

Thermal neutrino interaction rates for decoupling temperatures^{1,2}

Greg Jackson

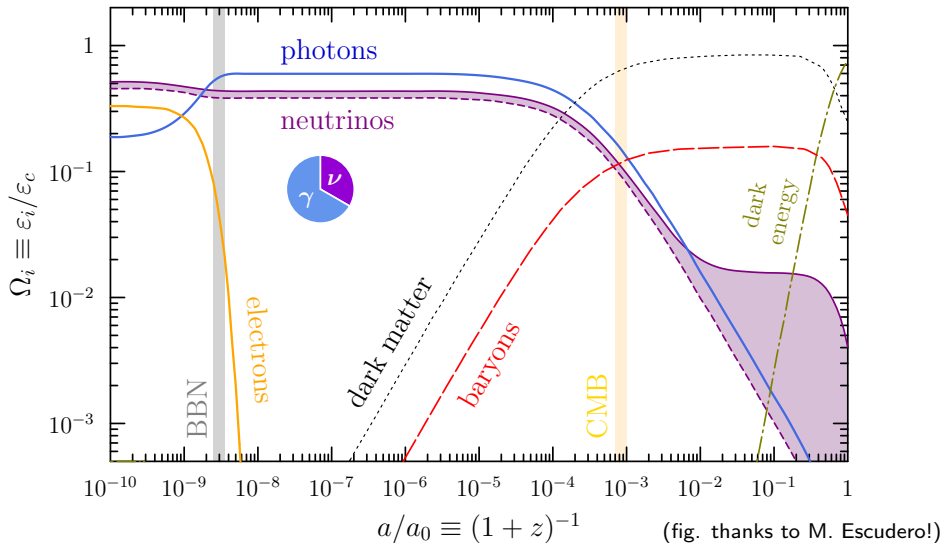
Subatech, CNRS/Nantes U./IMT-Atlantique

– CosmoFONDUE • Geneva • June 2025 –

¹ based on collaboration w/ M. Laine

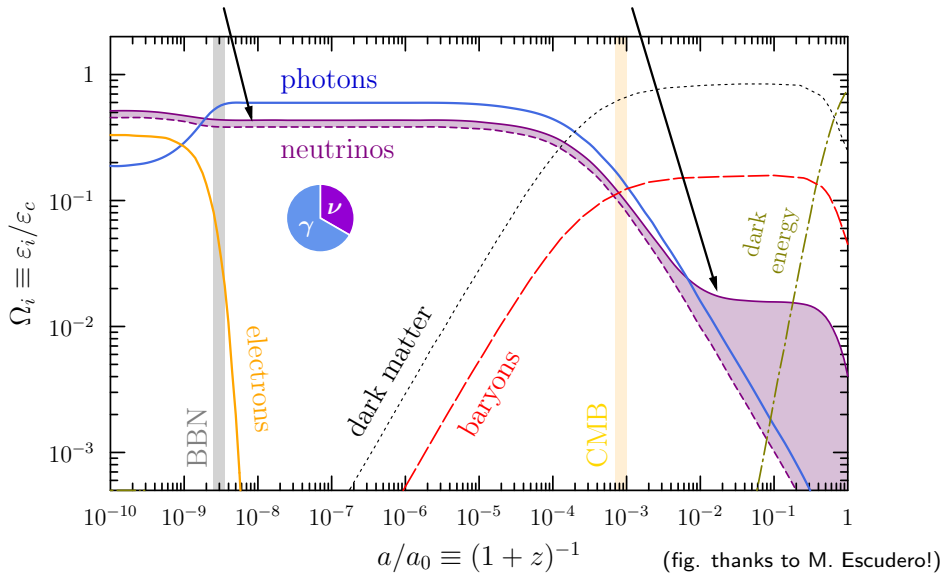
² supported by the ANR under grant No. 22-CE31-0018

Neutrinos are always relevant in the universe's evolution!



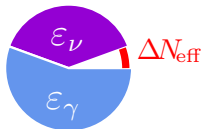
$$N_{\text{eff}} = 3.0 \pm 0.3$$

$$\Sigma m_\nu < 0.2 \text{ eV}$$



radiation energy budget:

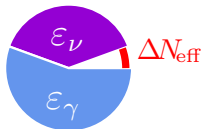
$$\frac{\varepsilon_\nu}{\varepsilon_\gamma} \equiv \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}}^{\text{SM}}$$



(zeroth order approx. $N_{\text{eff}} \simeq 3 =$ number of neutrino species)

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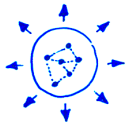


(zeroth order approx. $N_{\text{eff}} \simeq 3 =$ number of neutrino species)

observation	{	$N_{\text{eff}}^{\text{BBN}} = 2.86 \pm 0.28$	[Pisanti, et al (2020)]
		$N_{\text{eff}}^{\text{CMB}} = 2.99 \pm 0.17$	[Planck Collab. (2018)]
theory		$N_{\text{eff}}^{\text{SM}} = 3.044 \pm 0.001$	$\Rightarrow$ subject of this talk

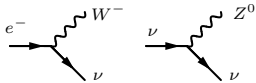
note: CMB S4 could measure N_{eff} with sub-1% accuracy...

$\Rightarrow \Delta N_{\text{eff}}$ is a probe of BSM physics! J. Ghiglieri, (Wed, 15:20)



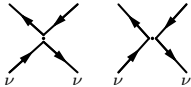
competition between *interaction* and *expansion* !

neutrinos feel only weak interactions:

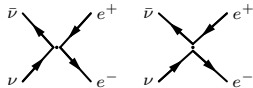


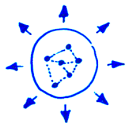
$$G_F = \frac{g_2^2}{4\sqrt{2}m_W^2} \quad (\text{can use } \mathbf{Fermi\ model} \text{ at low energies})$$

elastic:



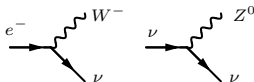
, inelastic:



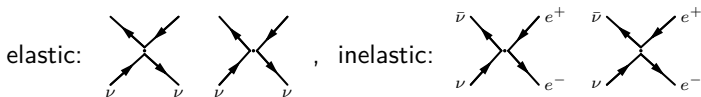


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$$G_F = \frac{g_2^2}{4\sqrt{2}m_W^2} \quad (\text{can use } \mathbf{Fermi \ model} \text{ at low energies})$$



- the interaction rate is $\Gamma \sim \int d\Omega |\mathcal{M}|^2 \left(n_F n_F - \dots \right) \sim G_F^2 T^5$
- the Hubble rate is $H \sim \frac{T^2}{m_{Pl}}$ where $m_{Pl} \sim 10^{19} \text{ GeV}$ (Planck mass)

$$\Gamma = H \Leftrightarrow T_{\text{dec}} \sim \left(\frac{1}{m_{Pl} G_F^2} \right)^{1/3} \sim 1 \text{ MeV}$$

Why is N_{eff} in the SM not 3?

quantum kinetic equations: [Sigl, Raffelt (1993)]

$$\dot{\rho} \simeq -i[\mathcal{H}, \rho] + \mathcal{C}[\rho] \quad , \quad \rho_{ab} = e^{i\phi_{ab}(t)} \left\langle \frac{\hat{w}_a^\dagger \hat{w}_b}{V} \right\rangle$$



(approx/numerical solution, e.g. FortEPiaNO [Gariazzo, de Salas, Pastor (2019)])

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- some $e^+e^- \rightarrow \nu\bar{\nu}$ heating since $T_{\text{dec}} \approx m_e$

[Dicus, *et al.* (1982)] [Dolgov, *et al.* (1997)]

$$\delta N_{\text{eff}} \simeq +0.03$$

- corrections to equation of state $P_{\text{int}}(T)$

[Heckler (1994)] [Bennet, *et al.* (2020)]

$$\delta N_{\text{eff}} \simeq +0.01$$

- neutrino oscillations

[Mangano, *et al.* (2005)] [de Salas, Pastor (2016)]

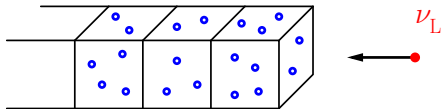
$$\delta N_{\text{eff}} \simeq +0.001$$

- QED corrections to interaction rates

[Bennet, *et al.* (2020)] [Cielo, *et al.* (2023)]

$$\delta N_{\text{eff}} = ??$$

start with simpler problem: interaction rate for 'tagged' particle

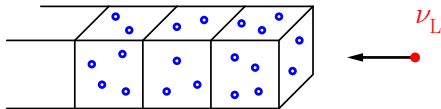


calculate the rate in thermal field theory [Bödeker, *et al.* (2016)]

$$\Gamma = \frac{1}{2\omega} \text{tr} [\mathcal{K} \text{Im} \Sigma(\mathcal{K})] , \quad \mathcal{K} = (\omega, \mathbf{k})$$

$\Sigma(K) =$  fermion self-energy

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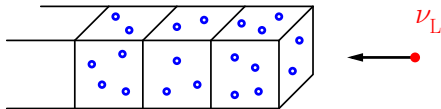
$$\Sigma(K) = \text{---} \bullet \text{---} \quad \text{fermion self-energy}$$

$\Rightarrow$ EFT of neutrinos, electrons, positrons and photons at $T \sim \text{MeV}$

$$L = \frac{G_F}{4\sqrt{2}} \left\{ 2 \bar{\nu}_a \gamma_\mu (1 - \gamma_5) \nu_a \bar{\ell}_e \gamma_\mu [2\delta_{a,e} - 1 + 4s_W^2 + (1 - 2\delta_{a,e})\gamma_5] \ell_e \right. \\ \left. + \bar{\nu}_a \gamma_\mu (1 - \gamma_5) \nu_a \bar{\nu}_b \gamma_\mu (1 - \gamma_5) \nu_b \right\}$$

sum over $a, b = \{e, \mu, \tau\}$

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sum over $a, b = \{e, \mu, \tau\}$

1-loop level operator!

Feynman diagrams

$\Rightarrow$ address perturbatively, expansion in G_F (and $\alpha_{\text{em}} = \frac{e^2}{4\pi}$)

$$\Sigma = aG_F + bG_F^2 + \dots = \text{diagram 1} + \text{diagram 2} + \dots$$

The first diagram is a circle with two external lines labeled ν and ν , and a vertex labeled $\nu, e^\pm$. The second diagram is a circle with a shaded blob in the center and two external lines.

$$\text{diagram 2} = \text{diagram 1} + \underbrace{\text{diagram 3} + \text{diagram 4} + \text{diagram 5}}_{\text{QED corrections}} + \text{diagram 6}$$

Diagram 3 is a circle with a wavy line inside. Diagram 4 is a circle with a wavy line and a shaded blob. Diagram 5 is a circle with a wavy line and a shaded blob. Diagram 6 is a circle with a wavy line and two external lines.

one-particle irreducible [\[1910.07552\]](#)

Feynman diagrams

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The first diagram shows a fermion line with a self-energy loop labeled $\nu, e^\pm$. The second diagram shows a fermion line with a self-energy loop that is shaded gray, representing a higher-order correction.

QED corrections

$$\text{shaded blob} = \text{empty circle} + \underbrace{\text{circle with wavy line} + \text{circle with wavy line} + \text{circle with wavy line}}_{\text{one-particle irreducible [1910.07552]}} + \text{two circles connected by wavy line}$$

The diagram shows a shaded blob equal to an empty circle plus a bracketed sum of three circles with wavy lines (representing photon corrections) plus a diagram with two circles connected by a wavy line.

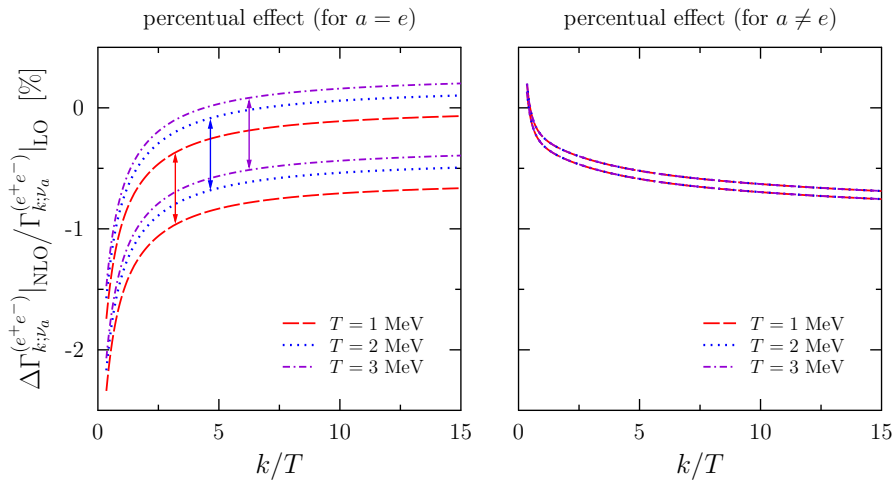
$$\text{diagram} \simeq \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5}$$

The diagram shows a fermion line with a self-energy loop and a wavy line, which is approximately equal to the sum of five diagrams. The first diagram shows a fermion line with a self-energy loop and a wavy line. The second diagram shows a fermion line with a self-energy loop and a wavy line, with labels ℓ_a and W . The third diagram shows a fermion line with a self-energy loop and a wavy line, with labels ℓ_a and W . The fourth diagram shows a fermion line with a self-energy loop and a wavy line, with labels Z and γ . The fifth diagram shows a fermion line with a self-energy loop and a wavy line, with a label 'any charged particle'.

more EFT details in [\[Hill, Tomalak \(2020\)\]](#)

numerical impact of the NLO corrections

[GJ, Laine (2024)]



$\Rightarrow$ we find tiny QED corrections to Γ , relatively less than $\sim 1 \%$

the rate Γ describes the *approach* to equilibrium:

$$\dot{f}_{\mathbf{k}} \simeq -\Gamma(k) [f_{\mathbf{k}} - n_{\text{F}}(k)]$$

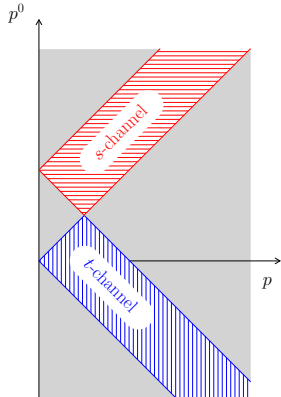
$$\Gamma = \frac{G_{\text{F}}^2 k}{8\pi^2} \left(\int d\Omega^{(t)} + \int d\Omega^{(s)} \right) \mathcal{P}^2 [1 - n_{\text{F}}(k - p^0) + n_{\text{B}}(p^0)]$$

$$\times \underbrace{L_{\mu\nu}(\mathcal{P}, \mathcal{K} - \mathcal{P})}_{\text{leptonic tensor}} \underbrace{\text{Im } \Pi^{\mu\nu}(p^0, p)}_{\text{self-energy}}$$

integration measures are defined by

$$\int d\Omega^{(s)} \equiv -\frac{1}{k^3} \int_0^k dp_- \int_k^\infty dp_+ p$$

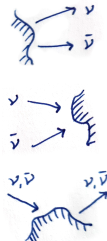
$$\int d\Omega^{(t)} \equiv \frac{1}{k^3} \int_{-\infty}^0 dp_- \int_0^k dp_+ p$$



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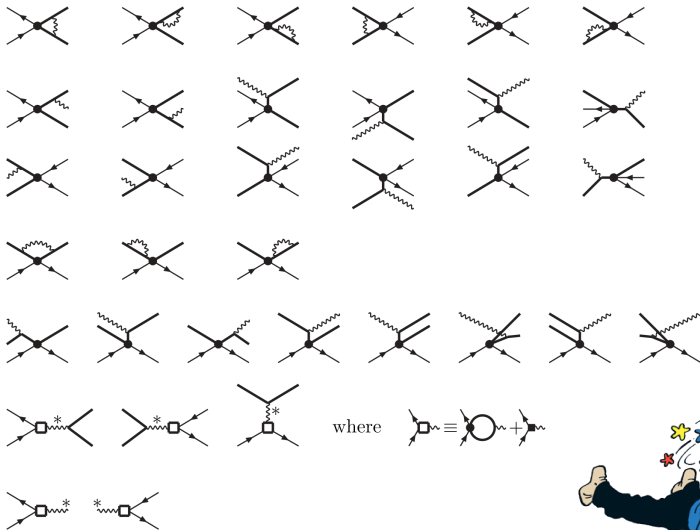
... but to properly obtain N_{eff} , energy transfer rates are needed:

$$\begin{aligned} \dot{\varepsilon}_{\nu+\bar{\nu}} = & \int_{\mathbf{k}_{\nu}, \mathbf{q}_{\bar{\nu}}} (k_{\nu} + q_{\bar{\nu}}) \Psi(\mathbf{k}_{\nu}, \mathbf{q}_{\bar{\nu}}) (1 + f_{\mathbf{k}_{\nu}})(1 + f_{\mathbf{q}_{\bar{\nu}}}) \\ & - \int_{\mathbf{k}_{\nu}, \mathbf{q}_{\bar{\nu}}} (k_{\nu} + q_{\bar{\nu}}) \tilde{\Psi}(\mathbf{k}_{\nu}, \mathbf{q}_{\bar{\nu}}) f_{\mathbf{k}_{\nu}} f_{\mathbf{q}_{\bar{\nu}}} \\ & + \int_{\mathbf{k}_{\nu}, \mathbf{q}_{\nu}} (k_{\nu} - q_{\nu}) \Theta(\mathbf{q}_{\nu} \rightarrow \mathbf{k}_{\nu}) f_{\mathbf{q}_{\nu}} (1 + f_{\mathbf{k}_{\nu}}) \end{aligned}$$


the “double-differential” rates Ψ , $\tilde{\Psi}$, Θ can be computed in thermal field theory, from the spectral function!

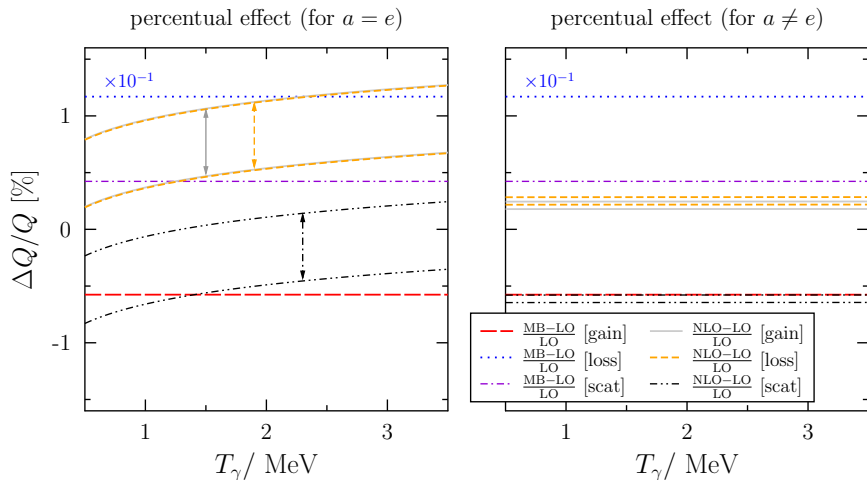
[GJ, Laine (2025)]

The spectral function $\text{Im } \Pi_{\mu\nu}$ neatly encodes many scatterings:



Results for $\dot{\mathcal{E}}_{\nu+\bar{\nu}}$ at NLO

$\Rightarrow$ reaffirms tiny QED corrections to energy transfers [\[GJ, Laine \(2024\)\]](#)



(in the figure above, $s_W^2 \simeq 0.2386$ and $T_\nu \simeq 0.714 T_\gamma$)

Results for $\dot{\epsilon}_{\nu+\bar{\nu}}$ at NLO

double-differential rates i.t.o. $\text{Im } \Pi(p^0, p)$ evaluated³ at...

$$\left. \begin{array}{l} \text{production: } \Psi(\mathbf{k}_\nu, \mathbf{q}_{\bar{\nu}}) \\ \text{annihilation: } \tilde{\Psi}(\mathbf{k}_\nu, \mathbf{q}_{\bar{\nu}}) \end{array} \right\} \begin{array}{l} p^0 = k + q, \quad p = |\mathbf{k}_\nu + \mathbf{q}_{\bar{\nu}}| \\ \\ \text{scattering: } \Theta(\mathbf{q}_\nu \rightarrow \mathbf{k}_\nu) \quad p^0 = k - q, \quad p = |\mathbf{k}_\nu - \mathbf{q}_\nu| \end{array}$$

$\delta N_{\text{eff}} \simeq 10^{-4}$ after inserting these coefficients into
momentum-averaged code [[Escudero \(2019\)](#)] [[Escudero \(2020\)](#)]

³ tabulation & interpolation code for the e^+e^- spectral function made available in relevant kinematic domains: [zenodo.14217713](https://zenodo.org/record/14217713)

Summary

era of precision neutrino cosmology ...

ultimate goal: N_{eff} in the Standard Model at NLO

- equilibration rate at decoupling [\[2312.07015\]](#)
- double-differential energy transfer rates [\[2412.03958\]](#)



... the devil's in the details!

- ΔN_{eff} projections from CMB

A. Challinor, (Tue, 14:30)

