Thermal neutrino interaction rates for decoupling temperatures[1,2]

Greg Jackson

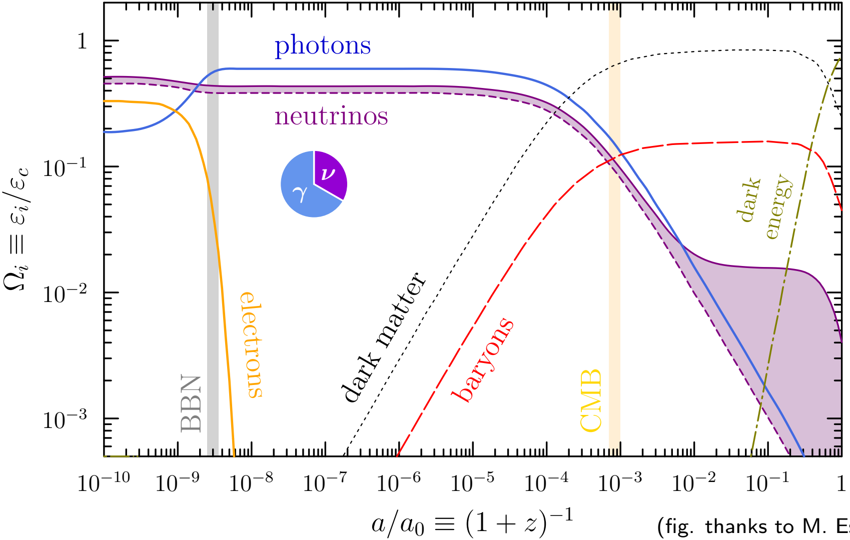
Subatech, CNRS/Nantes U./IMT-Atlantique

– CosmoFONDUE • Geneva • June 2025 –

[1] based on collaboration w/ M. Laine

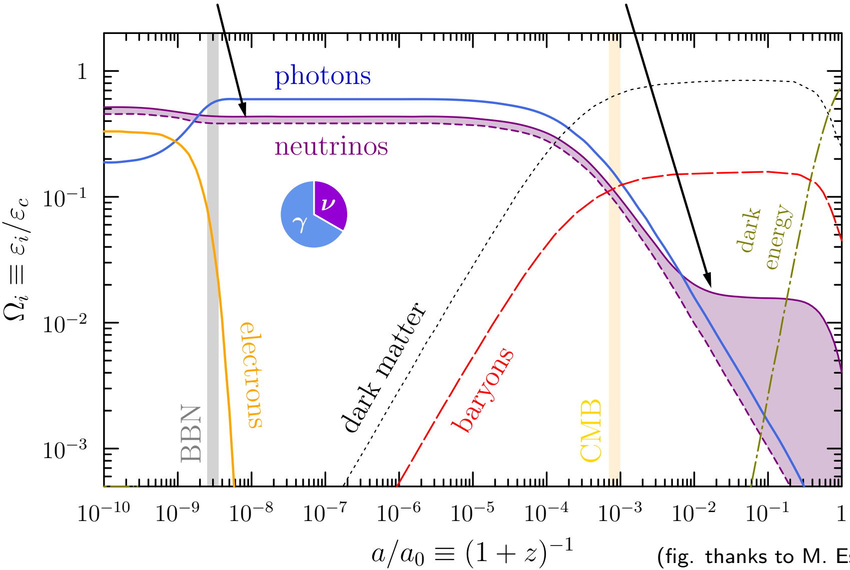
[2] supported by the ANR under grant No. 22-CE31-0018

Neutrinos are always relevant in the universe's evolution!



(fig. thanks to M. Escudero!)

$N_{\text{eff}} = 3.0 \pm 0.3$ $\qquad$ $\sum m_\nu < 0.2$ eV



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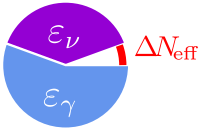
radiation energy budget:



$$\frac{\varepsilon_\nu}{\varepsilon_\gamma} \equiv \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}^{\rm SM}$$

(zeroth order approx. $N_{\rm eff} \simeq 3 =$ number of neutrino species) radiation energy budget:

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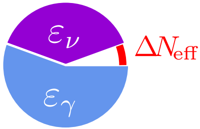


(zeroth order approx. $N_{\rm eff} \simeq 3 =$ number of neutrino species)

observation

$N_{\rm eff}^{\rm BBN} = 2.86 \pm 0.28$ [Pisanti, *et al* (2020)]

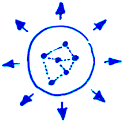
$N_{\rm eff}^{\rm CMB} = 2.99 \pm 0.17$ [Planck Collab. (2018)]

theory

$N_{\rm eff}^{\rm SM} = 3.044 \pm 0.001$ $\Rightarrow$ subject of this talk

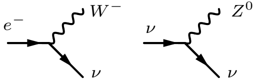
note: CMB S4 could measure $N_{\rm eff}$ with sub-1% accuracy...

$\Rightarrow$ $\Delta N_{\rm eff}$ is a probe of BSM physics! J. Ghiglieri, (Wed, 15:20)



competition between *interaction* and *expansion* !

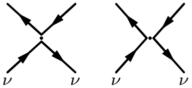
neutrinos feel only weak interactions:



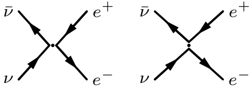
$$G_{\rm F} = \frac{g_2^2}{4\sqrt{2}m_W^2}$$

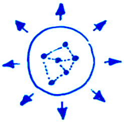
(can use **Fermi model** at low energies)

elastic:



, inelastic:





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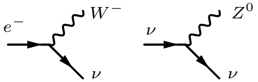

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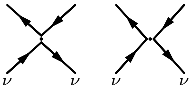

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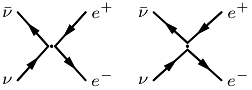

- the interaction rate is $\Gamma \sim \int \mathrm{d}\Omega \, |\mathcal{M}|^2 \left(n_\mathrm{F} n_\mathrm{F} - \ldots \right) \sim G_\mathrm{F}^2 T^5$
- the Hubble rate is $H \sim \dfrac{T^2}{m_\mathrm{Pl}}$ where $m_\mathrm{Pl} \sim 10^{19}$ GeV (Planck mass)

$$\Gamma = H \quad \Leftrightarrow \quad T_\mathrm{dec} \sim \left(\frac{1}{m_\mathrm{Pl} G_\mathrm{F}^2} \right)^{1/3} \sim 1 \text{ MeV}$$

Why is N_{eff} in the SM not 3?

quantum kinetic equations: [Sigl, Raffelt (1993)]

$$\dot{\rho} \;\simeq\; -i\big[\mathcal{H}, \rho\big] + \mathcal{C}[\rho] \quad , \quad \rho_{ab} = e^{i\phi_{ab}(t)} \left\langle \frac{\hat{w}_a^\dagger \hat{w}_b}{V} \right\rangle$$

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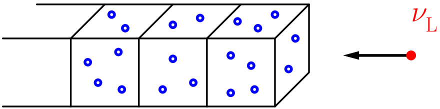
- some $e^+e^- \to \nu\bar{\nu}$ heating since $T_{\rm dec} \approx m_e$
  [Dicus, *et al.* (1982)] [Dolgov, *et al.* (1997)] — $\delta N_{\rm eff} \simeq +0.03$
- corrections to equation of state $P_{\rm int}(T)$
  [Heckler (1994)] [Bennet, *et al.* (2020)] — $\delta N_{\rm eff} \simeq +0.01$
- neutrino oscillations
  [Mangano, *et al.* (2005)] [de Salas, Pastor (2016)] — $\delta N_{\rm eff} \simeq +0.001$
- QED corrections to interaction rates
  [Bennet, *et al.* (2020)] [Cielo, *et al.* (2023)] — $\delta N_{\rm eff} = ??$

start with simpler problem: interaction rate for ‘tagged’ particle



calculate the rate in thermal field theory [Bödeker, *et al.* (2016)]

$$\Gamma \;=\; \frac{1}{2\omega}\,\mathrm{tr}\left[\not{\mathcal{K}}\,\mathrm{Im}\,\Sigma(\mathcal{K})\right]\,, \quad \mathcal{K} = (\omega, \boldsymbol{k})$$

$\Sigma(K) =$  fermion self-energy

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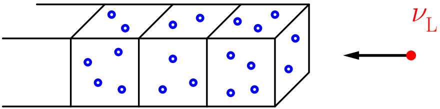

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$\Rightarrow$ EFT of neutrinos, electrons, positrons and photons at $T \sim$ MeV

$$L \;=\frac{G_{\rm F}}{4\sqrt{2}}\Big\{\, 2\,\bar{\nu}_a\gamma_\mu(1-\gamma_5)\nu_a\,\bar{\ell}_e\gamma_\mu\left[2\delta_{a,e} - 1 + 4s_W^2 + (1-2\delta_{a,e})\gamma_5\right]\ell_e + \bar{\nu}_a\gamma_\mu(1-\gamma_5)\nu_a\,\bar{\nu}_b\gamma_\mu(1-\gamma_5)\nu_b \Big\}$$

sum over $a, b = \{e, \mu, \tau\}$

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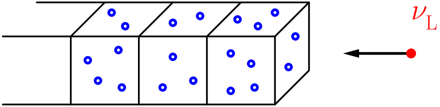

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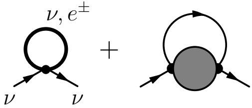
$$+\;\bar{\nu}_a\gamma_\mu(1-\gamma_5)\nu_a\,\bar{\nu}_b\gamma_\mu(1-\gamma_5)\nu_b + ieC_a\bar{\nu}_a\gamma_\mu(1-\gamma_5)\nu_a\,\partial_\nu F_{\nu\mu}\;\Big\}$$

sum over $a,b=\{e,\mu,\tau\}$ 1-loop level operator!

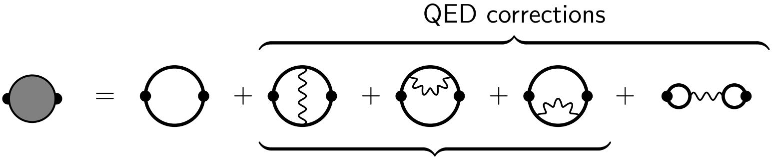
Feynman diagrams

$\Rightarrow$ address perturbatively, expansion in G_{F} $\left(\text{and } \alpha_{\mathrm{em}} = \dfrac{e^2}{4\pi}\right)$

$$\Sigma \quad = \quad aG_{\mathrm{F}} + bG_{\mathrm{F}}^2 + \dots \quad =$$



$+ \quad \dots$



one-particle irreducible [1910.07552]

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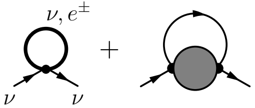
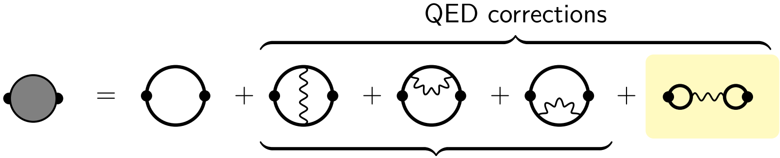


$+ \quad \ldots$



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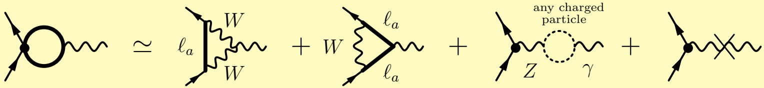
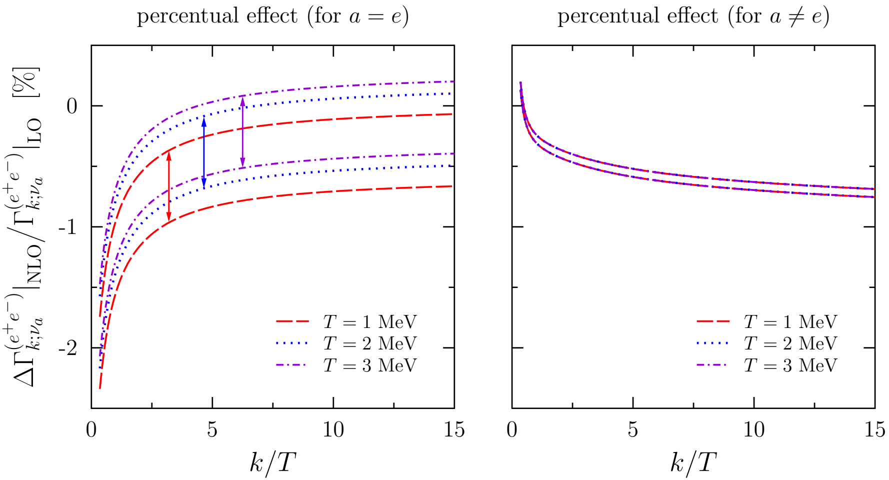


more EFT details in [Hill, Tomalak (2020)]

numerical impact of the NLO corrections

[GJ, Laine (2024)]



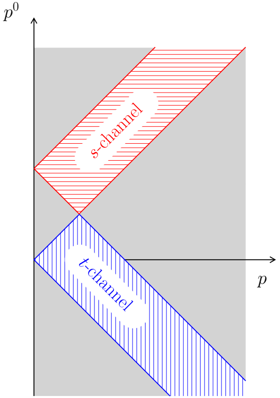
$\Rightarrow$ we find tiny QED corrections to Γ, relatively less than ~ 1 %

the rate Γ describes the *approach* to equilibrium:

$$\dot{f}_{\boldsymbol{k}} \simeq -\Gamma(k)\left[f_{\boldsymbol{k}} - n_{\mathrm{F}}(k) \right]$$

$$\Gamma = \frac{G_{\mathrm{F}}^2 k}{8\pi^2} \left(\int \mathrm{d}\Omega^{(t)} + \int \mathrm{d}\Omega^{(s)} \right) \mathcal{P}^2 \left[1 - n_{\mathrm{F}}(k - p^0) + n_{\mathrm{B}}(p^0) \right]$$

$$\times \underbrace{L_{\mu\nu}(\mathcal{P}, \mathcal{K} - \mathcal{P})}_{\text{leptonic tensor}} \, \mathrm{Im}\, \underbrace{\Pi^{\mu\nu}(p^0, p)}_{\text{self-energy}}$$



integration measures are defined by

$$\int \mathrm{d}\Omega^{(s)} \equiv -\frac{1}{k^3} \int_0^k \mathrm{d}p_- \int_k^\infty \mathrm{d}p_+ \; p$$

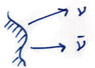
$$\int \mathrm{d}\Omega^{(t)} \equiv \frac{1}{k^3} \int_{-\infty}^0 \mathrm{d}p_- \int_0^k \mathrm{d}p_+ \; p$$

the rate Γ describes the *approach* to equilibrium:

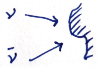
$$\dot{f}_{\boldsymbol{k}} \simeq -\Gamma(k)\big[\, f_{\boldsymbol{k}} - n_{\rm F}(k) \,\big]$$

... but to properly obtain $N_{\rm eff}$, energy transfer rates are needed:

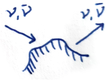
$$\dot{\varepsilon}_{\nu+\bar{\nu}} = \int_{\boldsymbol{k}_\nu,\boldsymbol{q}_{\bar\nu}} (k_\nu + q_{\bar\nu})\, \Psi(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar\nu})\, (1 + f_{\boldsymbol{k}_\nu})(1 + f_{\boldsymbol{q}_{\bar\nu}})$$



$$- \int_{\boldsymbol{k}_\nu,\boldsymbol{q}_{\bar\nu}} (k_\nu + q_{\bar\nu})\, \widetilde{\Psi}(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar\nu})\, f_{\boldsymbol{k}_\nu} f_{\boldsymbol{q}_{\bar\nu}}$$



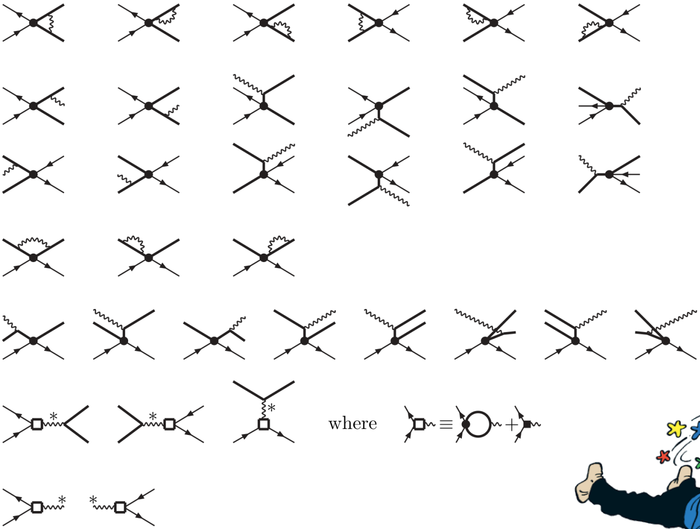
$$+ \int_{\boldsymbol{k}_\nu,\boldsymbol{q}_\nu} (k_\nu - q_\nu)\, \Theta(\boldsymbol{q}_\nu \to \boldsymbol{k}_\nu)\, f_{\boldsymbol{q}_\nu} (1 + f_{\boldsymbol{k}_\nu})$$



the "double-differential" rates Ψ, $\widetilde{\Psi}$, Θ can be computed in thermal field theory, from the spectral function!

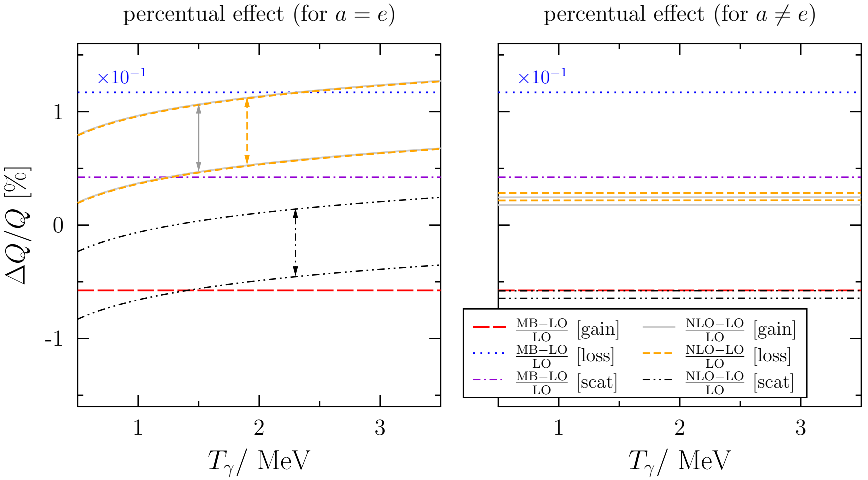
[GJ, Laine (2025)]

The spectral function $\mathrm{Im}\,\Pi_{\mu\nu}$ neatly encodes many scatterings:



Results for $\dot{\varepsilon}_{\nu+\bar{\nu}}$ at NLO

$\Rightarrow$ reaffirms tiny QED corrections to energy transfers [GJ, Laine (2024)]



(in the figure above, $s_W^2 \simeq 0.2386$ and $T_\nu \simeq 0.714\, T_\gamma$)

Results for $\dot{\varepsilon}_{\nu+\bar{\nu}}$ at NLO

double-differential rates i.t.o. $\mathrm{Im}\,\Pi(p^0, p)$ evaluated[3] at...

production: $\Psi(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}})$

annihilation: $\widetilde{\Psi}(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}})$

$$p^0 = k + q\,,\;\; p = |\boldsymbol{k}_\nu + \boldsymbol{q}_{\bar{\nu}}|$$

scattering: $\Theta(\boldsymbol{q}_\nu \to \boldsymbol{k}_\nu)$ $\quad p^0 = k - q\,,\;\; p = |\boldsymbol{k}_\nu - \boldsymbol{q}_\nu|$

$\delta N_{\rm eff} \simeq 10^{-4}$ after inserting these coefficients into momentum-averaged code [Escudero (2019)] [Escudero (2020)]

[3] tabulation & interpolation code for the e^+e^- spectral function made available in relevant kinematic domains: zenodo.14217713

Summary

era of precision neutrino cosmology ...

ultimate goal: $N_{\rm eff}$ in the Standard Model at NLO

- equilibration rate at decoupling [2312.07015]
- double-differential energy transfer rates [2412.03958]

... the devil's in the details!

- $\Delta N_{\rm eff}$ projections from CMB

A. Challinor, (Tue, 14:30)