Gravitational waves from phase transitions in the early Universe

COSMOFondue

University of Geneva, June 11, 2025

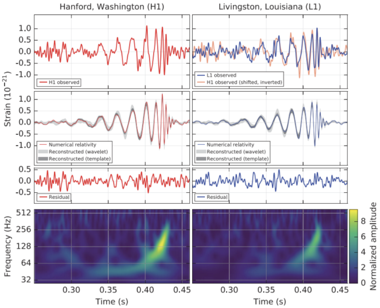
Alberto Roper Pol
University of Geneva

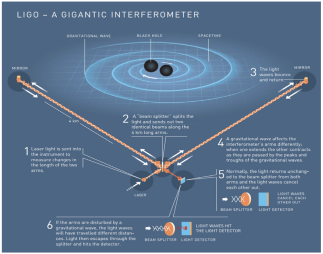
SNSF Ambizione grant (2023–2027): "*Exploring the early universe with gravitational waves and primordial magnetic fields.*"

Collaborators: A. Brandenburg (Nordita), C. Caprini (UniGe & CERN), R. Jinno (Kobe), T. Kahniashvili (CMU), T. Konstandin (DESY), A. Kosowsky (PittU), S. Mandal (SBU), A. S. Midiri (UniGe), A. Neronov (APC), S. Procacci (UniGe), H. Rubira (TUM), I. Stomberg (IFIC), D. Semikoz (APC)

Introduction and Motivation

- Gravitational waves are opening a new window into our understanding of the Universe [Michele Maggiore talk]
  - First event GW150914 detected by LIGO-Virgo collaboration[1]





[1] [LIGO-Virgo Collaboration], *Phys. Rev. Lett.* **116**, 061102 (2016)

GWs from the early Universe

GWs from the early Universe [Arttu Rajante, Daniel Figueroa talks] have the potential to provide us with *direct information on early universe physics* that is *not accessible via electromagnetic observations, possibly complementary to collider experiments*:

nature of first-order phase transitions (baryogenesis, BSM physics, high-energy physics),
primordial origin of intergalactic magnetic fields [Ruth Durrer talk].

Probing the early Universe with GWs

Cosmological (pre-recombination) GW background

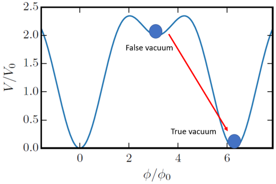
- Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe.

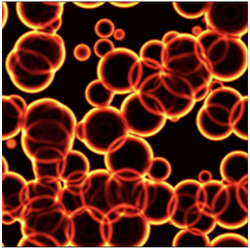
$$f_* \simeq 1.64 \times 10^{-3} \frac{100}{R_* \mathcal{H}_*} \frac{T_*}{100\,\mathrm{GeV}}\,\mathrm{Hz}$$

- Phase transitions
  - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz
    Peccei-Quinn, B-L, left-right symmetries $\sim 10^7, 10^8$ GeV.
  - Space-based detectors **(LISA)** frequencies are 10^{-5}–10^{-2} Hz
    Electroweak phase transition $\sim$ 100 GeV
  - Pulsar Timing Array **(PTA)** frequencies are 10^{-9}–10^{-7} Hz
    Quark confinement (QCD) phase transition $\sim$ 100 MeV

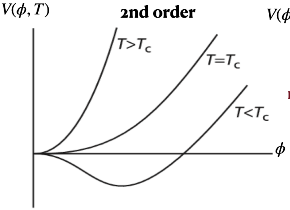
First–order phase transition
[Rajantie talk, Bernardo, Schicho posters]

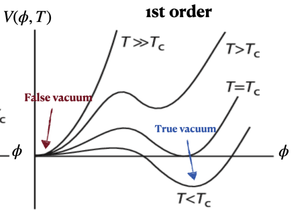
$$V(\phi, T) = \frac{1}{2}M^2(T)\phi^2 - \frac{1}{3}\delta(T)\phi^3 + \frac{1}{4}\lambda\phi^4$$





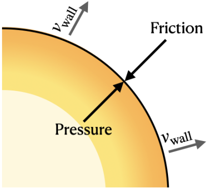
Credits: I. Stomberg





Hydrodynamics of first-order phase transitions[2]

- Broken-phase bubbles are nucleated and expand
- Friction from particles yield a terminal velocity ξ_w of the bubbles
- The bubble can run away when the friction is not enough to stop the bubble's acceleration

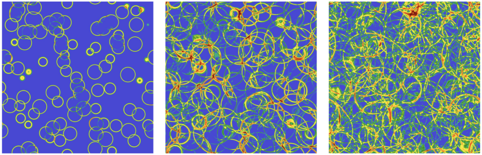


$$\nabla_\mu T^{\mu\nu}_{\rm field} = \frac{\partial V}{\partial \phi}\partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi \,,$$
$$\nabla_\mu T^{\mu\nu}_{\rm fluid} = -\frac{\partial V}{\partial \phi}\partial^\nu \phi - \eta u^\mu \partial_\mu \phi \partial^\nu \phi \,,$$

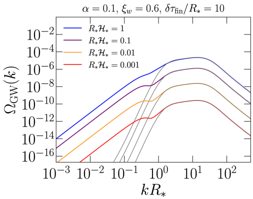
[2] Espinosa, Konstandin, No, Servant, *JCAP* **06** (2010) 028.

GWs from sound waves[3]

- Numerical simulations of the scalar + fluid system performed by the Sussex/Helsinki group via an effective friction term indicate sound-wave regime to dominate for weak/intermediate phase transitions.



- Sound-shell model. Two scales are found that determine the GW spectrum: R_* and ΔR_* (sound-shell thickness).



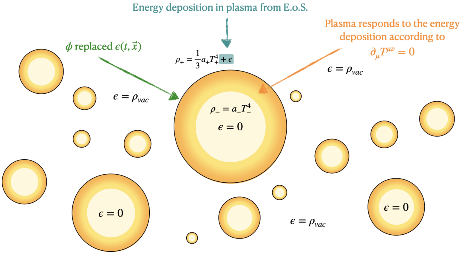
$$\Omega_{\rm GW}(f) = 3\,\tilde{\Omega}_{\rm GW}\,K^2\,\Upsilon(\tau_{\rm sw})(H_* R_*)\,S(f\,R_*)$$

$$\Upsilon(\tau_{\rm sw}) = \frac{H_*\tau_{\rm sw}}{1 + H_*\tau_{\rm sw}}$$

[3] Hindmarsh *et al.*, 2013, 2015, 2017, Cutting *et al.*, 2019, Correia *et al.*, 2025
Hindmarsh, 2016, Hindmarsh, Hijazi, 2019, ARP, Procacci, Caprini, 2023.

GWs from sound waves: Higgsless simulations[4]

- Difficulty on simulations is due to the different scales of the scalar field ϕ and the fluid shell, so one can consider a nucleation history and set the pressure and energy density by knowing the value of ϵ and setting it during the simulation.
- Effect of bubble collisions on GWs is subdominant when sound waves are produced, so one can ignore the scalar field.
- Nucleation history is produced from an exponential probability distribution $P(t) \propto \exp\left[\beta(t - t_*)\right]$.

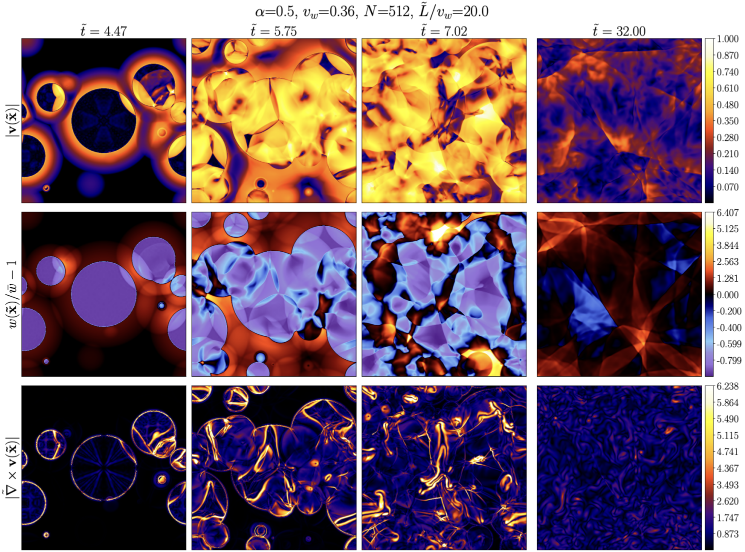


Credit: I. Stomberg

---

[4] Jinno *et al.* JCAP 02 (2023) 011, 2209.04369,
ARP, Stomberg et al., arXiv:2409.03651 (2024).

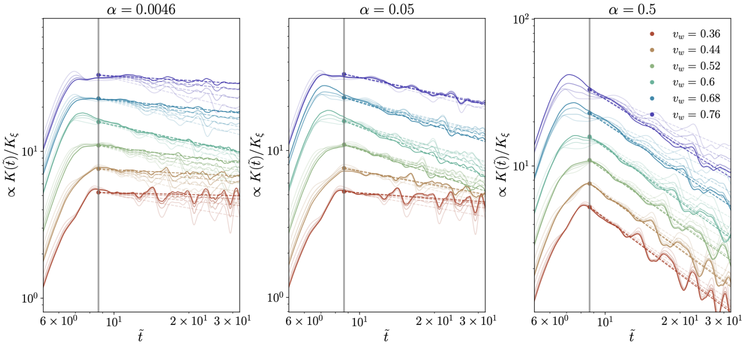
Higgsless simulations of strong PTs[5]



[5] ARP, Stomberg et al., arXiv:2409.03651.

Higgsless simulations (results)[6]

- Kinetic energy decay is observed in the simulations.
- For weak and strong PTs, increasing discretization enhances the decay.
- Potential indication of the development of non-linearities (turbulence).



[6] ARP, Stomberg et al., arXiv:2409.03651 (2024).

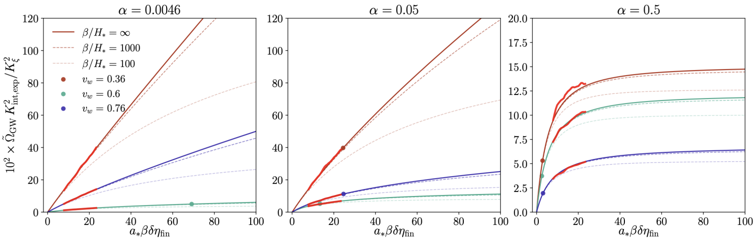
Higgsless simulations (results)[7]

- In the literature, the GW spectrum from sound waves is usually assumed to be

$$\Omega_{\rm GW}(f) = 3\,\tilde\Omega_{\rm GW}\,K^2\,\Upsilon(\tau_{\rm sw})\,(H_* R_*)\,S(f\,R_*)$$

- The linear growth, which only appears when expansion is neglected, is modified when the decay of the source is significant (e.g., due to the development of non-linearities).
- Extended model to proposed locally stationary UETC

$$\Omega_{\rm GW}(f) = 3\,\tilde\Omega_{\rm GW}\,\mathbf{K}^2_{\rm int,exp}\,(H_* R_*)\,S(f\,R_*)$$



[7] ARP, Stomberg et al., arXiv:2409.03651 (2024).

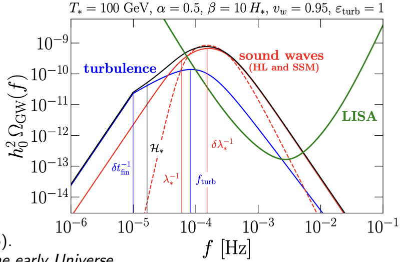
MHD sources of GWs in the early Universe

- Magnetohydrodynamic (MHD) sources of GWs:
  - Compressional motion (e.g. sound waves) generated from first-order phase transitions.
  - (M)HD turbulence from first-order phase transitions.
  - Primordial magnetic fields.

- High-conductivity of the early universe leads to a high-coupling between magnetic and velocity fields.

- Plasma dominated by radiation-like particles can be described by a traceless stress-energy tensor and the fluid equations become conformal invariant.[8]

- Other sources of GWs include
  - Bubble collisions.
  - Cosmic strings.
  - Primordial black holes.
  - Inflation.



[8] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996).
ARP, Midiri, *Relativistic magnetohydrodynamics in the early Universe*, arXiv:2501.05732 (2025).

Primordial magnetic fields

- Magnetic fields can either be produced at or present during cosmological phase transitions.
- The magnetic fields are strongly coupled to the primordial plasma and effectively produce vortical motion, inevitably leading to the development of MHD turbulence.[9]
- Present magnetic fields can be amplified by primordial turbulence via dynamo.[10]

---

[9] J. Ahonen and K. Enqvist, *Phys. Lett. B* **382**, 40 (1996).

[10] A. Brandenburg *et al.* (incl. ARP), *Phys. Rev. Fluids* **4**, 024608 (2019).

Generation of primordial magnetic fields

- Bubble collisions and velocity fields induced by first-order phase transitions can amplify seed magnetic fields.
- Parity-violating processes during the EWPT are predicted by SM extensions that account for baryogenesis and can produce helical magnetic fields through sphaleron decay or B+L anomalies.[11]

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - i\frac{2\sin\theta_w}{gv^2}\boldsymbol{\nabla}\Phi^\dagger \times \boldsymbol{\nabla}\Phi$$

- Axion fields can amplify and produce magnetic field helicity.[12]

$$\mathcal{L} \supset \frac{\phi}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

[11] T. Vachaspati, *Phys. Rev. B* **265**, 258 (1991), T. Vachaspati, *Phys. Rev. Lett.* **87**, 251302 (2001), J. M. Cornwall, *Phys. Rev. D* **56**, 6146 (1997).

[12] M. M. Forbes and A. R. Zhitnitsky, *Phys. Rev. Lett.* **85**, 5268 (2000).

GWs from (M)HD turbulence

- Direct numerical simulations using the PENCIL CODE[13] to solve:
  1. Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken).
  2. Gravitational waves equation.

- In general, large-scale simulations are necessary to solve the MHD nonlinearities (e.g., unequal-time correlators UETC and non-Gaussianities, which require simplifying assumptions in analytical studies).

[13] Pencil Code Collaboration, JOSS **6**, 2807 (2020), `https://github.com/pencil-code/`

Conservation laws for MHD turbulence

$$T^{\mu\nu}{}_{;\nu} = 0, \quad F^{\mu\nu}{}_{;\nu} = -J^{\mu}, \quad \tilde{F}^{\mu\nu}{}_{;\nu} = 0$$

In the limit of subrelativistic bulk flow:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to[14]

$$\begin{aligned}
\frac{\partial \ln \rho}{\partial t} &= -\frac{4}{3}\left(\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho\right) + \frac{1}{\rho}\left[\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \eta \boldsymbol{J}^2\right], \\
\frac{D\boldsymbol{u}}{Dt} &= \frac{\mathbf{u}}{3}\left(\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho\right) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \eta J^2\right] \\
&\quad - \frac{1}{4}\boldsymbol{\nabla}\ln\rho + \frac{3}{4\rho}\boldsymbol{J}\times\boldsymbol{B} + \frac{2}{\rho}\boldsymbol{\nabla}\cdot(\rho\nu\boldsymbol{S}), \\
\frac{\partial \boldsymbol{B}}{\partial t} &= \boldsymbol{\nabla}\times(\boldsymbol{u}\times\boldsymbol{B} - \eta\boldsymbol{J}), \quad \boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B},
\end{aligned} \tag{1}$$

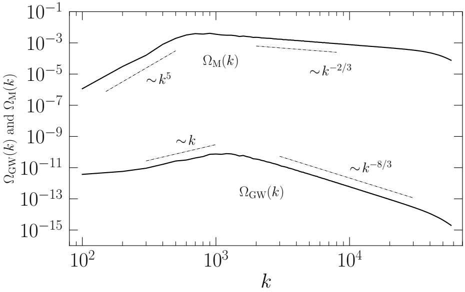
for a flat expanding universe with comoving and normalized $p = a^4 p_{\rm phys}, \rho = a^4 \rho_{\rm phys}, B_i = a^2 B_{i,{\rm phys}}, u_i$, and conformal time t $(\mathrm{d}t = a\mathrm{d}t_c)$.

---

[14] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996).
ARP, Midiri, *Relativistic magnetohydrodynamics in the early Universe*, arXiv:2501.05732 (2025).

Numerical results for decaying MHD turbulence[15]

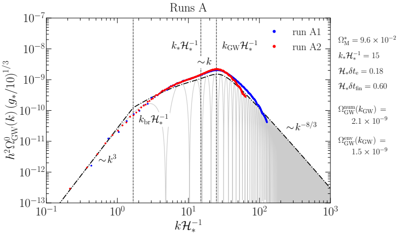
$1152^3, k_* = 2\pi \times 100, \Omega_{\rm M} \sim 10^{-2}, \sigma_{\rm M} = 1$

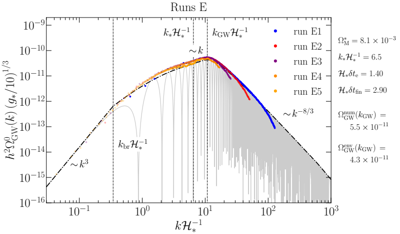


- **Characteristic k scaling in the subinertial range for the GW spectrum.**
- k^2 expected at scales $k < k_*$ and k^3 at $k < H_*$ according to the "top-hat" model (Caprini *et al.*, 2020).

[15] ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020).

Numerical results for nonhelical decaying MHD turbulence[16]





run	$\Omega_{\rm M}^*$	$k_*\mathcal{H}_*^{-1}$	$\mathcal{H}_*\delta t_{\rm e}$	$\mathcal{H}_*\delta t_{\rm fin}$	$\Omega_{\rm GW}^{\rm num}(k_{\rm GW})$	$[\Omega_{\rm GW}^{\rm env}/\Omega_{\rm GW}^{\rm num}](k_{\rm GW})$	n	$\mathcal{H}_*L$	$\mathcal{H}_*t_{\rm end}$	$\mathcal{H}_*\eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	–	–	–	–	–	–	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	–	–	–	–	–	–	512	10π	18	10^{-7}
E3	–	–	–	–	–	–	512	20π	61	10^{-7}
E4	–	–	–	–	–	–	512	30π	114	10^{-7}
E5	–	–	–	–	–	–	512	60π	234	10^{-7}

[16] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

Analytical model for GWs from decaying turbulence

- Assumption: magnetic or velocity field evolution $\delta t_{\rm e} \sim 1/(u_* k_*)$ is slow compared to the GW dynamics ($\delta t_{\rm GW} \sim 1/k$) at all $k \gtrsim u_* k_*$.
- We can derive an analytical expression for nonhelical fields of the envelope of the oscillations[17] of $\Omega_{\rm GW}(k)$.

$$\Omega_{\rm GW}(k, t_{\rm fin}) \approx 3 \left(\frac{k}{k_*}\right)^3 {\Omega_{\rm M}^*}^2 \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)}\, p_\Pi\left(\frac{k}{k_*}\right) \times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\, \delta t_{\rm fin} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\, \delta t_{\rm fin} \geq 1. \end{cases}$$

- p_Π is the anisotropic stress spectrum and depends on spectral shape, can be approximated for a von Kárman spectrum as[18]

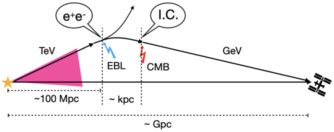
$$p_\Pi(k/k_*) \simeq \left[1 + \left(\frac{k}{2.2 k_*}\right)^{2.15}\right]^{-11/(3\times 2.15)}$$

[17] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022).

[18] ARP *et al.*, arXiv:2307.10744 (2023).

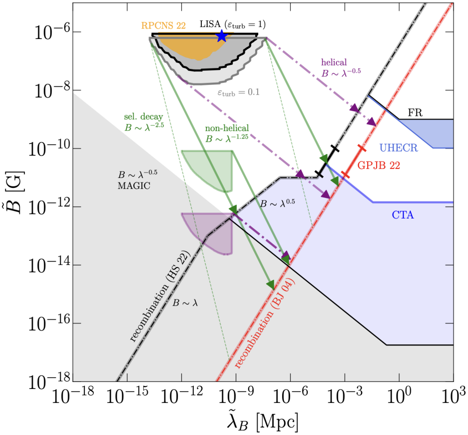
Primordial magnetic fields[30]

- Primordial magnetic fields would evolve through the history of the universe up to the present time and could explain the lower bounds in cosmic voids derived by the Fermi collaboration.[31]



- Maximum amplitude of primordial magnetic fields is constrained by the big bang nucleosynthesis.[32]

- Additional constraints from CMB, Faraday Rotation, ultra-high energy cosmic rays (UHECR).



---

[30] ARP *et al.*, arXiv:2307.10744 (2023).

[31] A. Neronov and I. Vovk, *Science* **328**, 73 (2010).

[32] V. F. Shvartsman, *Pisma Zh. Eksp. Teor. Fiz.* **9**, 315 (1969).

Conclusions

- Velocity and magnetic fields in the early universe can significantly contribute to the stochastic GW background (SGWB) via sound waves and (M)HD turbulence.
- The SGWB produced by non-linear motion requires, in general, performing high-resolution numerical simulations, which can be done using the Pencil Code.
- Since the SGWB is a superposition of different sources, it is extremely important to characterize the different sources, to be able to extract clean information from the early universe physics.
- The interplay between sound waves (acoustic motion) and the development of turbulence is not well understood. It plays an important role on the relative amplitude of both sources of GWs. On-going studies of phase transitions are required to understand this issue.
- LISA, PTA, and next-generation ground-based detectors can be used to probe the origin of magnetic fields in the largest scales of our Universe, which is still an open question in cosmology.
- γ-ray observations (Fermi LAT, CTA) can constrain intergalactic magnetic fields, providing a potential multi-messenger approach to study primordial magnetic fields.

Thank You!

alberto.roperpol@unige.ch

github.com/cosmoGW/cosmoGW
cosmology.unige.ch/users/alberto-roper-pol