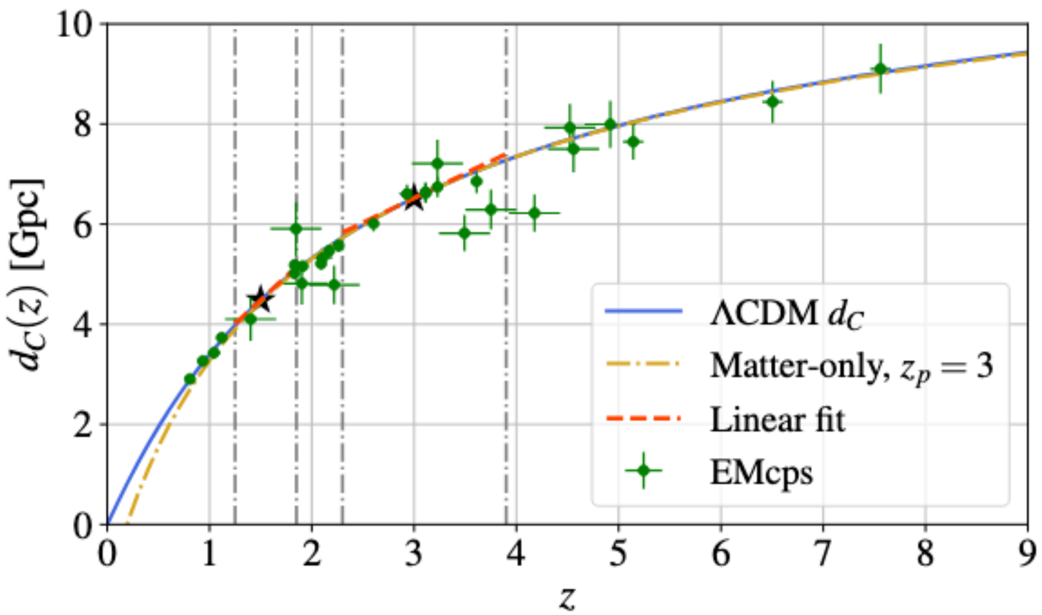
LISA Red Book arX

Massive black hole binaries

- MBHB in the centre of galaxies from galaxy mergers, $10^4 M_\odot$ to $10^7 M_\odot$
- the best LISA sources (other than the unexpected)
- expected rate still a bit uncertain: few to few hundreds per year
- signal duration: several months to few hours before merger
- LISA measures inspiral, merger, ring-down
- Probe the seeds of MBH, their accretion, the galaxy merger tree
- Tests of General Relativity
- **EM counterparts -> Cosmology at high redshift**

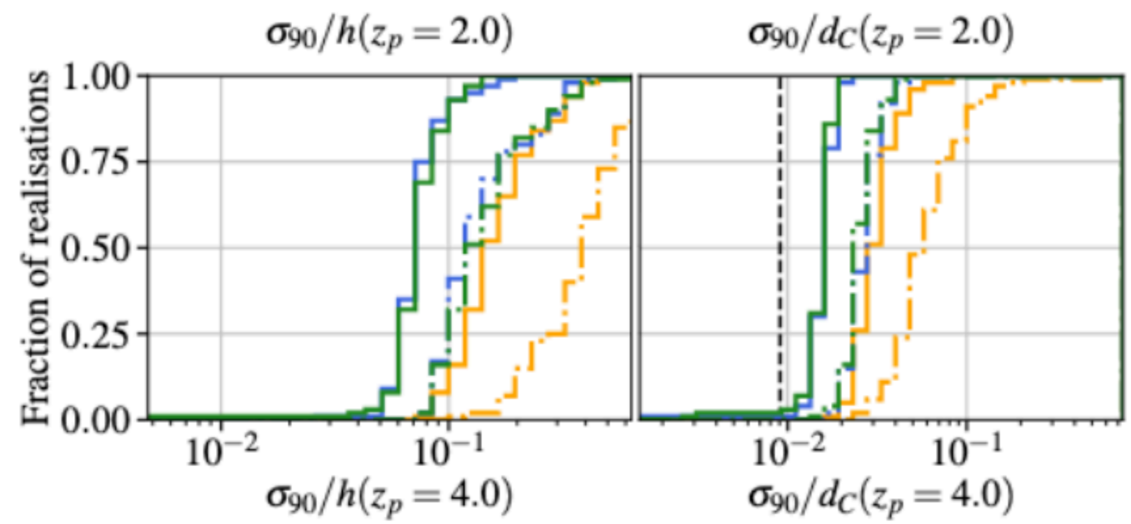
Mangiagli et al arXiv:2312.04632

LISA

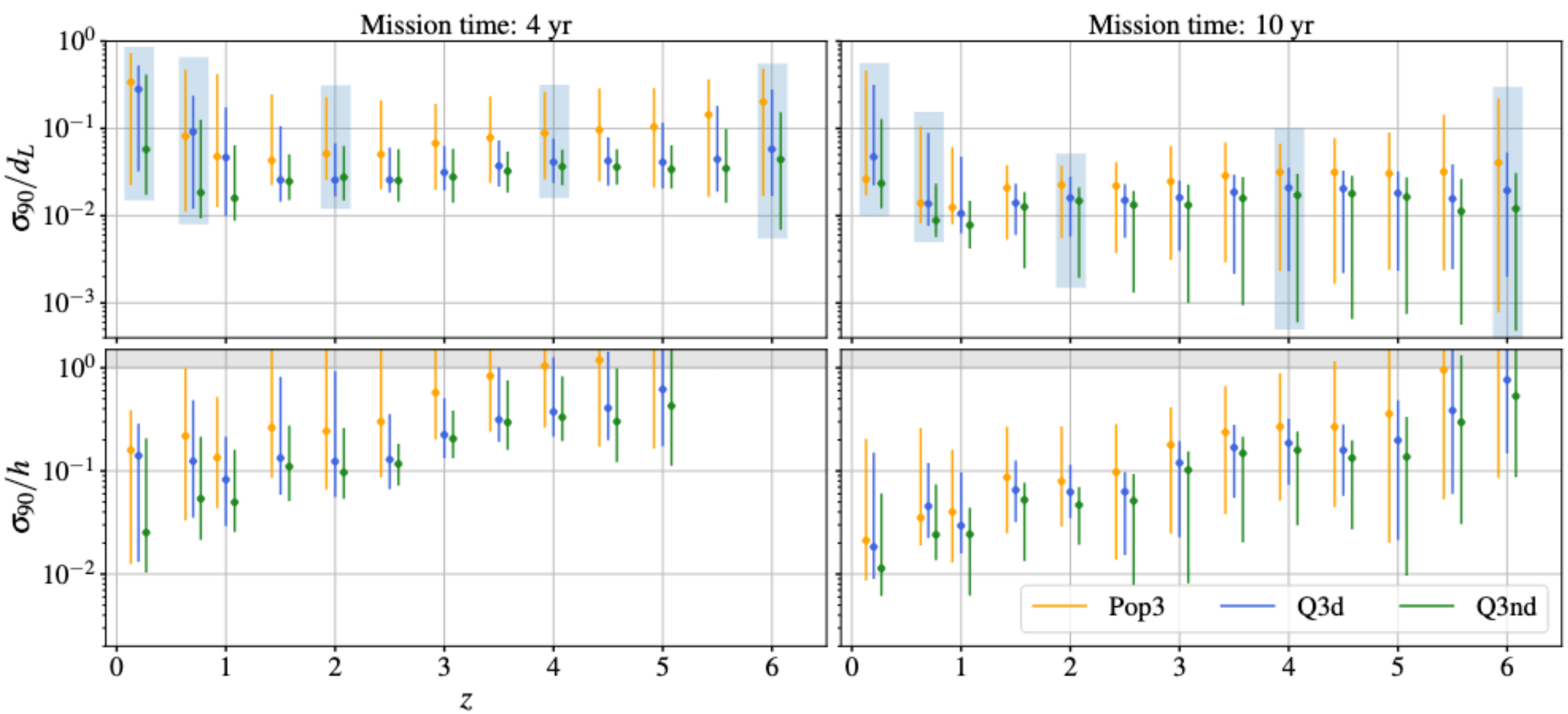


- **EM counterparts -> Cosmology at high redshift**

Assuming matter only, at redshift z = 4:
20% error on Hubble parameter
4% on luminosity distance
in 50% of the universe realisations

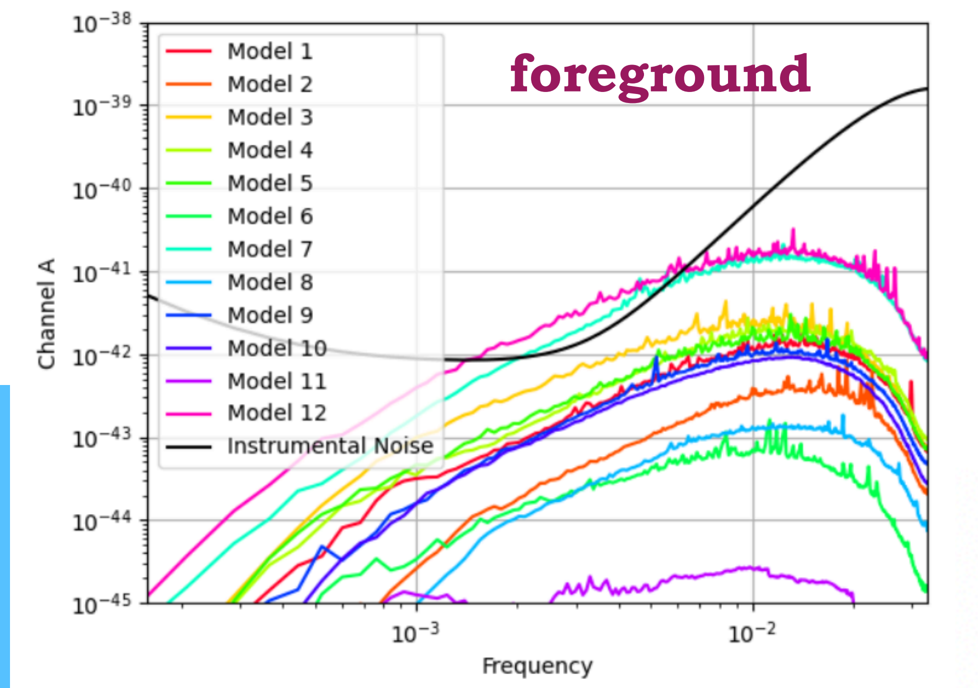


Assuming a model independent spline interpolation, at redshift z ≤ 3:
10% error on Hubble parameter (2% in 10 years)
5% on luminosity distance (1% in 10 years)
in 50% of the universe realisations



LISA

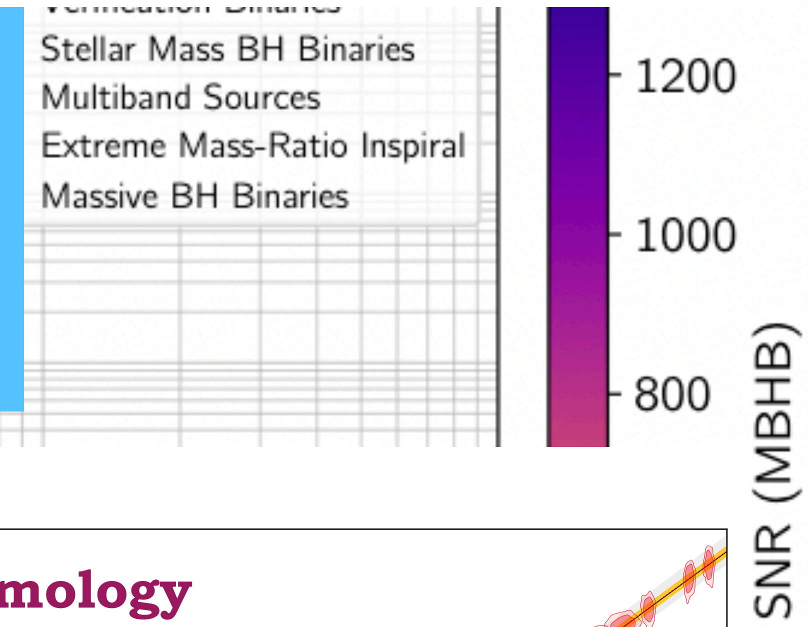
Pozzoli et al arXiv:2302.07043
Piarulli et al arXiv:2410.08862

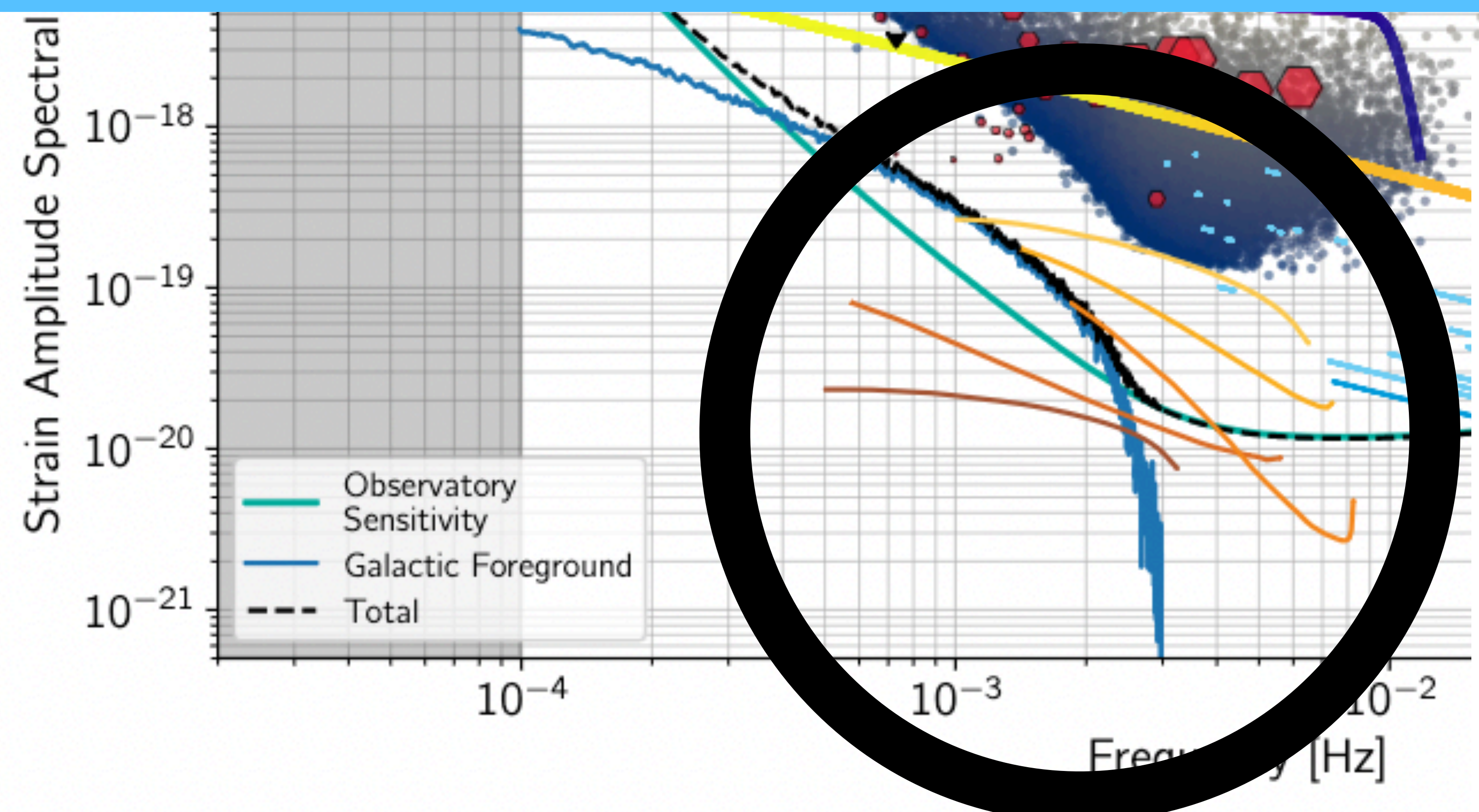


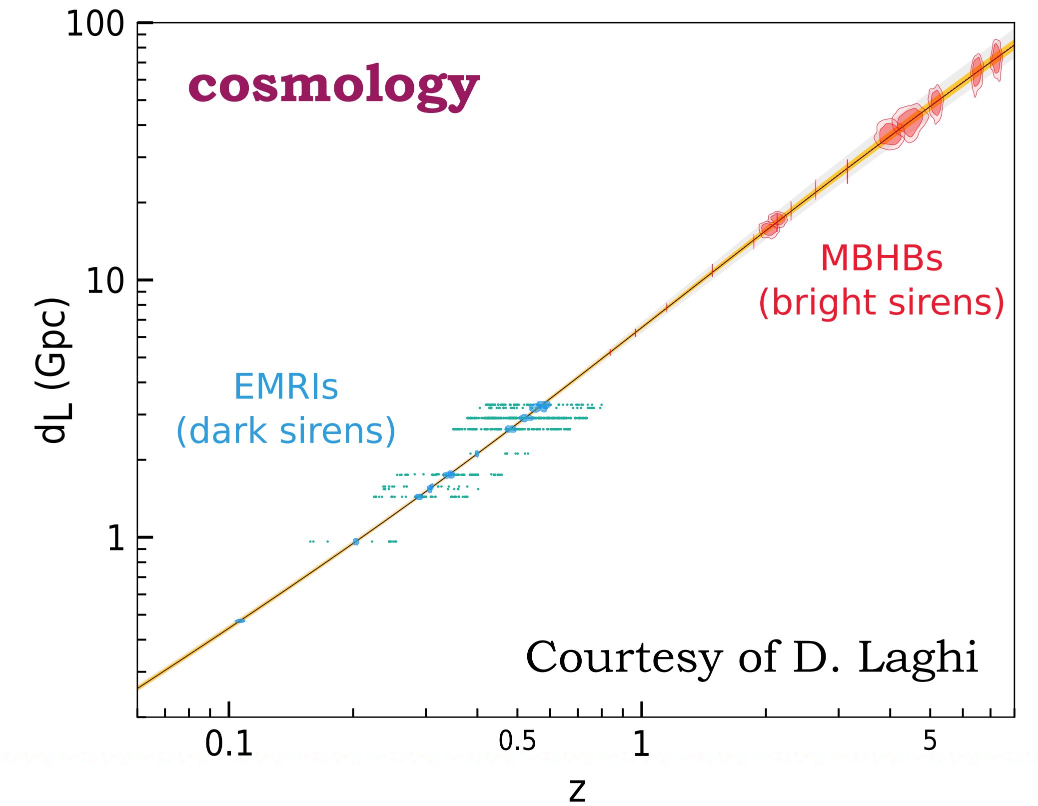
Extreme mass ratio inspirals

$m_1 = 10 - 60\, M_\odot \qquad m_2 = 10^5 - 10^6\, M_\odot$

- Inspiral and merger of a stellar mass BH into massive BHs
- Thousands of orbits, precise determination of binary parameters
- Rate very uncertain
- Test the environment of dense nuclear clusters in galaxies
- Mapping of space-time around MBH, tests of General Relativity
- **Dark sirens, but also possible foreground for cosmology**







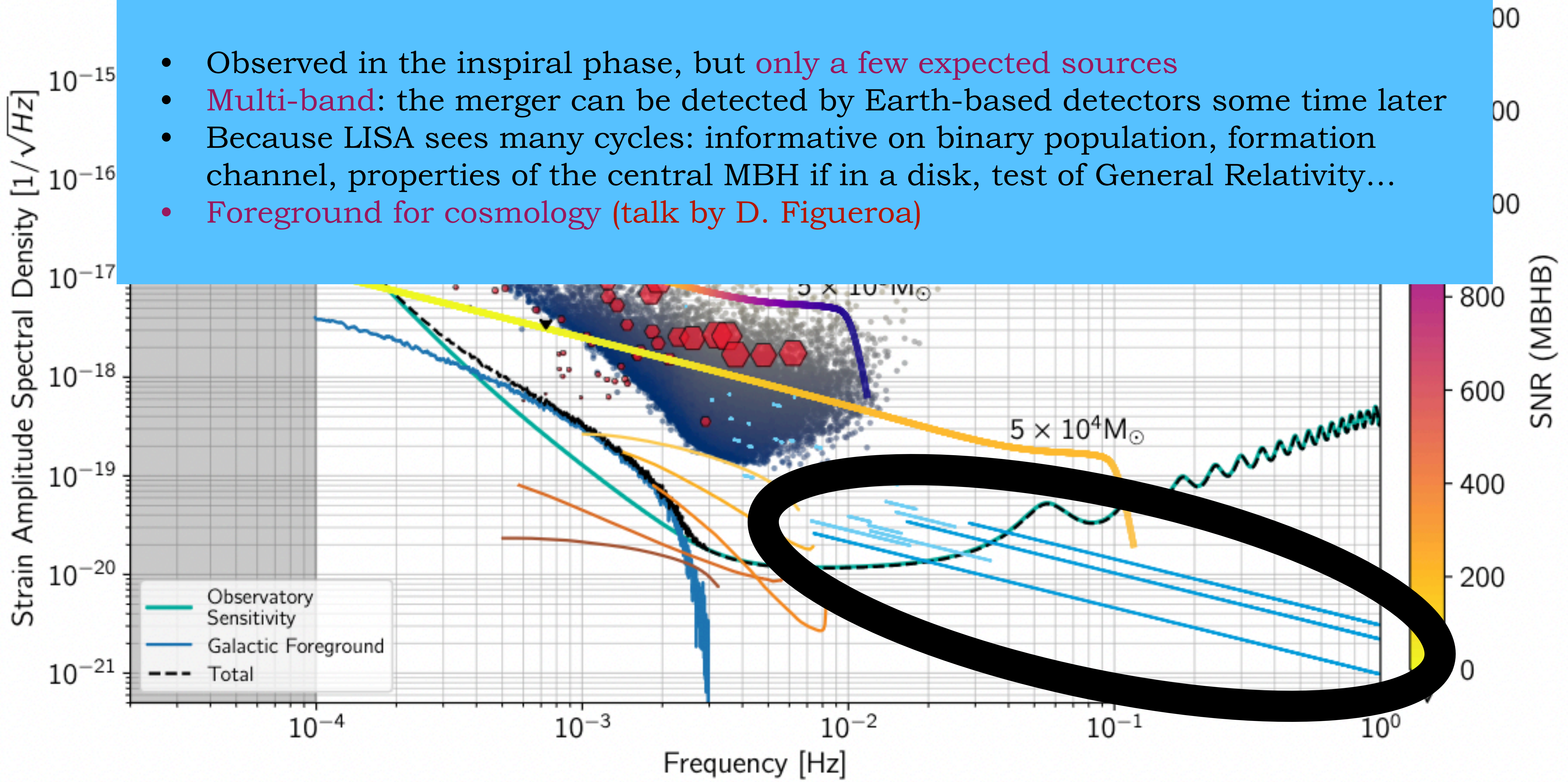
LISA Red Book arXiv:2402.07571

LISA

Stellar mass Black Hole Binaries

$$M_c = 25\,\mathrm{M}_\odot \quad \tau = 10\,\mathrm{year} \quad \longrightarrow \quad f = 0.01\,\mathrm{Hz}$$

- Observed in the inspiral phase, but only a few expected sources
- Multi-band: the merger can be detected by Earth-based detectors some time later
- Because LISA sees many cycles: informative on binary population, formation channel, properties of the central MBH if in a disk, test of General Relativity…
- Foreground for cosmology (talk by D. Figueroa)



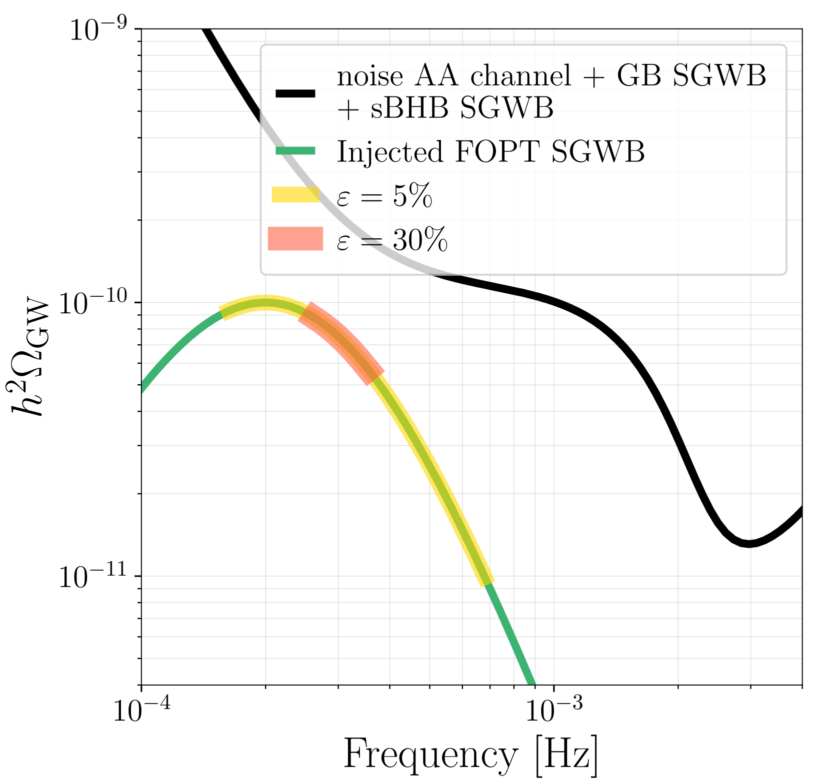
LISA Red Book arXiv:2402.07571

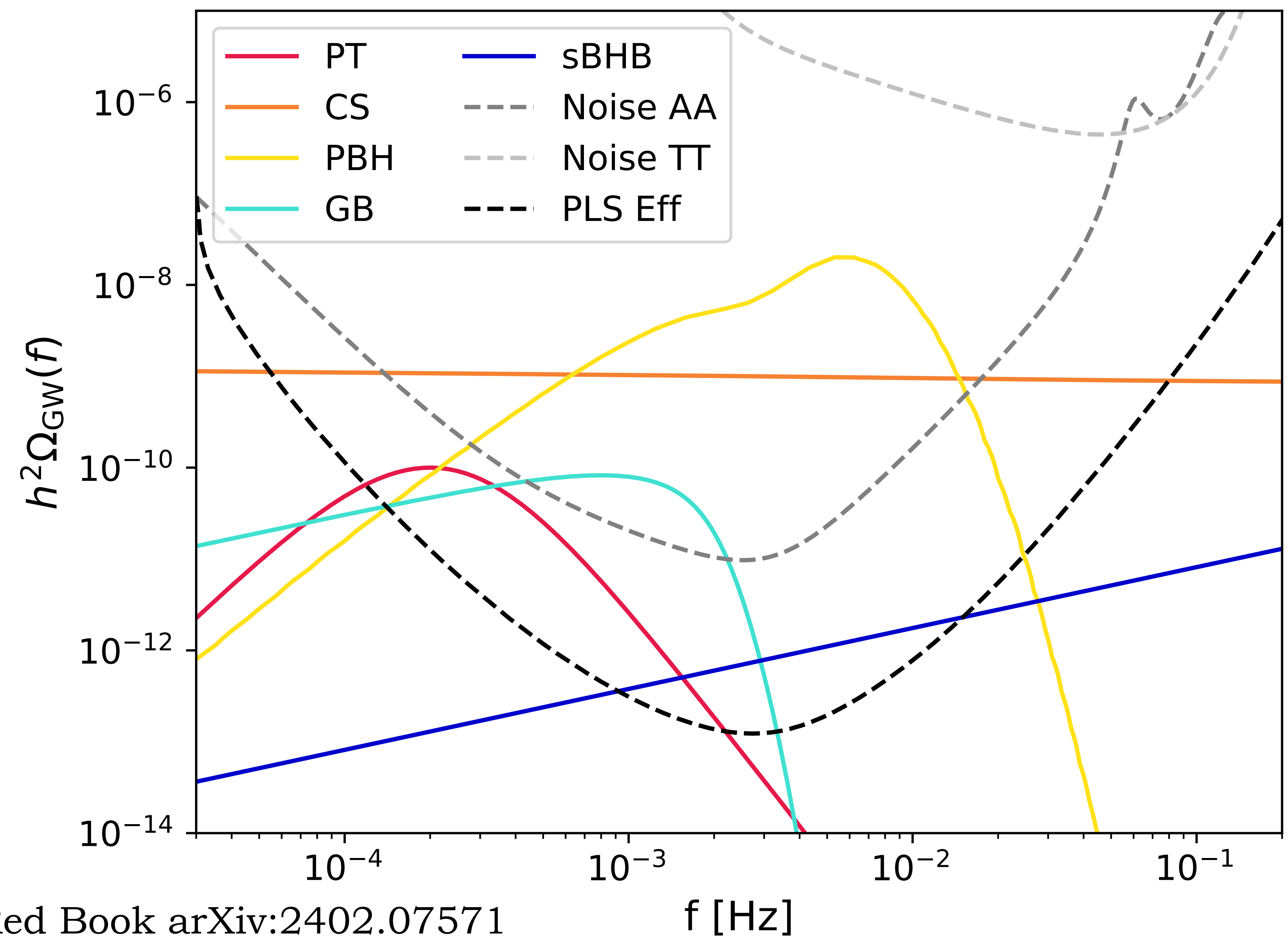
LISA

Stochastic GW background

- Possible access to many interesting phenomena and fundamental physics constraints, high potential for discoveries (talk by D. Figueroa)
- **Vey challenging to detect: how to separate it from the detector noise?**
- LISA frequency band -> EW scale
- PBH in the mass window in which they can be the totality of the Dark Matter

N. Karnesis, arXiv:1906.09027





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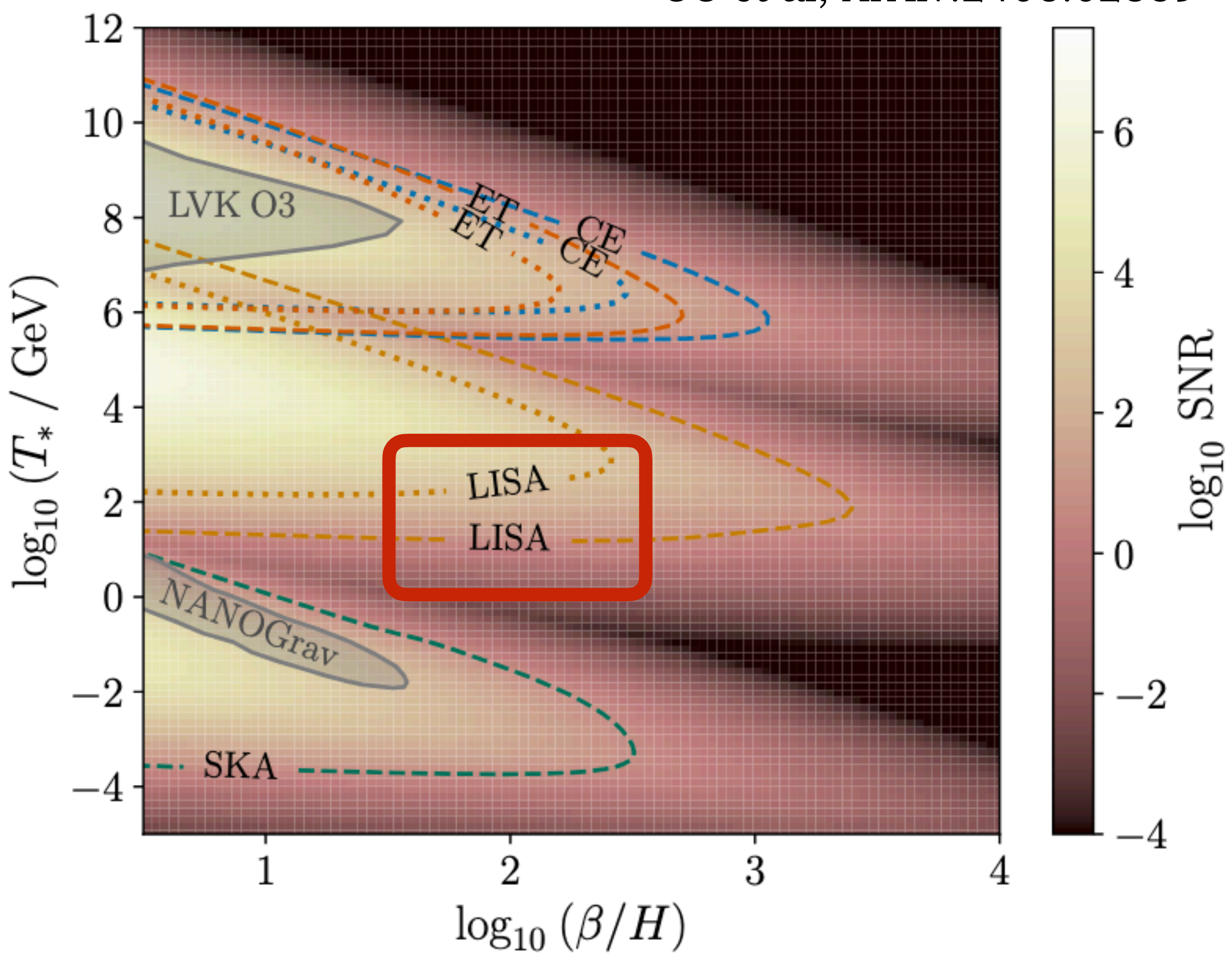
$$10^{-5}\,\mathrm{Hz} < f < 0.1\,\mathrm{Hz} \quad \longrightarrow \quad 10\,\mathrm{GeV} \lesssim T_* \lesssim 10^5\,\mathrm{GeV}$$

Talk by A. Rajantie
Talk by A. Roper Pol

CC et al, ArXiv:2406.02359



Parameter to which the signal amplitude is *inversely* proportional

Progress is needed:

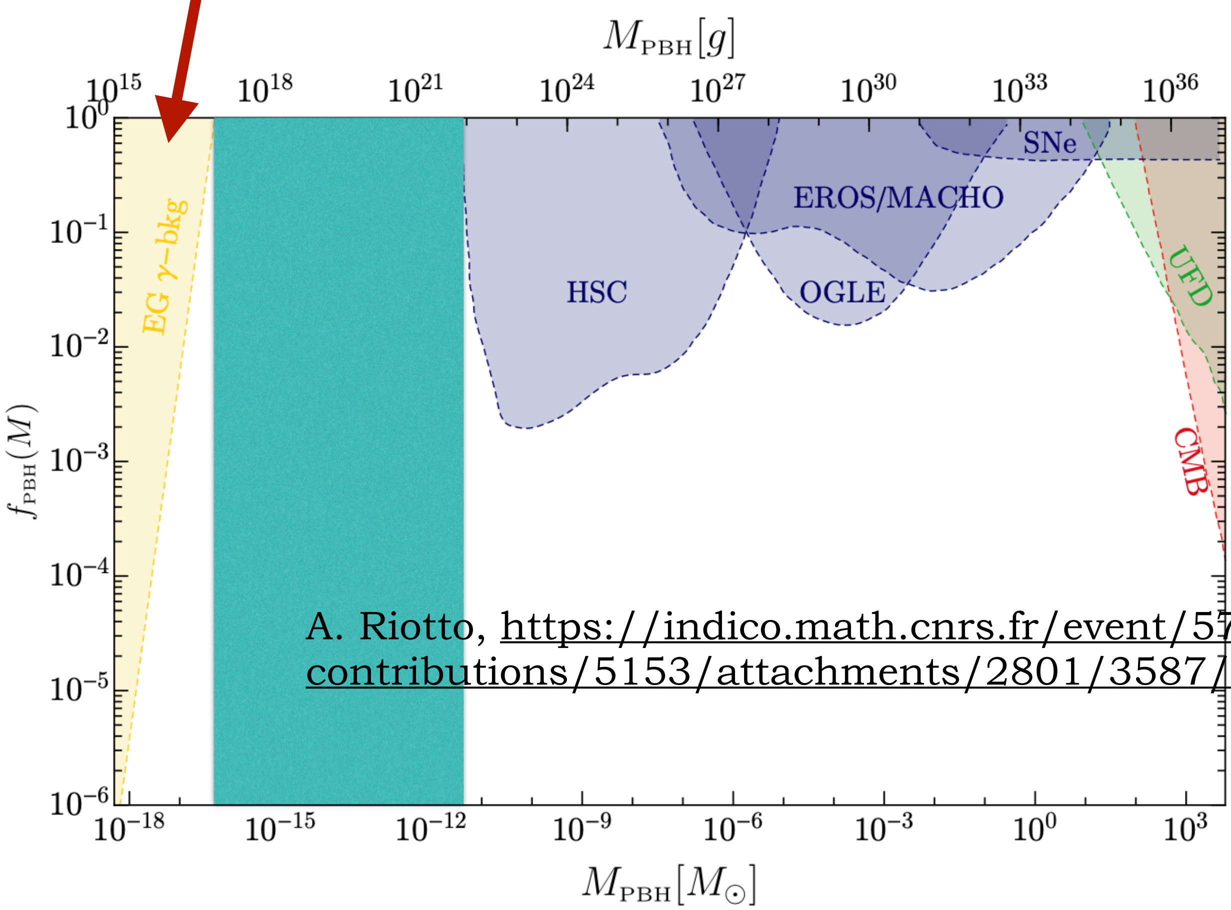
- One can *constrain* models but not *infer* them, because big degeneracy between signal characteristics and model parameters -> reduce degeneracy!
- Better prediction of the signal is required at every level: from the effective potential to the PT parameters to the bubble and fluid thermodynamics to the spectral shape of the signal
- Complementarity with colliders must be thoroughly studied

LISA

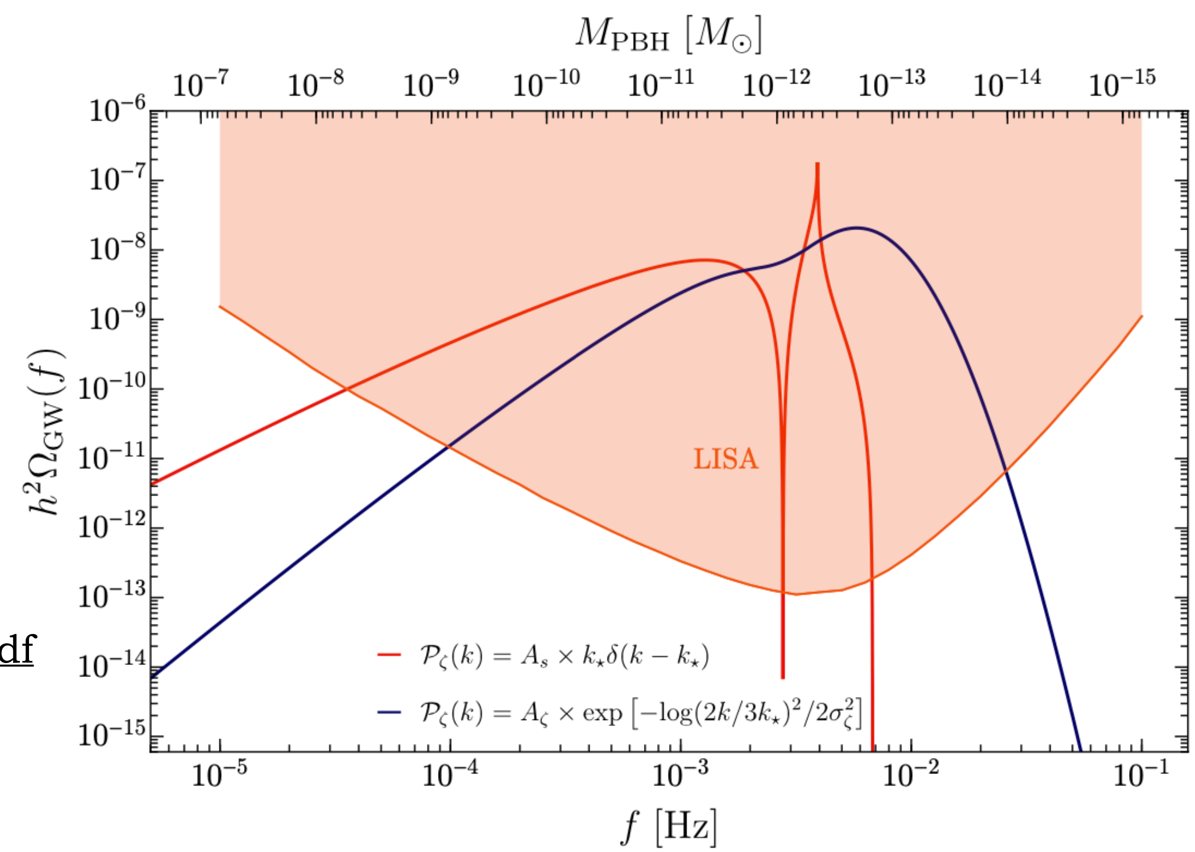
Stochastic GW background

- Possible access to many interesting phenomena and fundamental physics constraints, high potential for discoveries (talk by D. Figueroa)
- Vey challenging to detect: how to separate it from the detector noise?
- LISA frequency band -> EW scale
- **PBH in the mass window in which they can be the totality of the Dark Matter**

talk by S. Zell

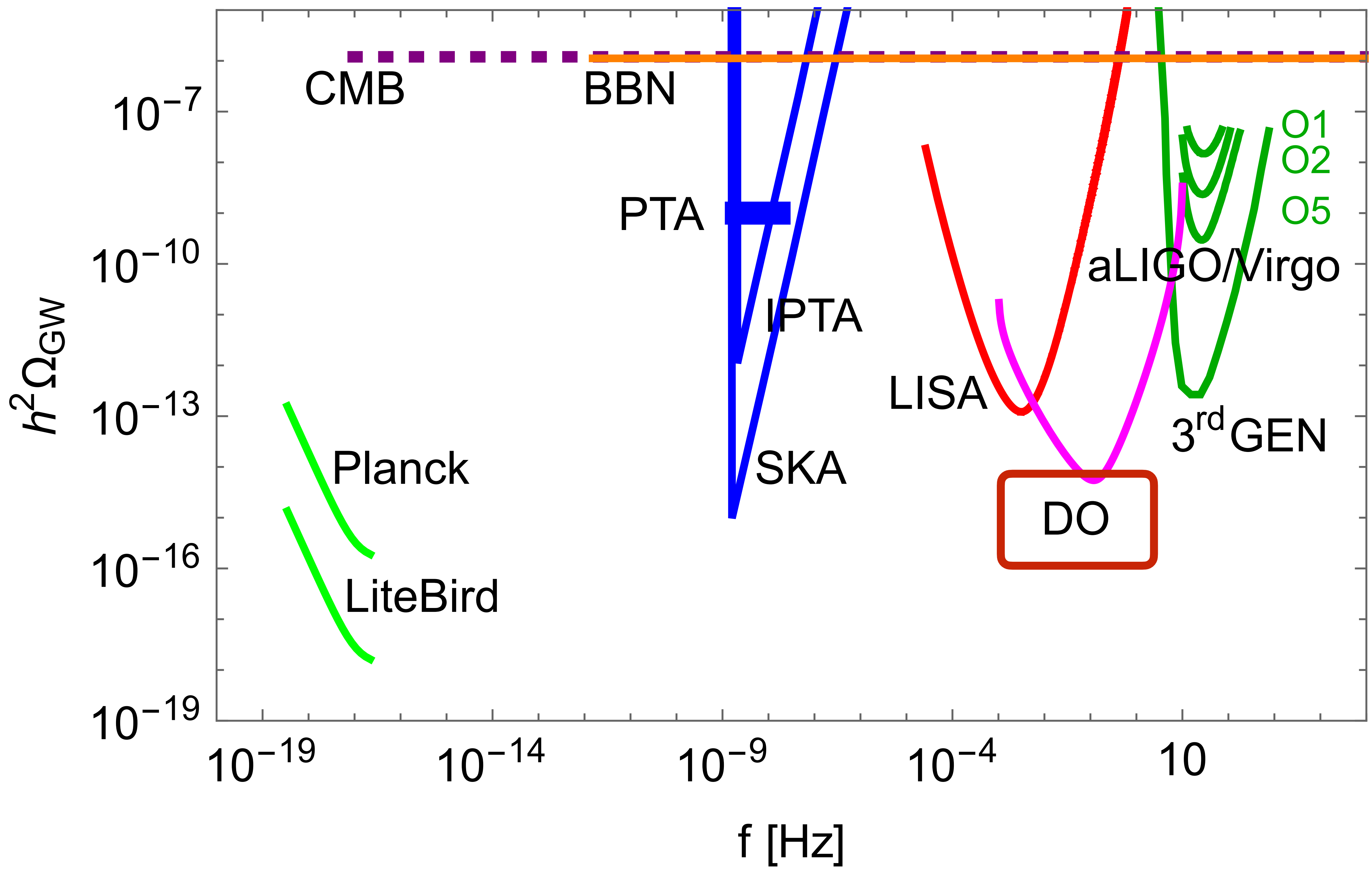


A. Riotto, https://indico.math.cnrs.fr/event/5766/contributions/5153/attachments/2801/3587/Paris2021.pdf



N. Bartolo et al, arXiv:1810.12218, arXiv:1810.12224

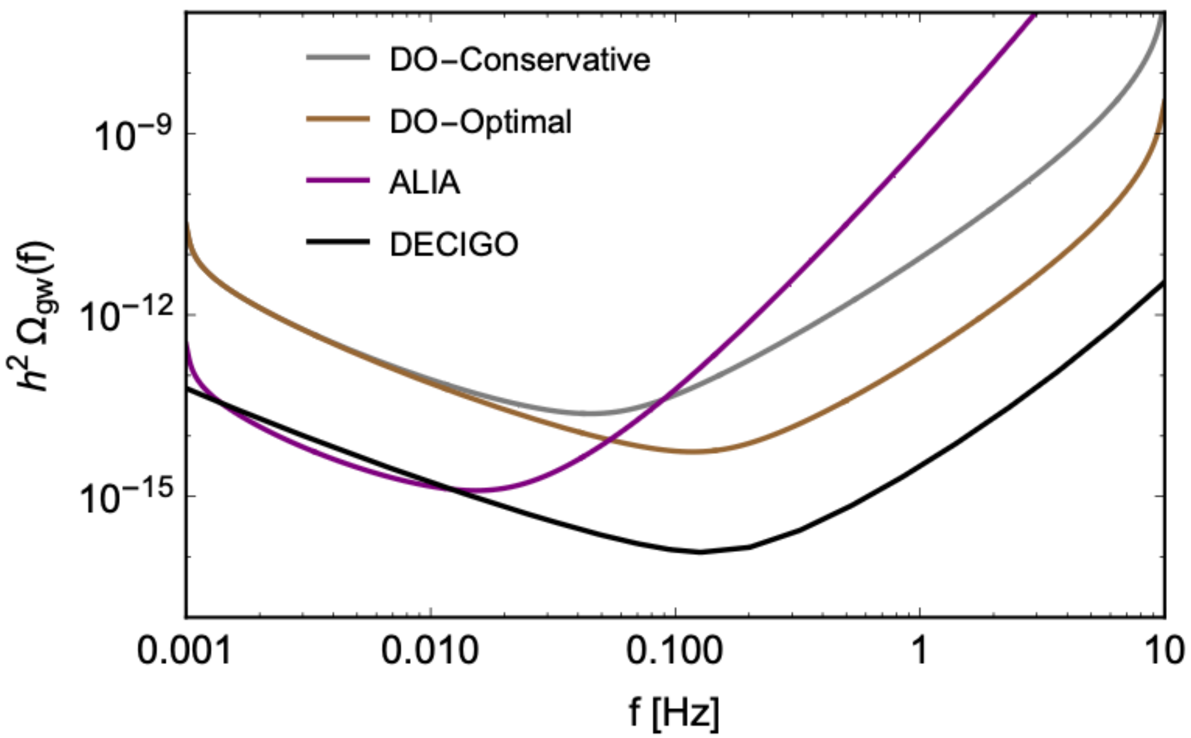
DeciHertz



CMB
BBN
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
$h^2\Omega_{GW}$
PTA
IPTA
SKA
Planck
LiteBird
LISA
DO
aLIGO/Virgo
O1
O2
O5
3^{rd}GEN
10^{-19}
10^{-14}
10^{-9}
10^{-4}
10
f [Hz]

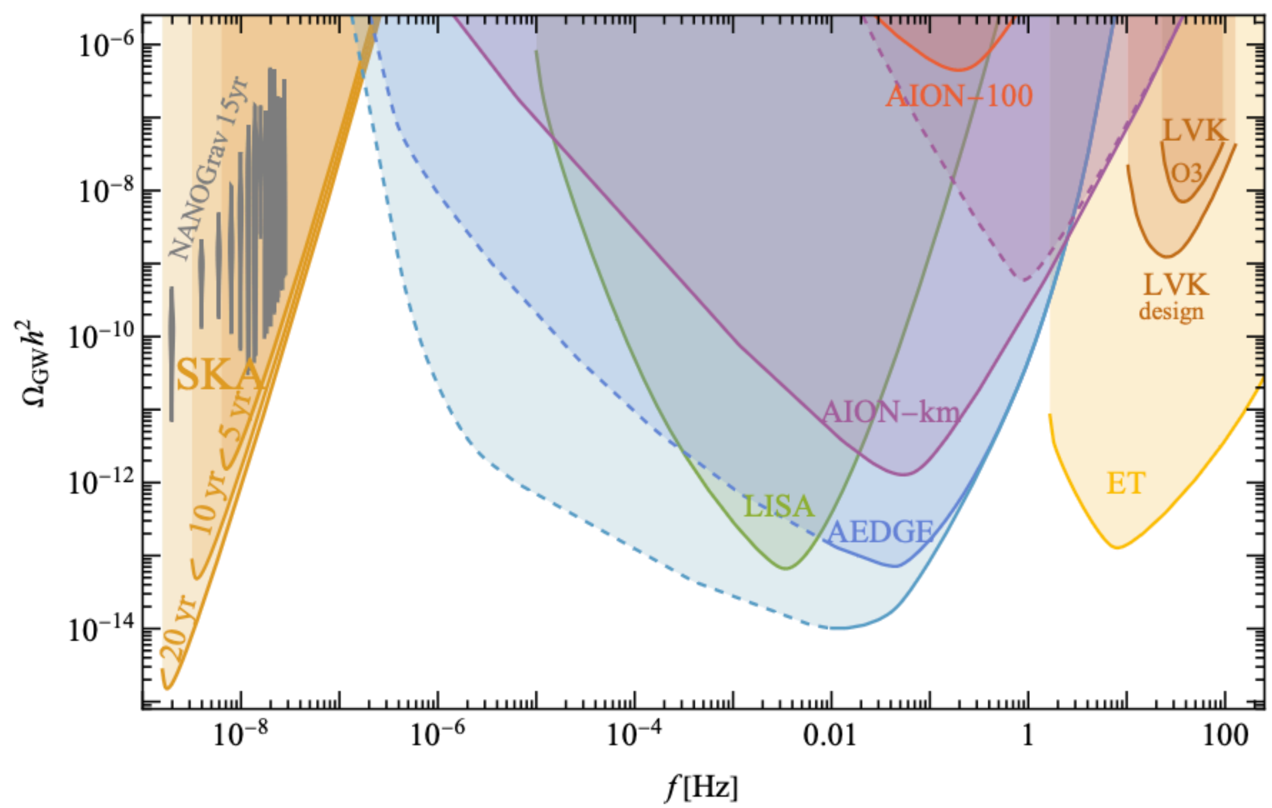
DeciHertz

Several detector concepts have been proposed

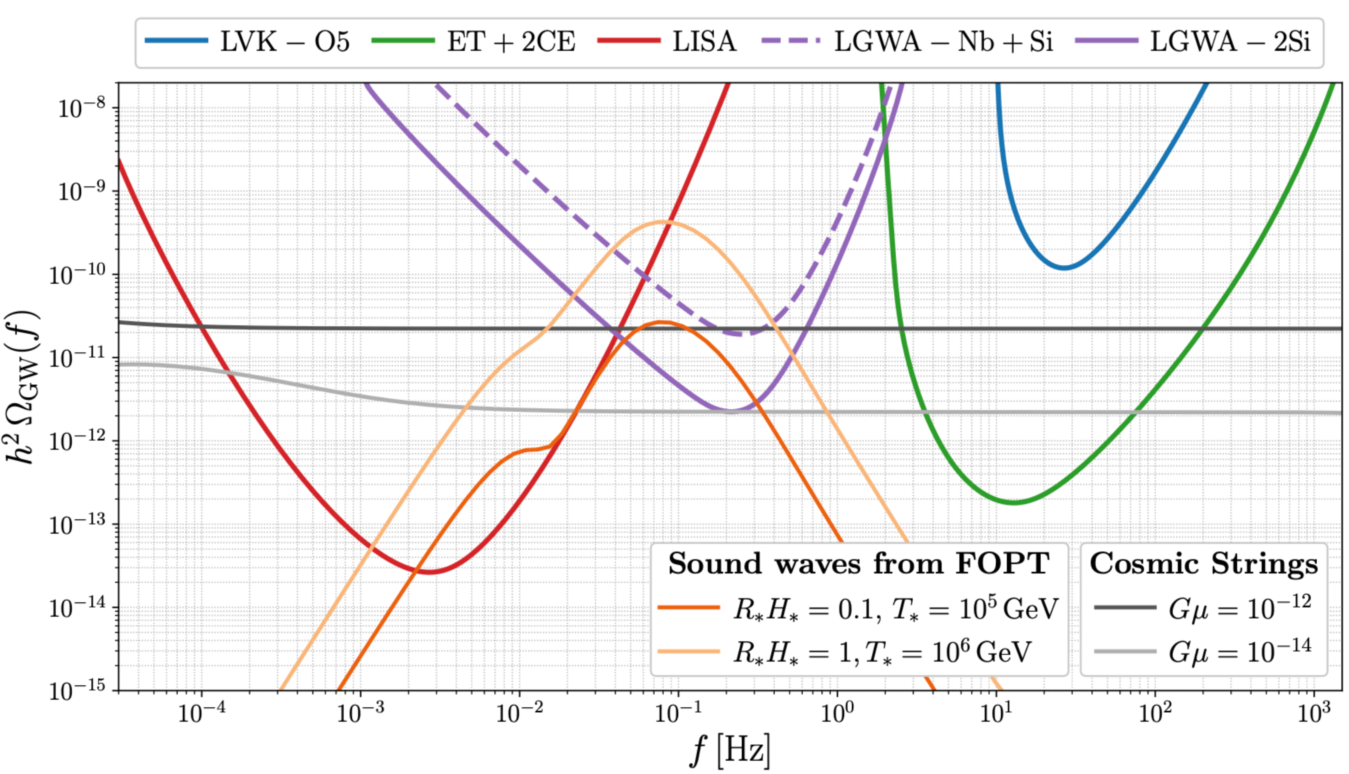


DECIGO-like interferometers
(e.g. M. Arca Sedda et al, arXiv:2104.14583)

Atom interferometry (e.g. Abend et al, arXiv:2310.08183)



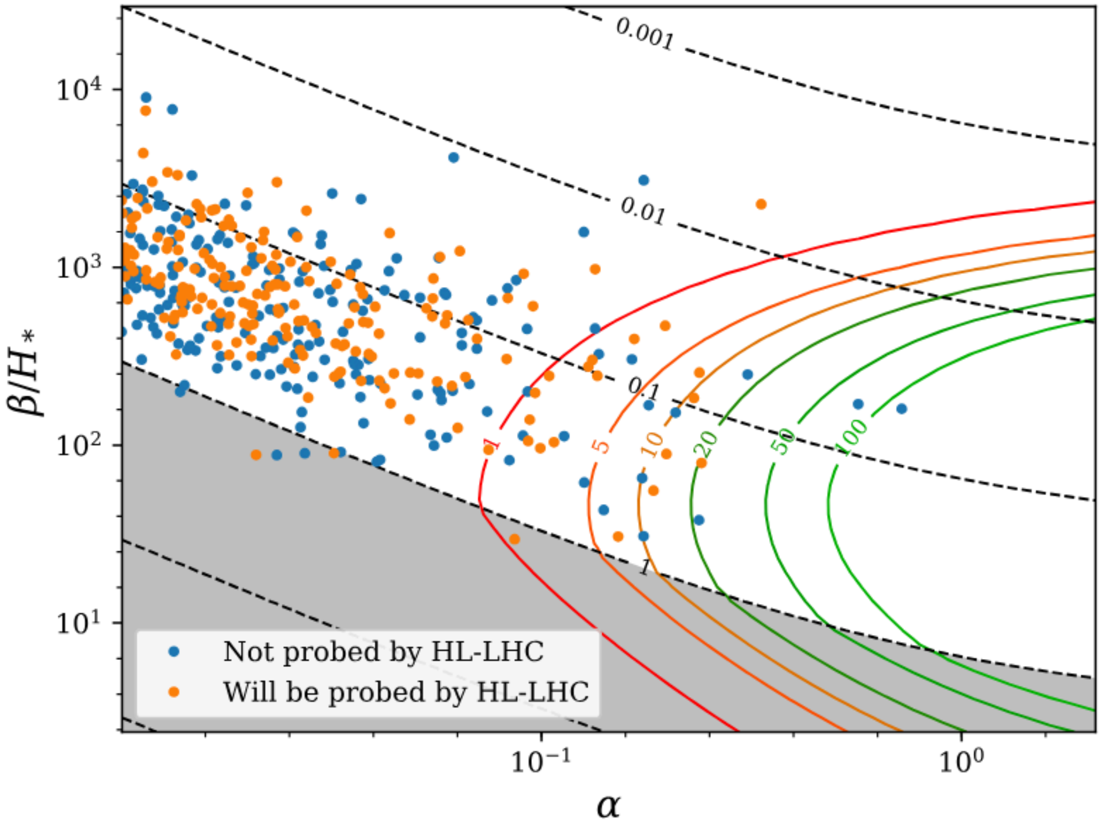
Lunar GW antenna (Ajith et al, arXiv:2404.09181)



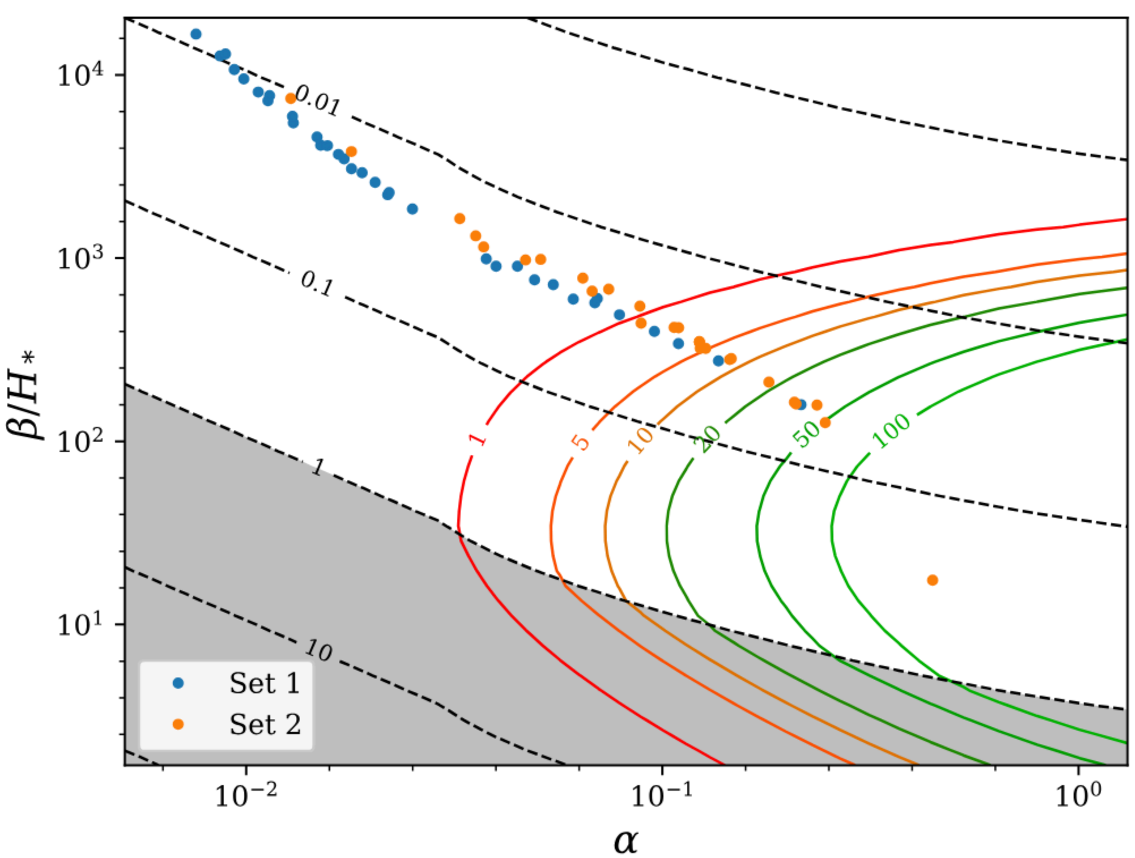
Science case:

- Stellar binaries (white dwarfs, neutron stars…)
- Intermediate BHBs + multi-band (full probe of BH masses in the universe)
- Early source localisation for Earth-based: multi-messenger and test of the universe expansion
- Early Universe signals

DeciHertz



LISA CosWG arXiv:1910.13125



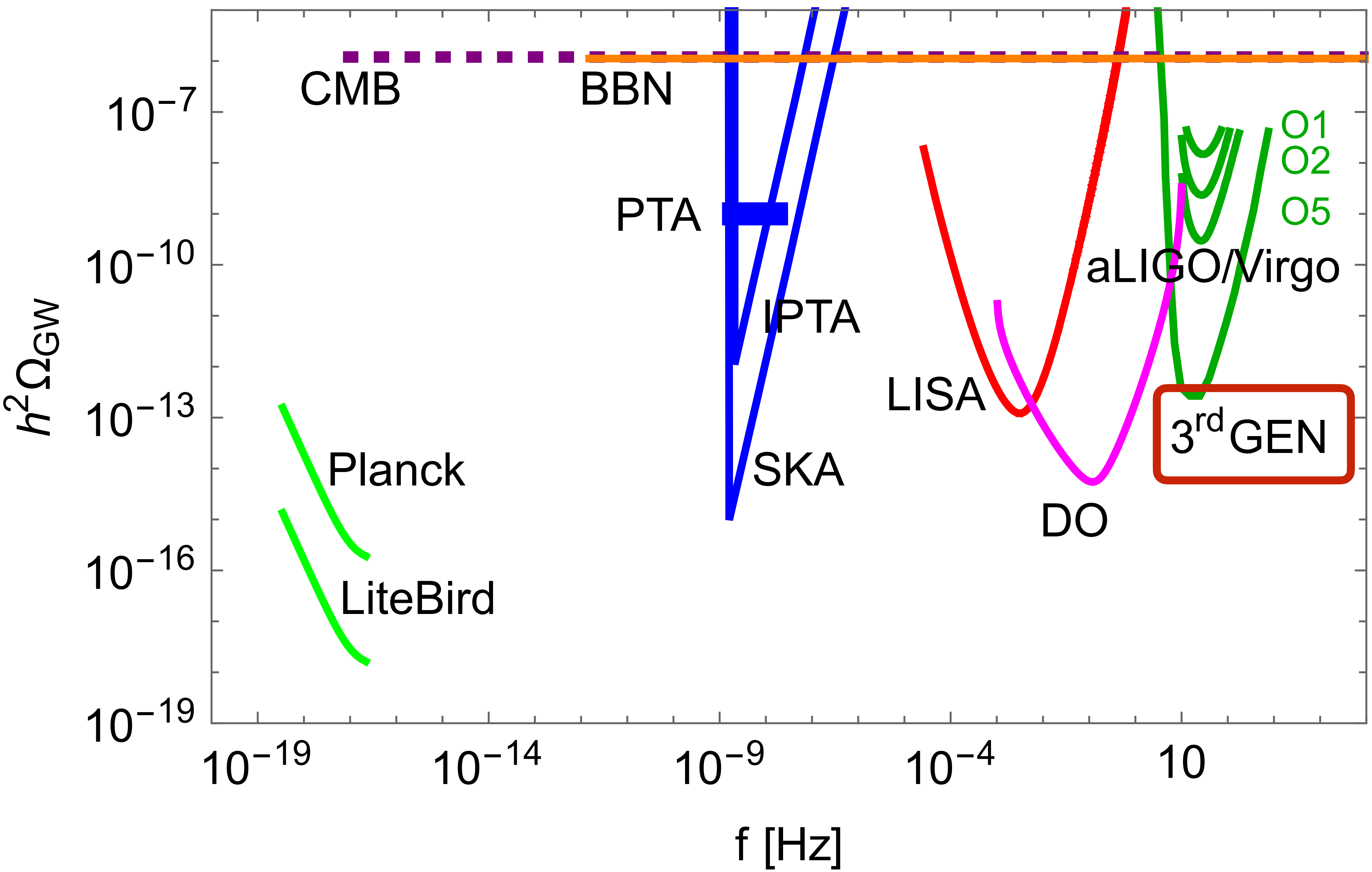
Just one example:
weakly first order PTs at the EW to TeV scale

A DO detector can possibly probe the most populated region of parameter space

Science case:

- Stellar binaries (white dwarfs, neutron stars…)
- Intermediate BHBs + multi-band (full probe of BH masses in the universe)
- Early source localisation for Earth-based: multi-messenger and test of the universe expansion
- **Early Universe signals**

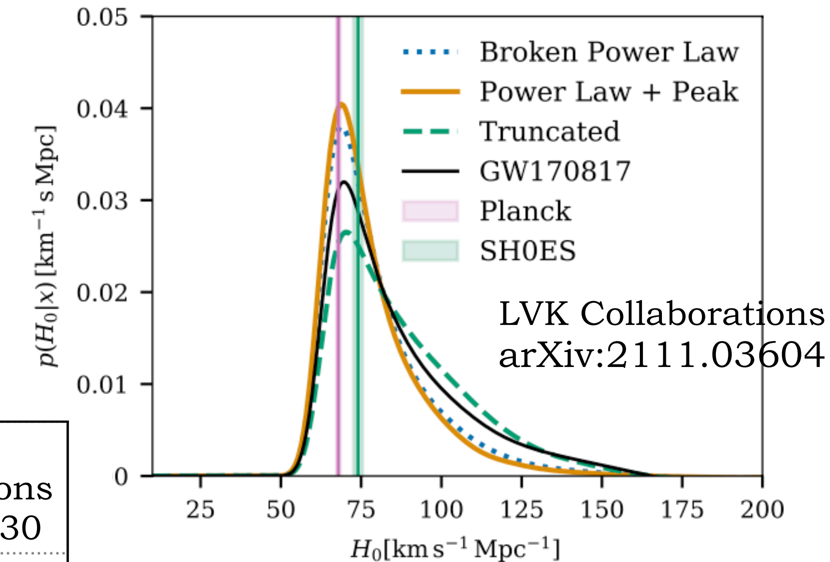
Earth-based 3G detectors: ET and CE

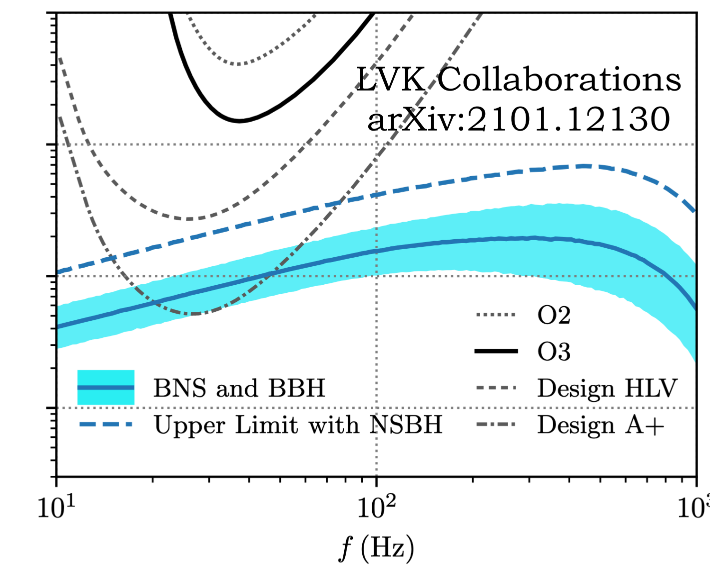


Earth-based 3G detectors: ET and CE

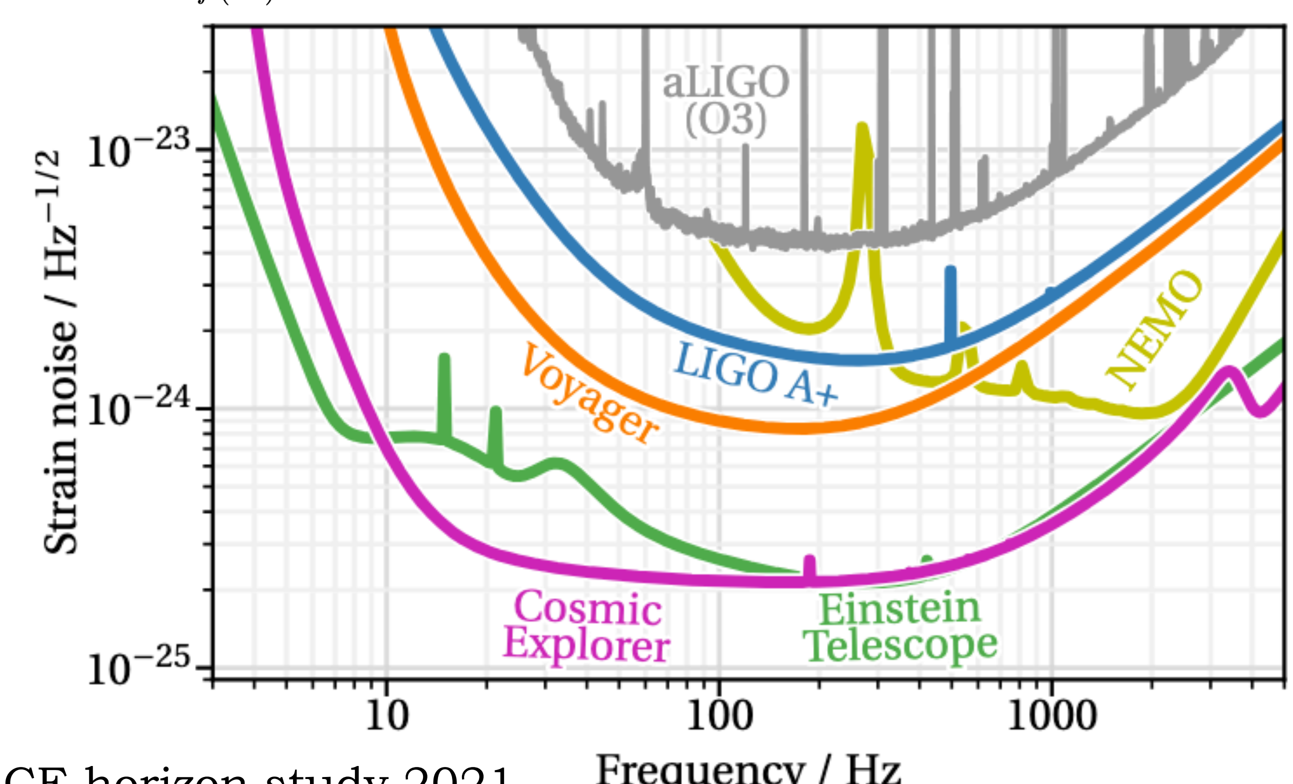
Cosmology with Earth-based detectors is already a reality

- LVK has already provided constraints on H0 with both one event with counterpart and with dark sirens
- However, most probably no cosmological SGWB detection with LVK because masked by astrophysical foreground detection, expected for ~2030





~2035 3rd generation projects:
Einstein Telescope and Cosmic Explorer,
with a factor up to 20 improvement in
sensitivity will be a game changer



Ewans et al, CE horizon study 2021

Earth-based 3G detectors: ET and CE

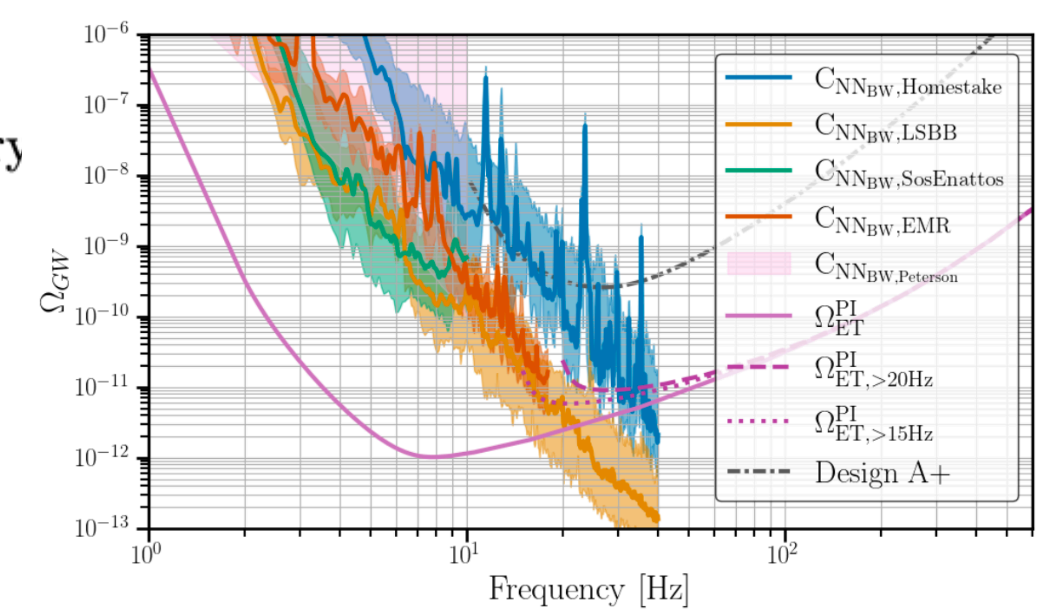
Talk by M. Maggiore

Abac et al, arXiv:2503.12263

2 Cosmology with ET

- 2.1 Stochastic gravitational-wave backgrounds
  - 2.1.1 Definition and characterisation
  - 2.1.2 Anisotropies of the GWB
  - 2.1.3 Polarization of the GWB and parity violation
  - 2.1.4 Source separability
  - 2.1.5 Impact of correlated noise on GWB
  - 2.1.6 Reconstruction of GWBs in presence of correlated noise
- 2.2 Probing the early Universe
  - 2.2.1 GWs from inflation
  - 2.2.2 GWs from phase transitions
  - 2.2.3 GWs from cosmic strings
  - 2.2.4 GWs from domain walls
  - 2.2.5 Primordial black holes
  - 2.2.6 GWs as probes of the early Universe expansion history
- 2.3 Probing the late Universe with Einstein Telescope
  - 2.3.1 Cosmography with coalescing binaries
  - 2.3.2 Modified GW propagation
  - 2.3.3 GW lensing
- 2.4 Probing the large scale structure of the Universe
  - 2.4.1 Cross-correlation GWxLSS
  - 2.4.2 Cross-correlation of AGWB with CMB
  - 2.4.3 Probing LSS with GWs alone

From noise dominated to signal dominated detector (LISA-like): challenges from global fitting of many sources and residuals. Possibility of signal cross-correlation if two or more detectors are present, however, need to account for correlated noises



K. Janssens et al
arXiv:2402.17320

Earth-based 3G detectors: ET and CE

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Abac et al, arXiv:2503.12263

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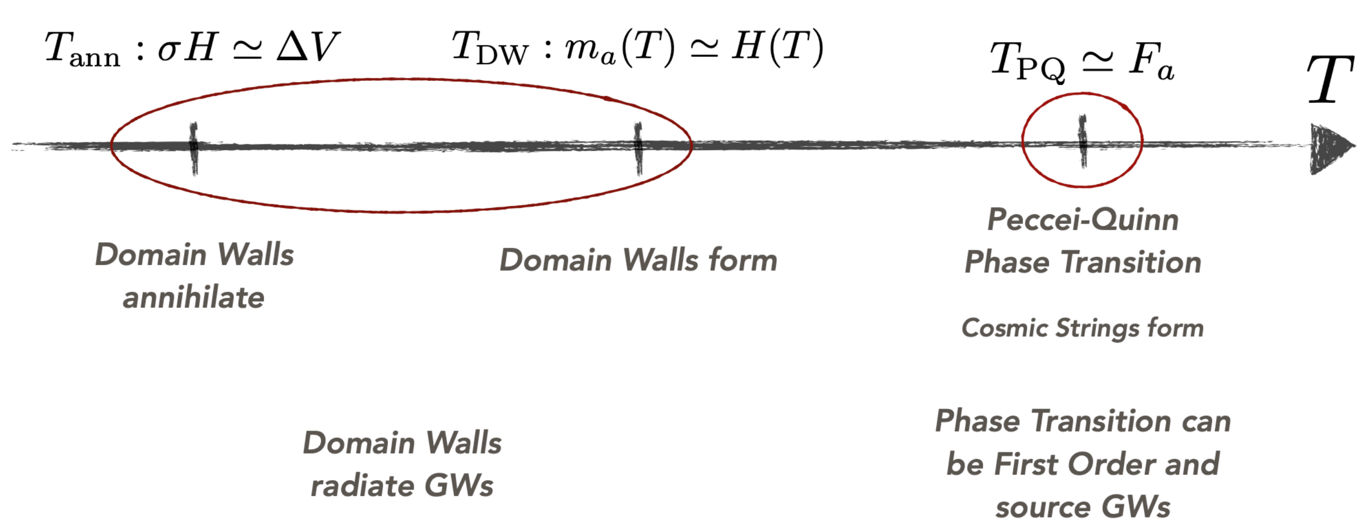
$1\,\mathrm{Hz} < f < 1000\,\mathrm{Hz}$ ➡ $10^6\,\mathrm{GeV} \lesssim T_* \lesssim 10^{10}\,\mathrm{GeV}$

Peccei-Quinn phase transition



$10^{7-8}\,\mathrm{GeV} \lesssim F_a \lesssim 10^{10-11}\,\mathrm{GeV}$

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Abac et al, arXiv:2503.12263

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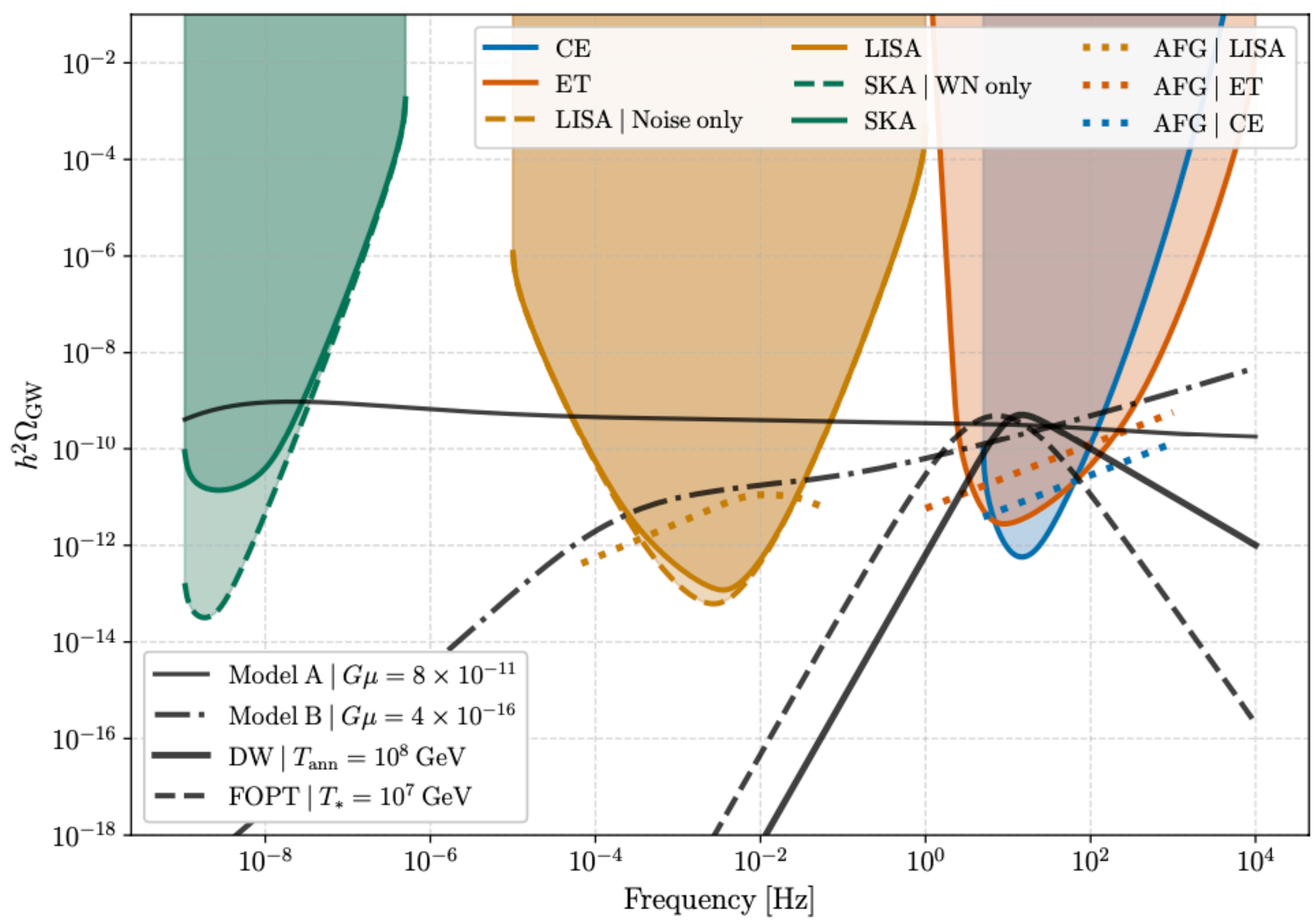

CC et al, arXiv:2406.02359

Earth-based 3G detectors: ET and CE

Talk by M. Maggiore

Abac et al, arXiv:2503.12263

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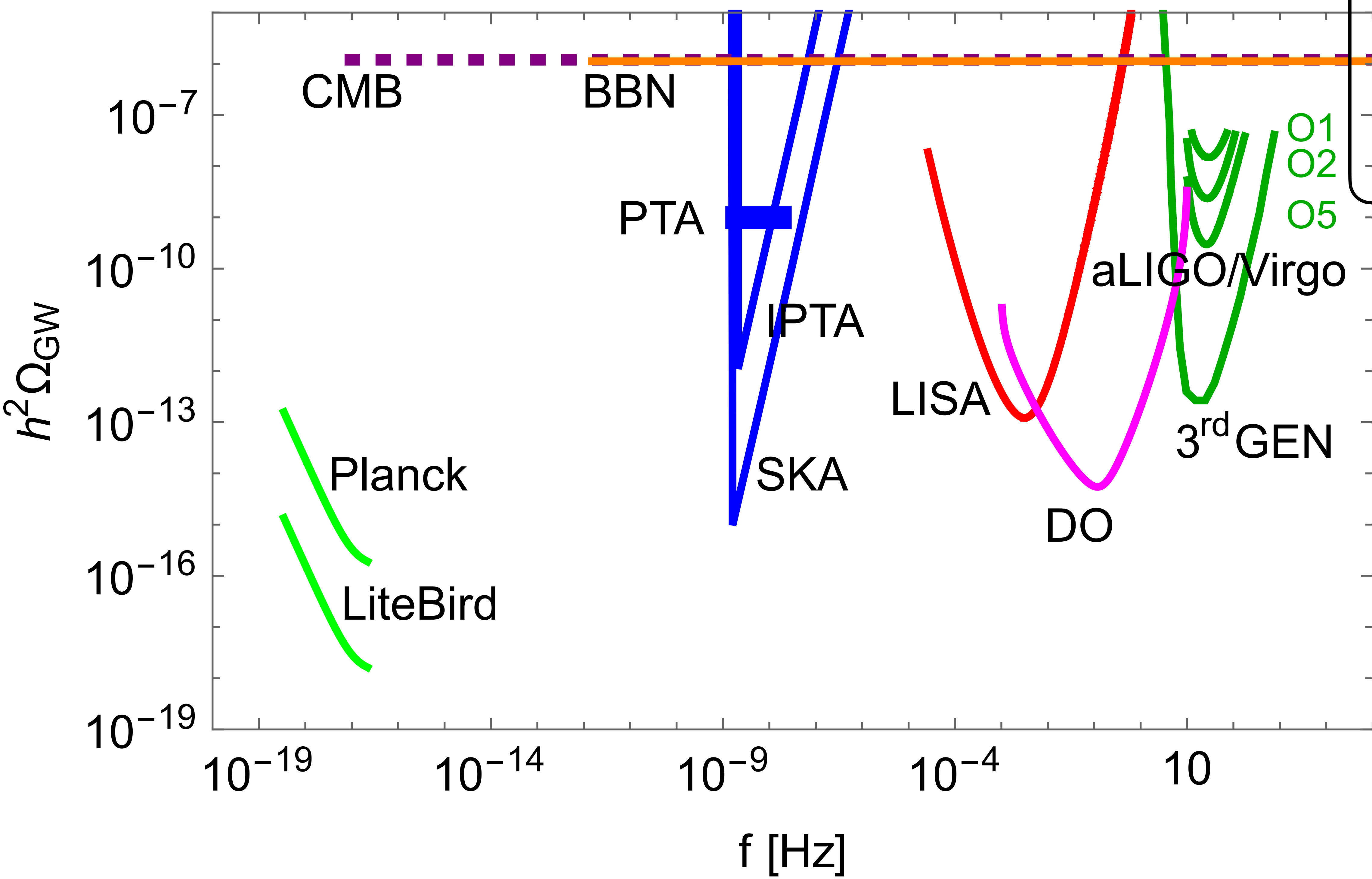
The huge increase in the number of binaries expected wrt LVK will vastly improve cosmological constraints hopefully up to sub-percent level relative error on H0

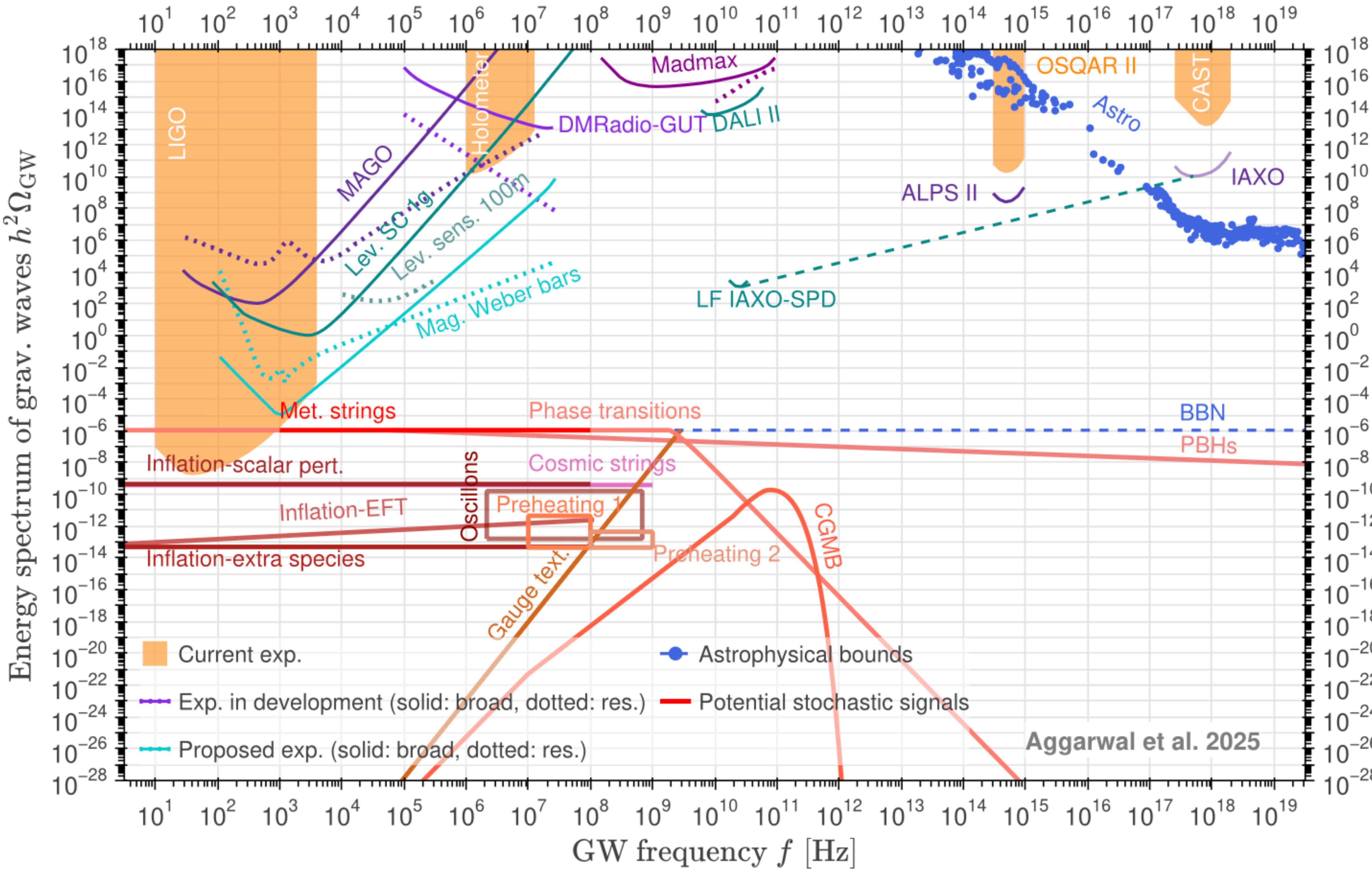
High-frequency



high frequency detectors up to 10^{18} GeV, sensitivity still above BBN and CMB bounds

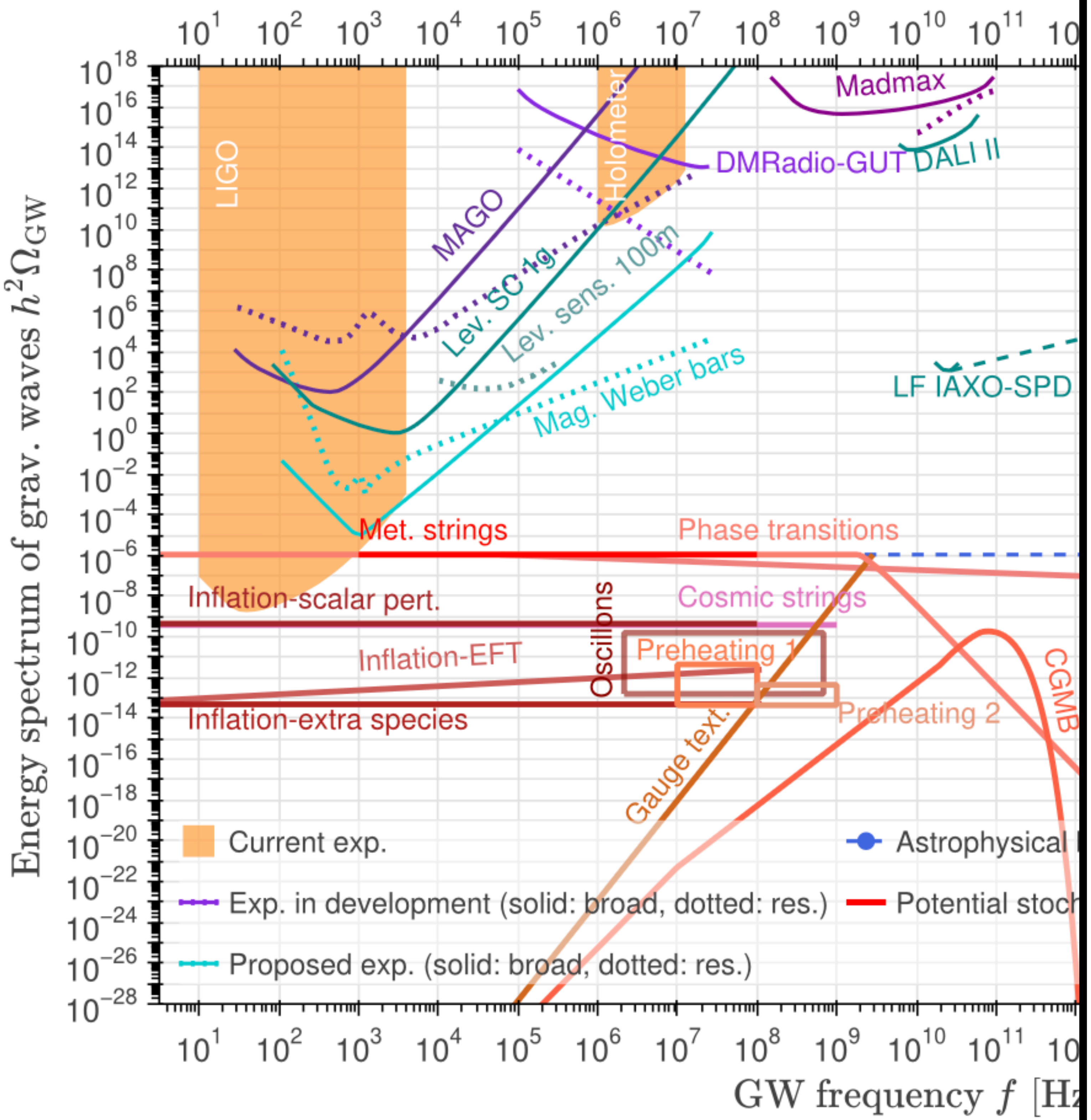
High-frequency

Aggarwal et al, arXiv:2501.11723



High-frequency

Aggarwal et al, arXiv:2501.11723



Quite a few interesting sources?

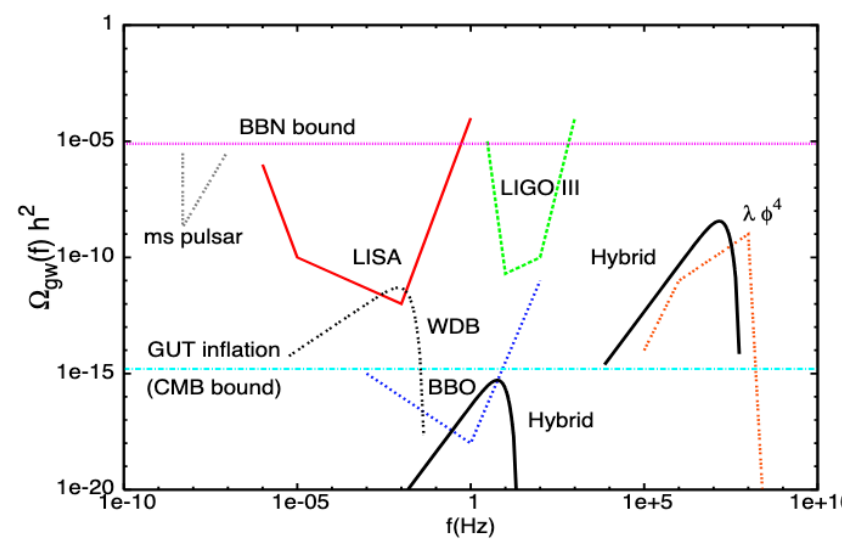
- Mergers of light PBH

$$f_{\rm ISCO} = 4400\,{\rm Hz}\,\frac{M_\odot}{M}$$

- FOPT in neutron stars (MHz)
- Preheating, natural scale

Talks by D. Figueroa and A. Rajantie

Garcia-Bellido and Figueroa, astro-ph/0701014



- SGWB from the hot thermal plasma in the standard model

$$f_{\rm peak}^{\Omega_{\rm CGMB}} \approx 79.8\,{\rm GHz}\left(\frac{106.75}{g_{\star s}(T_{\rm max})}\right)^{1/3}$$

Ghiglieri and Laine, arXiv:1504.02569