Multi-Messenger Signals from Memory Burden: Detectable Small Primordial Black Holes as DM

Sebastian Zell

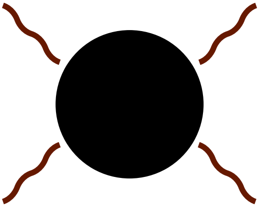
Ludwig Maximilian University & Max Planck Institute for Physics, Munich

Work[1] with Will Barker, Gia Dvali, Benjamin Gladwyn, Marco Michel and Michael Zantedeschi

11^{th} June 2025

[1] *Transitioning to Memory Burden: Detectable Small Primordial Black Holes as Dark Matter*, arXiv:2503.21740.
Inflationary and Gravitational Wave Signatures of Small Primordial Black Holes as Dark Matter, arXiv:2410.11948.
The Timescales of Quantum Breaking, Fortschr. Phys. **71** (2023) 2300163, arXiv:2306.09410. [News article "Where is the boundary to the quantum world?"].

What happens to an evaporating black hole?



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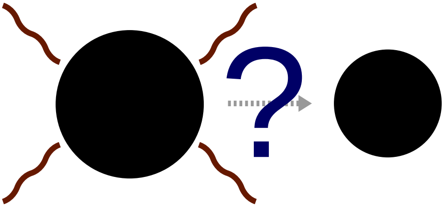

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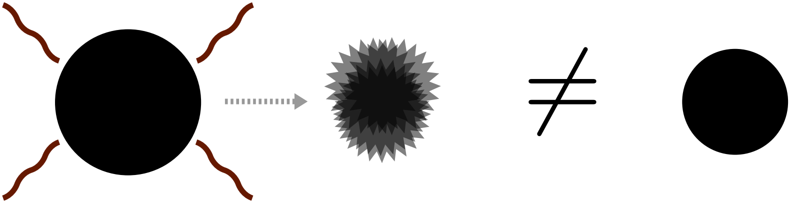

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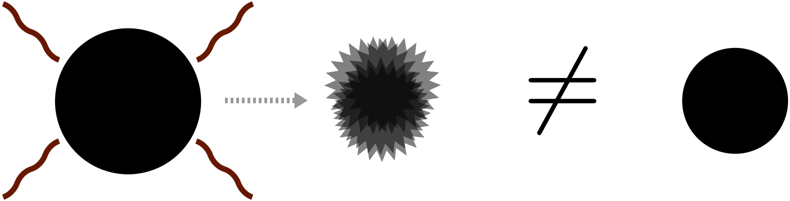

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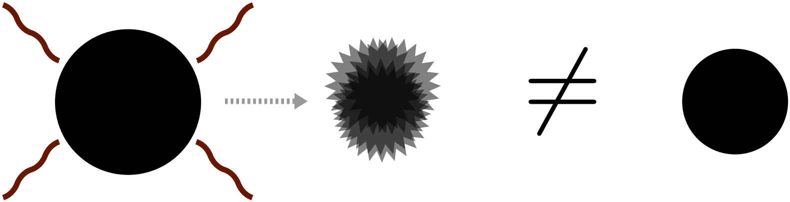

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- Primordial black holes (PBHs) below 10^{15} g as dark matter

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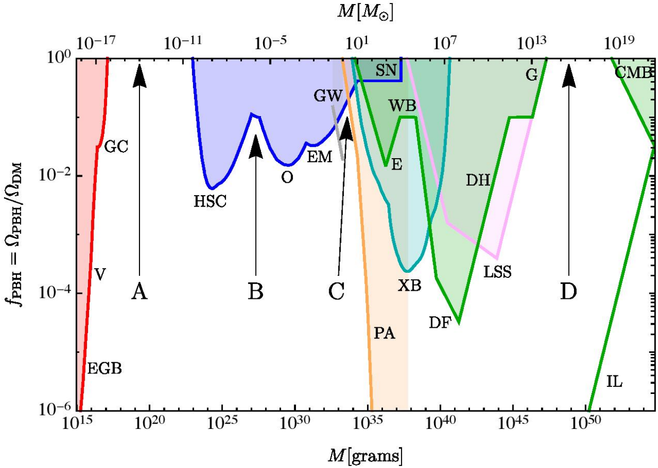

Figure from: B. Carr, F. Kühnel, *Primordial Black Holes as Dark Matter: Recent Developments*, `arXiv:2006.02838`.

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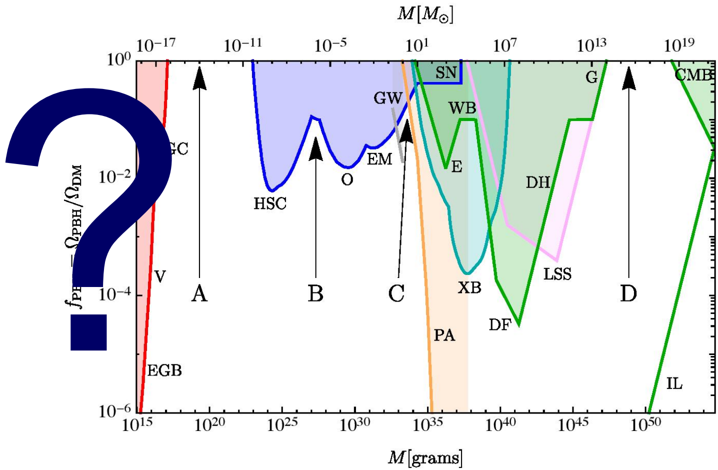

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A. Alexandre, G. Dvali, E. Koutsangelas, *New Mass Window for Primordial Black Holes as Dark Matter from Memory Burden Effect*, `arXiv:2402.14069`.
V. Thoss, A. Burkert, K. Kohri, *Breakdown of Hawking Evaporation opens new Mass Window for Primordial Black Holes as Dark Matter Candidate*, `arXiv:2402.17823`.

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1. Review of memory burden & slowdown of evaporation
2. High-energetic particles from the transition to memory burden
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- Single scale: $r_g \sim GM$

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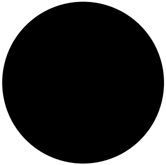

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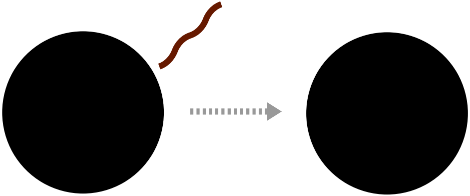

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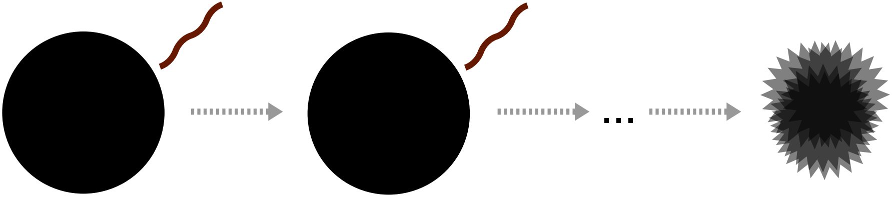

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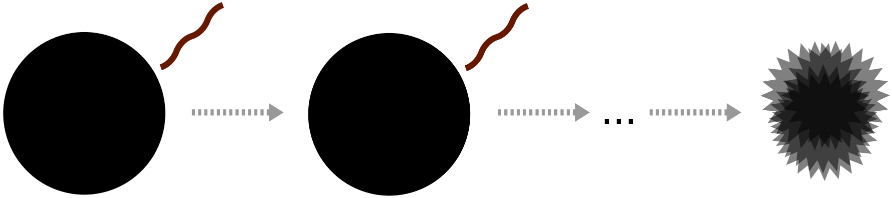

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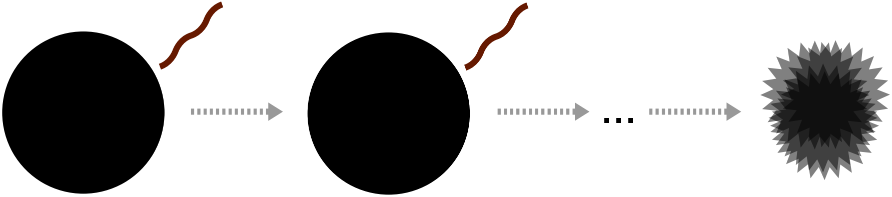

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additional material: MB

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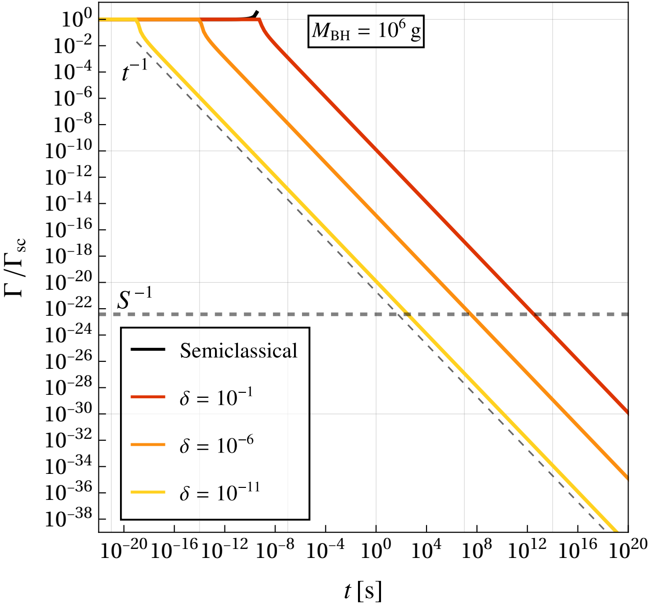

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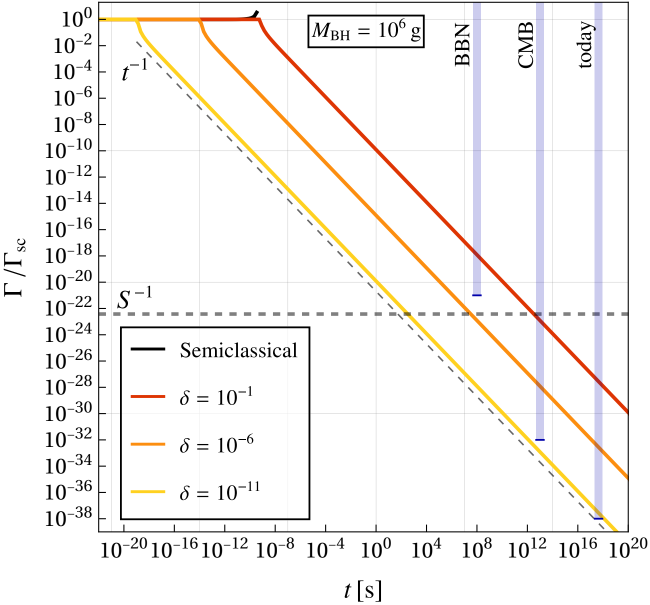

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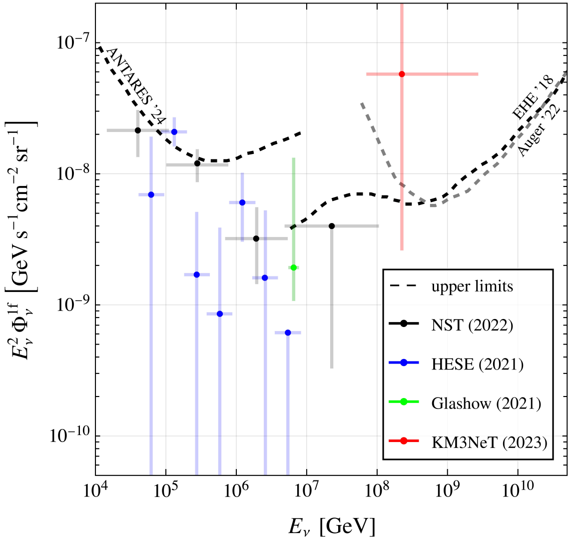
High-energetic neutrino signals[9]



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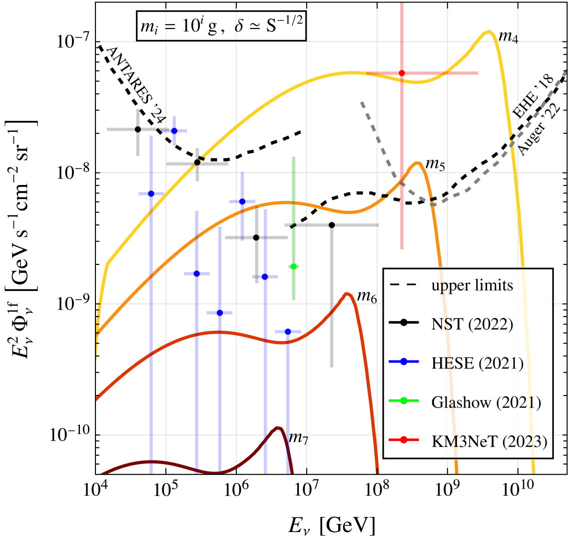

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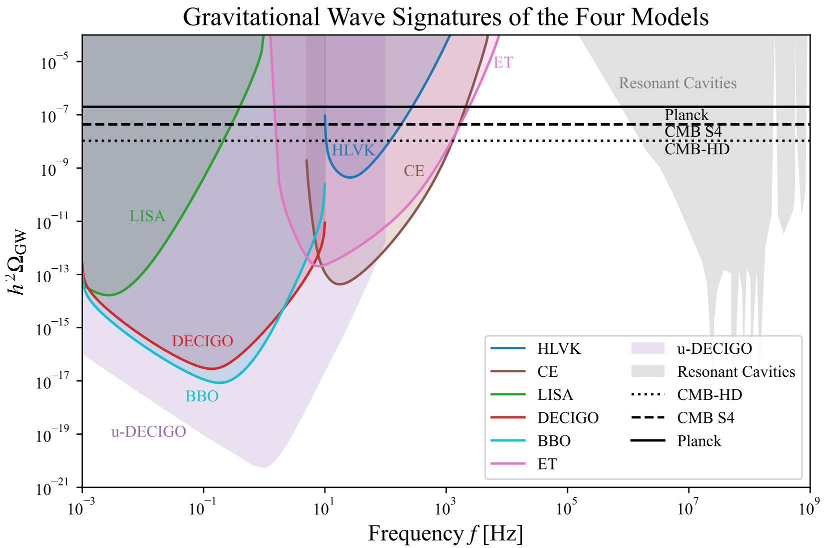

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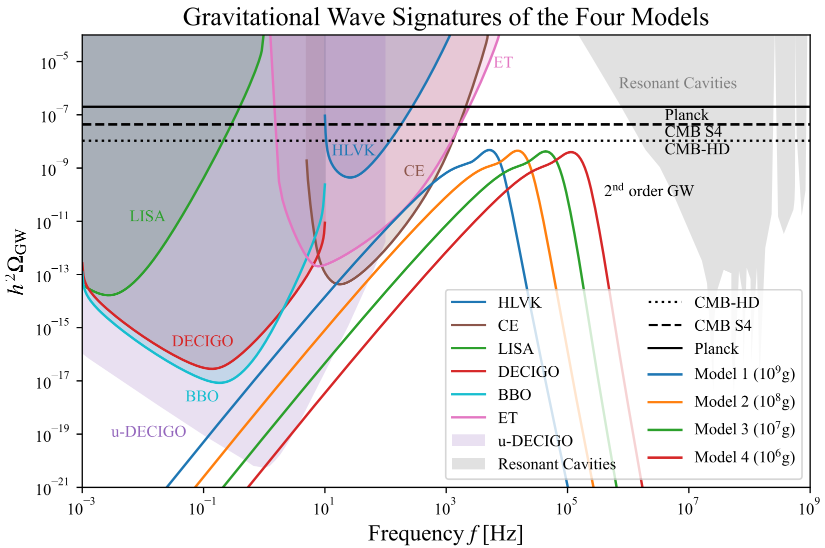

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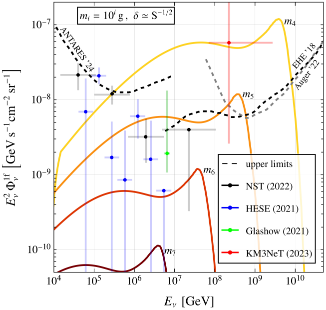

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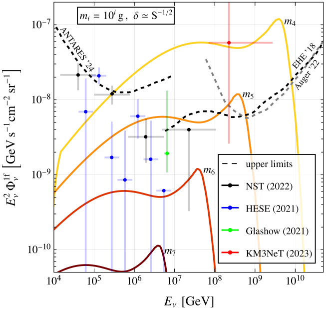

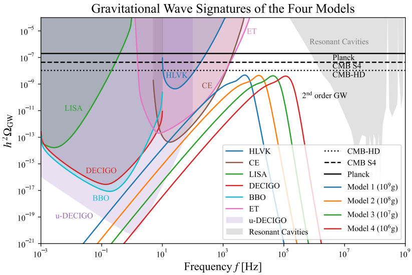

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[2] Memory burden also in inflation: G. Dvali, L. Eisemann, M. Michel, S. Z., *Universe's Primordial Quantum Memories*, arXiv:1812.08749.

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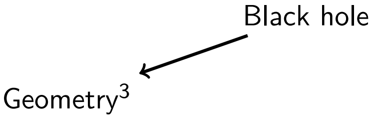

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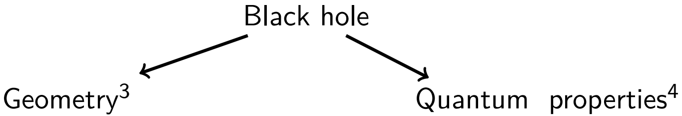

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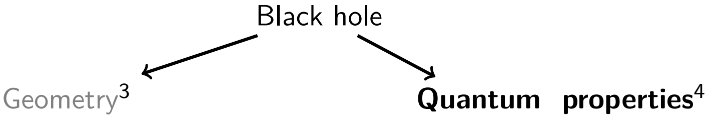
[4] G. Dvali, C. Gomez, *Black Holes as Critical Point of Quantum Phase Transition*, `arXiv:1207.4059`.

Analogue gravity

- Ideally: study evaporation without semi-classical limit
- Easier: analogue systems
  - Share important properties with gravity
  - Accessible for computations and experiments



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- Crucial: all microstates must have similar energy

Enhanced memory storage[5]

- Hamiltonian

$$\frac{\hat{\mathcal{H}}_S}{r_g^{-1}} = \sqrt{S}\sum_{k=1}^{S} \underbrace{\hat{n}_k}_{\hat{a}_k^\dagger \hat{a}_k}$$

[5] G. Dvali, *Critically excited states with enhanced memory and pattern recognition capacities in quantum brain networks: Lesson from black holes*, arXiv:1711.09079.
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Enhanced memory storage[5]

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- Dictionary

$\hat{n}_0$: carries mass

$\langle \hat{n}_0 \rangle = S$: black hole state

$\hat{n}_k$: carry entropy

$\hat{n}_b$: Hawking quanta

[5] G. Dvali, *Critically excited states with enhanced memory and pattern recognition capacities in quantum brain networks: Lesson from black holes*, `arXiv:1711.09079`.
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Time evolution

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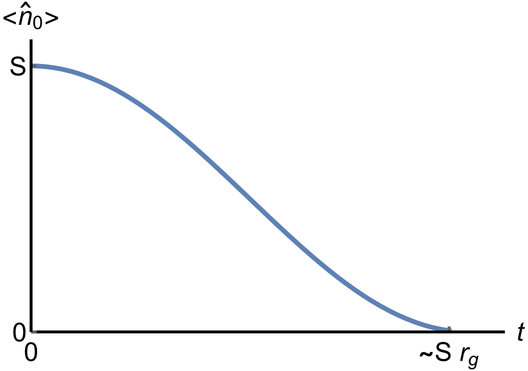

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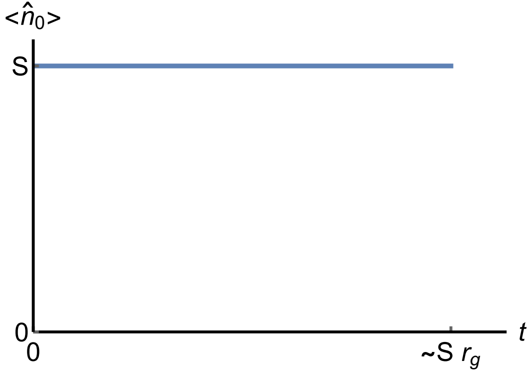

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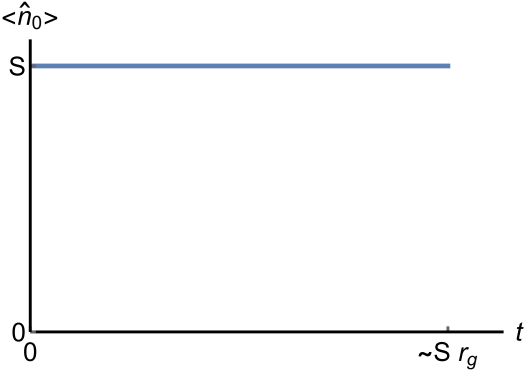

Memory burden:[6] entropy prevents evaporation

[6] G. Dvali, *A Microscopic Model of Holography: Survival by the Burden of Memory*, arXiv:1810.02336.

Full model[15]

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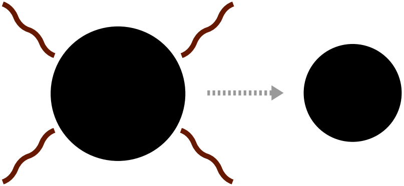

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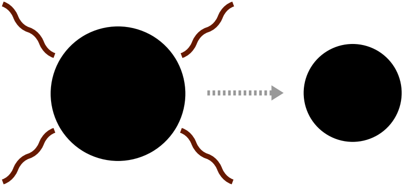

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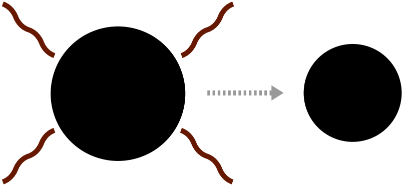

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- Exact time evolution:[16] transition suppressed dynamically

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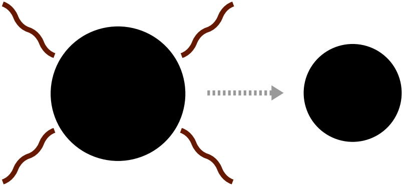

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- Exact time evolution:[16] transition suppressed dynamically
- Slowdown at the latest after half evaporation back

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Black hole criticality

- Gravitational coupling

$$\alpha = \hbar G r_g^{-2}$$

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- Critical collective coupling[17]

$$\alpha N = 1$$

[17] G. Dvali, C. Gomez, *Black Holes as Critical Point of Quantum Phase Transition*, arXiv:1207.4059.

Imitate criticality

- Prototype model

$$\hat{\mathcal{H}} = \sum_{k=1}^{Q} \left(\hat{n}_k - \frac{\alpha}{4} \left(2\hat{n}_0 \hat{n}_k + \hat{a}_0^{\dagger\,2} \hat{a}_k^2 + \hat{a}_k^{\dagger\,2} \hat{a}_0^2 \right) \right)$$

$$+ \frac{C_m}{2} \sum_{k=1}^{Q} \sum_{l=k+1}^{Q} f(k,l) \left(\hat{a}_k^{\dagger\,2} \hat{a}_l^2 + \text{h.c.} \right) \, .$$

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- Critical point

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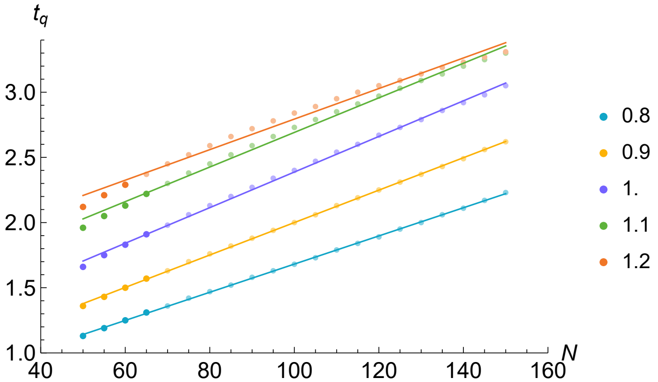
- Study quantum break-time:[18]
  timescale of breakdown of semi-classical approximation

[18] G. Dvali, C. Gomez, D. Flassig, A. Pritzel, *Scrambling in the Black Hole Portrait*, `arXiv:1307.3458`.

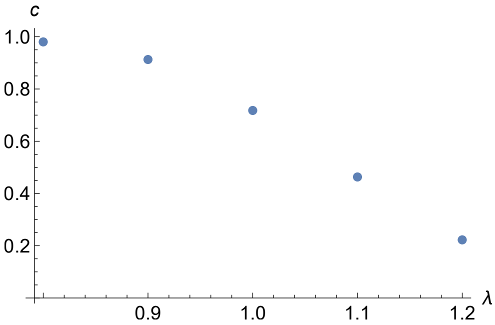
Result[19]



[19] M. Michel, S. Z., *TimeEvolver: A Program for Time Evolution With Improved Error Bound*, `arXiv:2205.15346`.
M. Michel, S. Z., *The Timescales of Quantum Breaking*, `arXiv:2306.09410`.

Result[19]

$$t_q = a \cdot N^c + b \; ,$$



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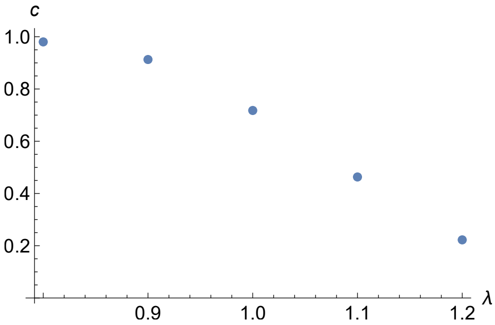

$$\lambda = \alpha N = 1 \quad \Rightarrow \quad c \approx 0.5$$

back

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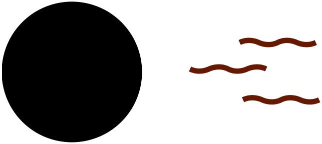
Page time[20]



[20] D. Page, *Information in black hole radiation*, arXiv:hep-th/9306083.

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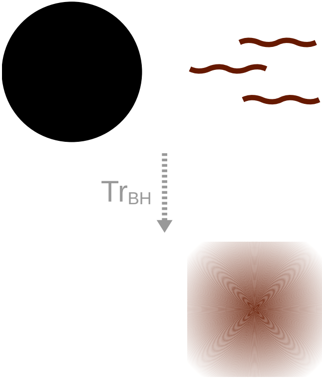

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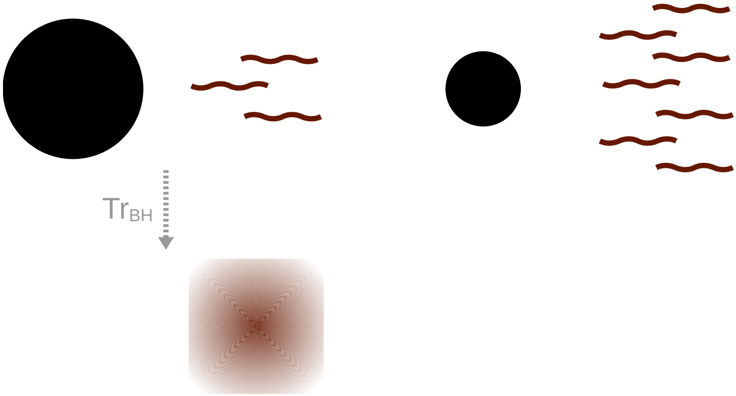

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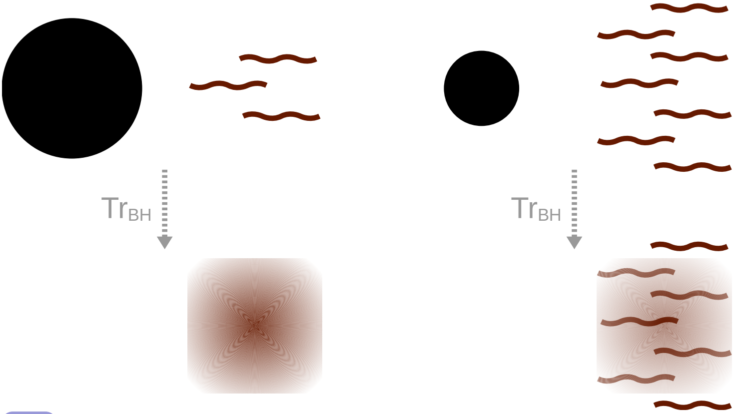

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Classical black hole: no hair

- Geometry fully determined by mass

$$r_g \sim GM$$

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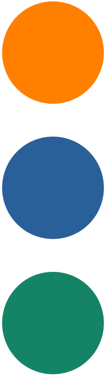

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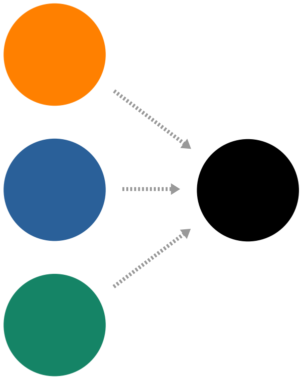

Classical black hole: no hair

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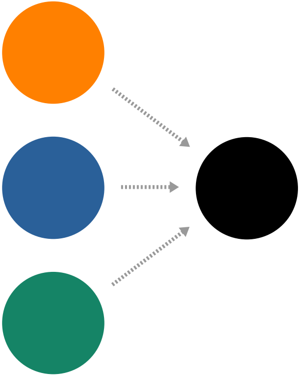
$$r_g \sim GM$$

- No hair outside[21]



[21] See e.g., P. Chrusciel, J. Costa, M. Heusler, *Stationary Black Holes: Uniqueness and Beyond*, arXiv:1205.6112.

Quantum black hole: entropy[22]

- Entropy

$$S \sim \frac{r_g^2}{\hbar G}$$

[22] J. Bekenstein, *Black holes and entropy*, Phys. Rev. D **7** (1973).

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- Black holes quantum-mechanically distinct

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Quantum black hole: entropy[22]

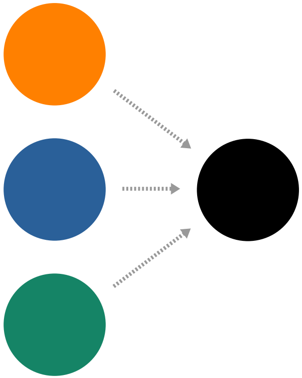
- Entropy

$$S \sim \frac{r_g^2}{\hbar G}$$

- Black holes quantum-mechanically distinct
- $\exp(S)$ different versions of a black hole of mass M

[22] J. Bekenstein, *Black holes and entropy*, Phys. Rev. D **7** (1973).

Add evaporation[17]



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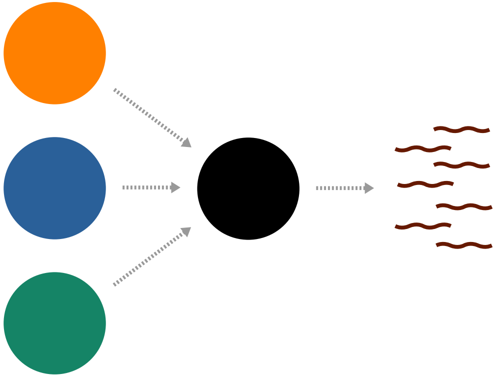

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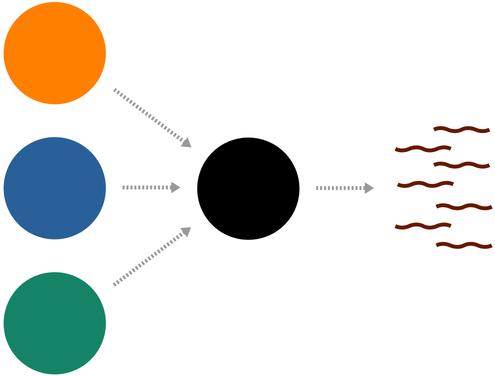

- Not unitary: information about initial state lost[18]

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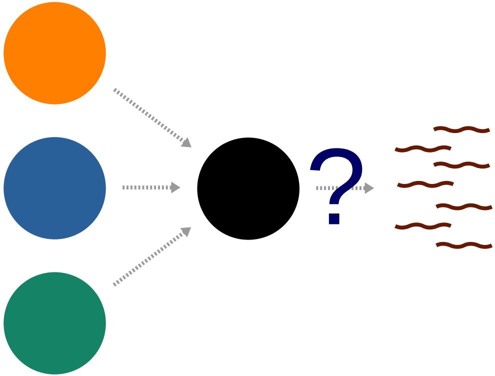

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- Question: how long is Hawking evaporation valid?

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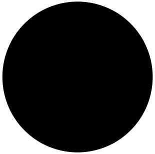

$|\mathrm{BH}; M\rangle$

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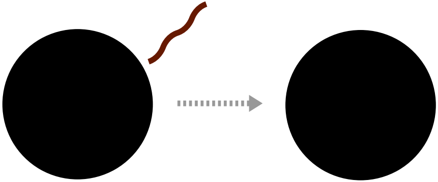

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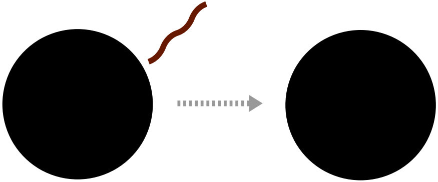

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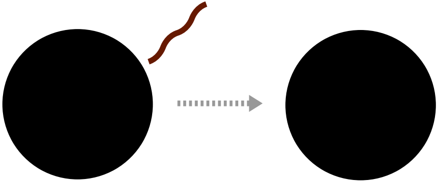

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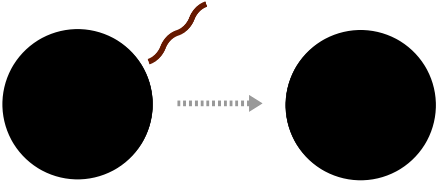

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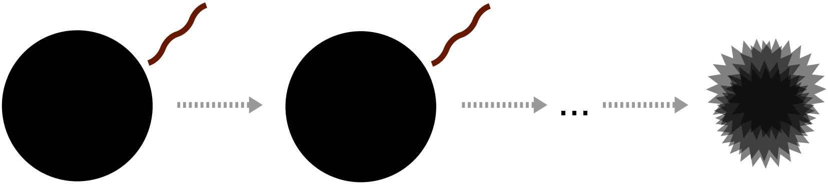

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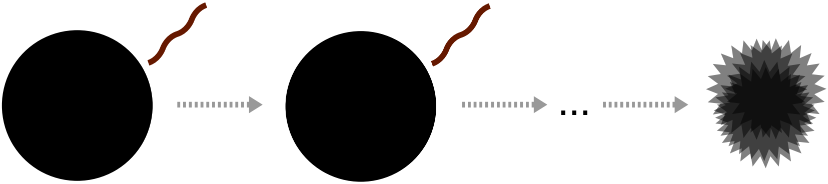

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- Full breakdown of semi-classical description back

Microscopic MB[19]

- Microscopic model

$$\frac{\hat{\mathcal{H}}}{r_g^{-1}} = \hat{n}_0 + \sqrt{S}\left(1 - \frac{\hat{n}_0}{S}\right)^p \sum_{k=1}^{S} \hat{n}_k$$

[19] G. Dvali, L. Eisemann, M. Michel, S. Z., *Black Hole Metamorphosis and Stabilization by Memory Burden*, arXiv:2006.00011.

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- Critical value

$$q \equiv \frac{(S - n_0)_{\text{crit}}}{S} = \left(p\sqrt{S}\right)^{-1/(p-1)}$$

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Transition to MB[20]

- Increased energy gap

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The transition[21]

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- Result: “slow” change of rate

$$\frac{\mathrm{d}M(t)}{\mathrm{d}t} \sim -r_g^{-1}\,\Gamma_{\rm sc}\frac{\delta\,\tau_{\rm SC}}{t}$$

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No dependence on q