Finite-temperature bubble nucleation with shifting scale hierarchies

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Classical scale invariant models

exhibit large supercooling with strong phase transition since barrier persists until low temperatures

$$m_\varphi^2(T) = [\mu_0^2] + m_T^2\,, \tag{1}$$

and the field φ is trapped in the false vacuum $\varphi_{\rm F}$ for a long time until percolation with temperature $T_{\rm p} \ll T_{\rm c}$.

The new-scalar vacuum expectation value (vev) is much larger than the Higgs vev. Need RG improvement to treat vast scale separation:

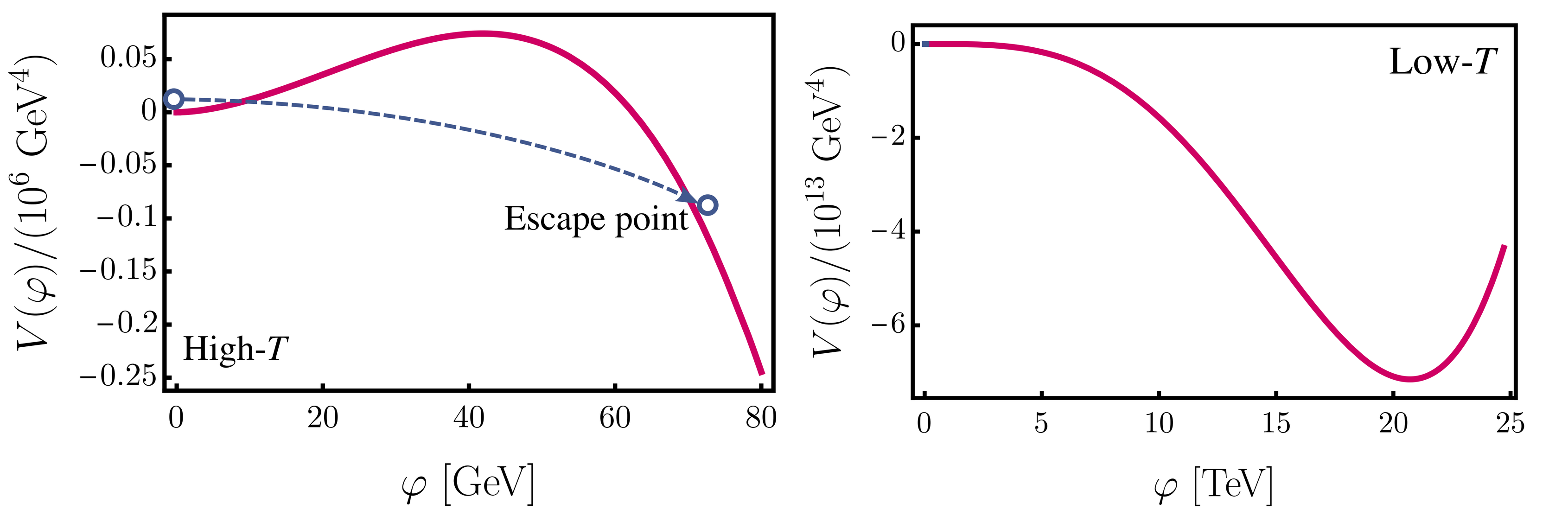

Figure 1. RG-improved potential for benchmark $g = 0.9$, $M = 10^4$ GeV at $T_{\rm n}$ [1].

Split computation into high- and low-temperature regimes:

- **High-T:** Small field regime $M(\varphi) < T$
- **Low-T:** Large field regime $M(\varphi) > T$

BSM extensions without explicit mass term. Example: SU(2)cSM,

$$V^{(0)}(h,\varphi) = \frac{1}{4}(\lambda_h h^4 + \lambda_{h\varphi} h^2\varphi^2 + \lambda_\varphi \varphi^4)\,. \tag{2}$$

To determine the bubble nucleation rate Γ

the action is evaluated on the *bounce* [2] and at finite temperature factorizes into *dynamical* and *statistical* parts

$$\Gamma = A_{\rm dyn} \times A_{\rm stat}\,, \tag{3}$$

with the naive estimates $A_{\rm dyn} \sim T$ and $A_{\rm stat} \sim T^3 e^{-S_{\rm eff}}$.

Since the escape point is in the high-temperature regime [3], there high-T expansion can be used to construct an EFT for nucleation [4].

Bubble tails causing trouble.

30–40% of contribution to $S_{\rm eff}$ from **scale shifters** (i.e. $M(\varphi) \sim \varphi$) in the tail where EFT is invalid. Use nucleation EFT prescription [4]:

- integrate out vector modes to obtain their barrier contribution,
- include their contribution in the prefactor of Γ,
- subtract double-counting from the exponential of Γ.

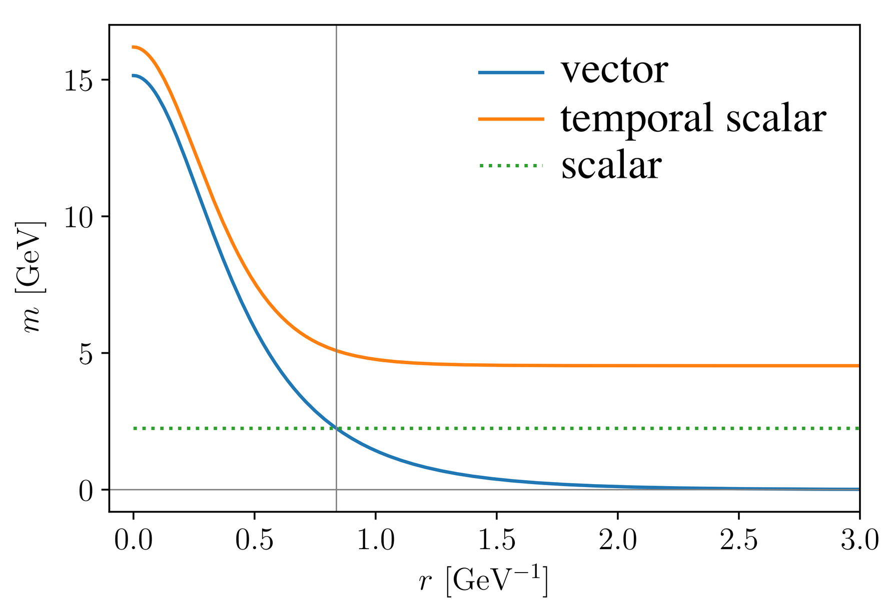

Figure 2. Field-dependent masses evaluated on the bounce for g, M as in figure 1.

Scale shifting

in the tail is *resummed* by computing the full 1-loop statistical part

$$A_{\rm stat} = \underbrace{\prod_a \mathcal{I}_a \mathcal{V}_a \left(\frac{\det \mathcal{O}_a(\varphi_{\rm F})}{\det' \mathcal{O}_a(\varphi_b)}\right)^{\frac{1}{2}}}_{\det_V} \times \underbrace{\mathcal{I}_\phi \left|\frac{\det \mathcal{O}_\phi(\varphi_{\rm F})}{\det' \mathcal{O}_\phi(\varphi_b)}\right|^{\frac{1}{2}}}_{\det_S} e^{-(S[\varphi_b]-S[\varphi_{\rm F}])}\,, \tag{4}$$

with $\mathcal{O}_a(\varphi) = -\partial^2 + m_a^2(\varphi)$. The gauge-mode fluctuation determinant $\det_V$ should be computed on the leading order bounce solution φ_b.

The fluctuation determinants

can be determined by expanding in spherical harmonics

$$\frac{\det \mathcal{O}(\varphi_b(r))}{\det \mathcal{O}(\varphi_{\rm F}(r))} = \prod_{l=0}^{\infty} \left(\frac{\det \mathcal{O}^l(\varphi_b(r))}{\det \mathcal{O}^l(\varphi_{\rm F}(r))}\right)^{\deg(l)}. \tag{5}$$

Gel'fand-Yaglom theorem reduces ratio of functional determinants to initial value problems. Implemented in `BubbleDet` [5]. Statistical rate evaluates to

$$A_{\rm stat} = \det_S \times \det_V \times e^{-S_3^{\rm LO}[\varphi_{3,b}]} \times e^{-\int_x V_3^{\rm NLO}[\varphi_{3,b}]}\,, \tag{6}$$

using the leading action $S_3^{\rm LO}$ and the next-to-leading potential $V_3^{\rm NLO}$.

Comparison of approximations to Γ.

Significant NLO differences between [NLO ∇] (with derivative expansion) and [NLO det] (without), driven by spatial gauge modes.

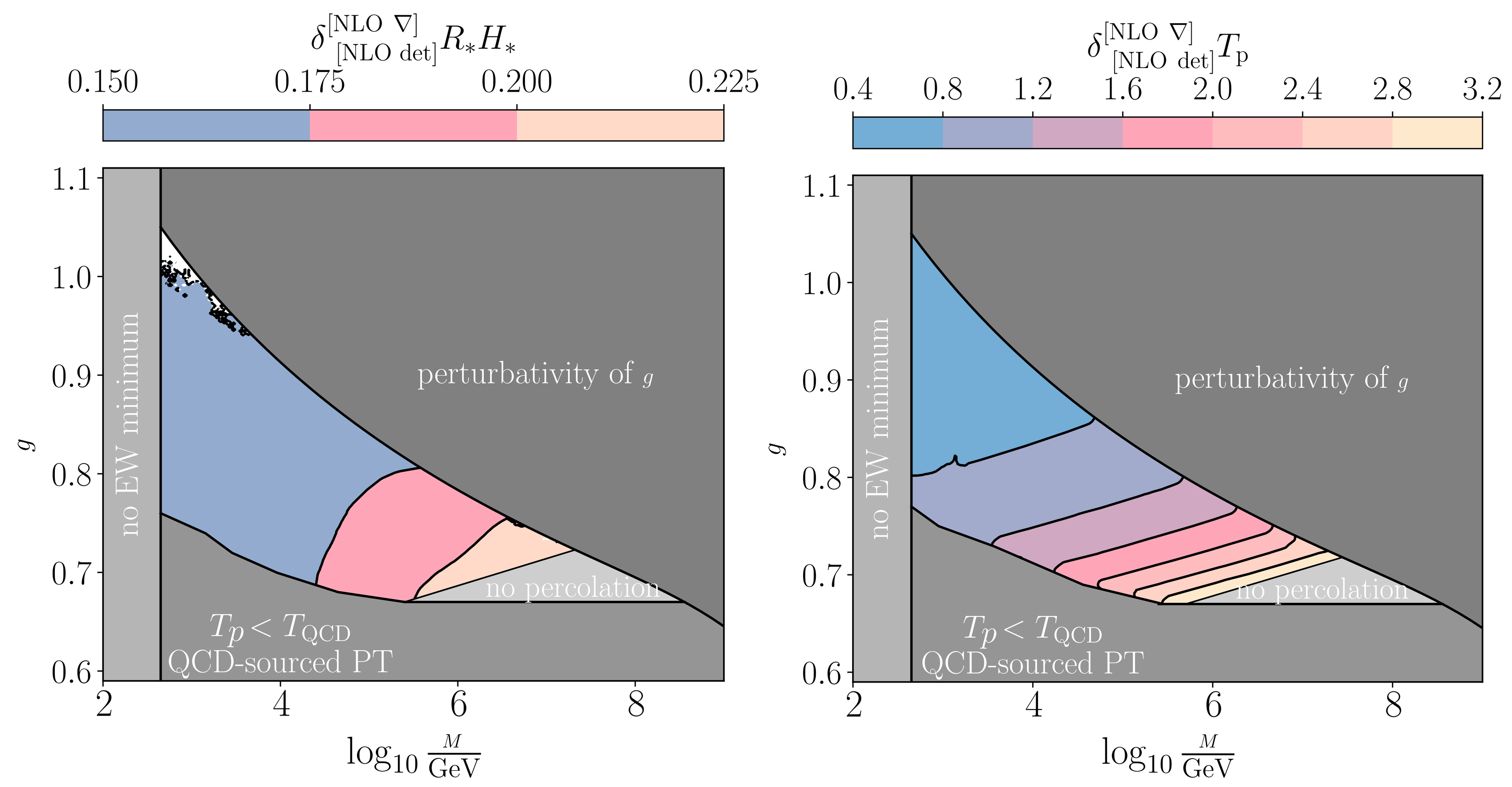

Figure 3. Relative difference of mean bubble radius $R_* H_*$ and T at percolation [6].

Conclusions.

- Higher-order terms in the soft expansion are mandatory for reliable predictions for supercooled phase transitions.
- ∇ Derivative expansion introduces significant errors.
- Full scalar contribution to $S_{\rm eff}$ significantly impacts results.
- $\mathcal{I}$ Jacobian prefactor performs worse than a simple T^4 ansatz.

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