Primordial Black Holes with Non-Gaussianities

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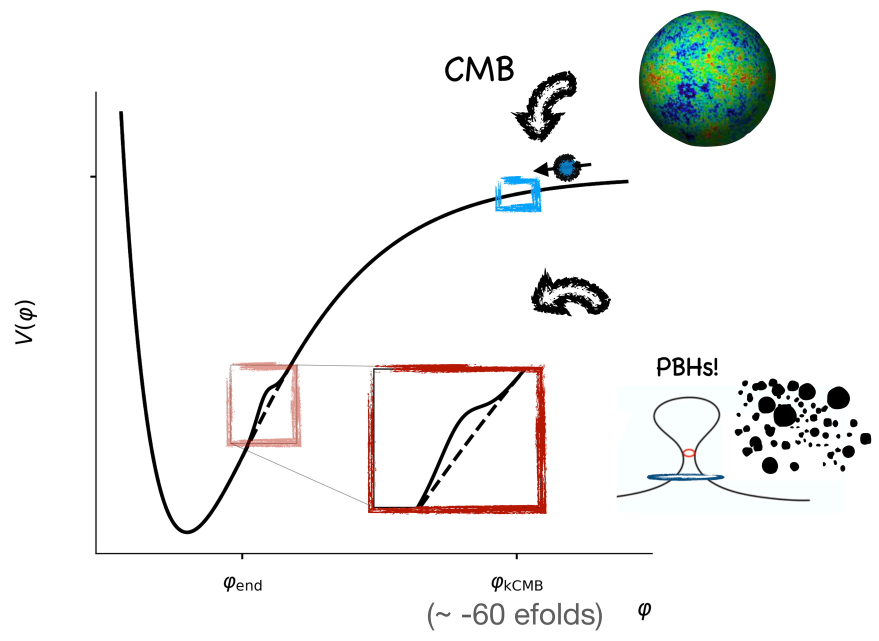
[Ref: Inui, **Joana,** Yada, Motohashi, Pi, Yokoyama. *arXiv:2411.07647*]

PBH as (perhaps all) Dark Matter

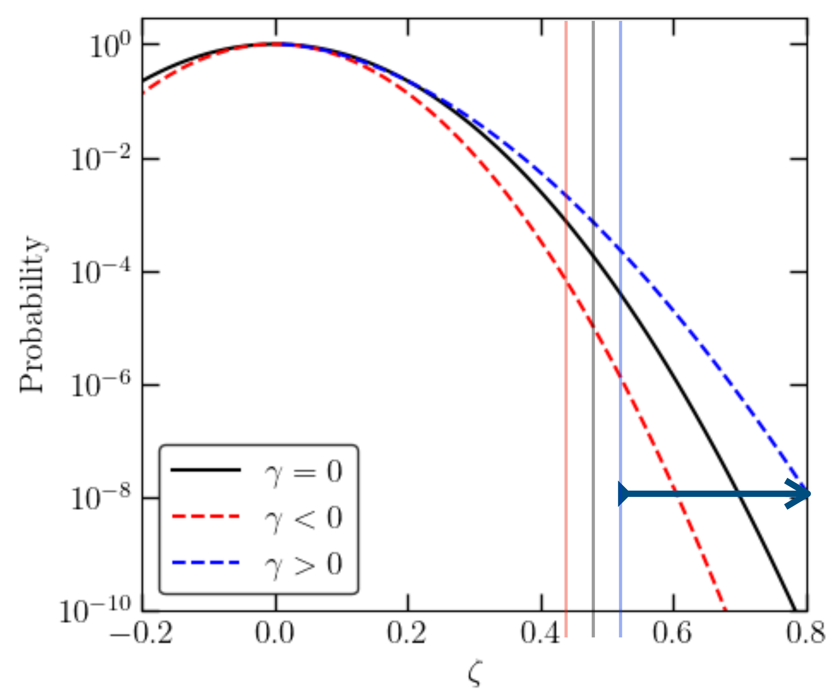
PBHs formed during the early Universe are exciting candidates for the dark matter. In this work, we focus on PBHs formed by the collapse of adiabatic curvature perturbation soon after the end of inflation, during the radiation domination. We take a model independent approach by exploiting the logarithmic duality [Pi, Sasaki 2211.13932] which can probe a variety of models (constant/ultra-slow roll, curvaton, etc.) while taking into account the role of local Non-Gaussianties:

Logarithmic curvature profile $\hat{\zeta}(\vec{x})$,
non-Gaussian parametrization γ

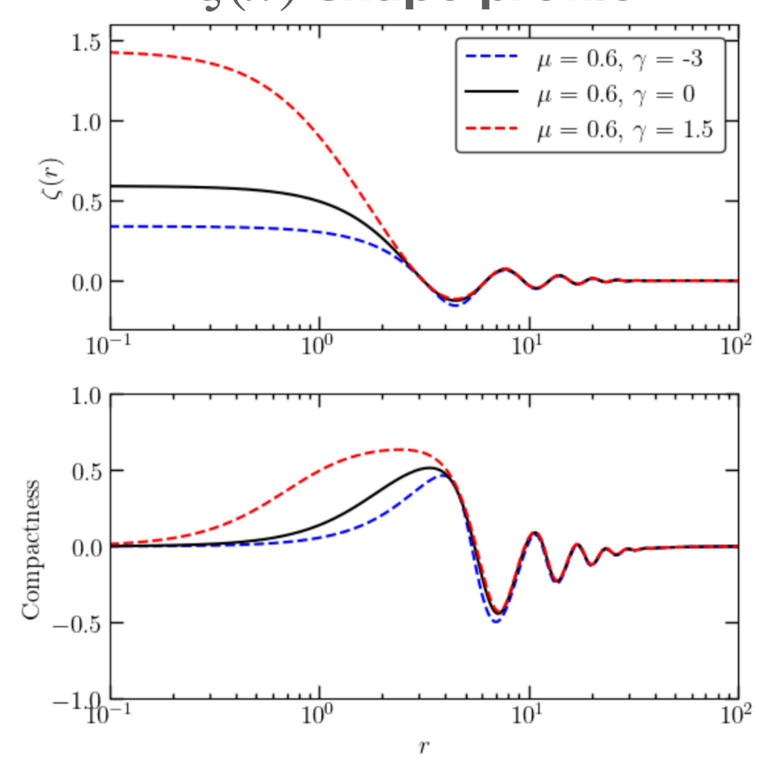
$$\hat{\zeta}(r) = -\frac{1}{\gamma}\ln\left(1-\gamma\zeta_G(r)\right)$$
$$\approx \zeta_G + f_{\rm nl}\zeta_G^2 + g_{\rm nl}\zeta^3 + h_{\rm nl}\zeta^4 + \dots$$
$$f_{\rm nl} = \frac{\gamma}{2},\ g_{\rm nl} = \frac{\gamma^2}{3},\ h_{\rm nl} = \frac{\gamma^3}{4},\ \dots$$



$\hat{\zeta}(\vec{x})$ Probability distribution



$\hat{\zeta}(\vec{x})$ shape profile



$$P(\zeta) = \left|\frac{d\zeta_G}{d\zeta}\right| P_G(\zeta_G) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{1}{2\gamma^2\sigma^2}\left(e^{-\gamma\zeta}-1\right)^2 - \gamma\zeta\right]$$

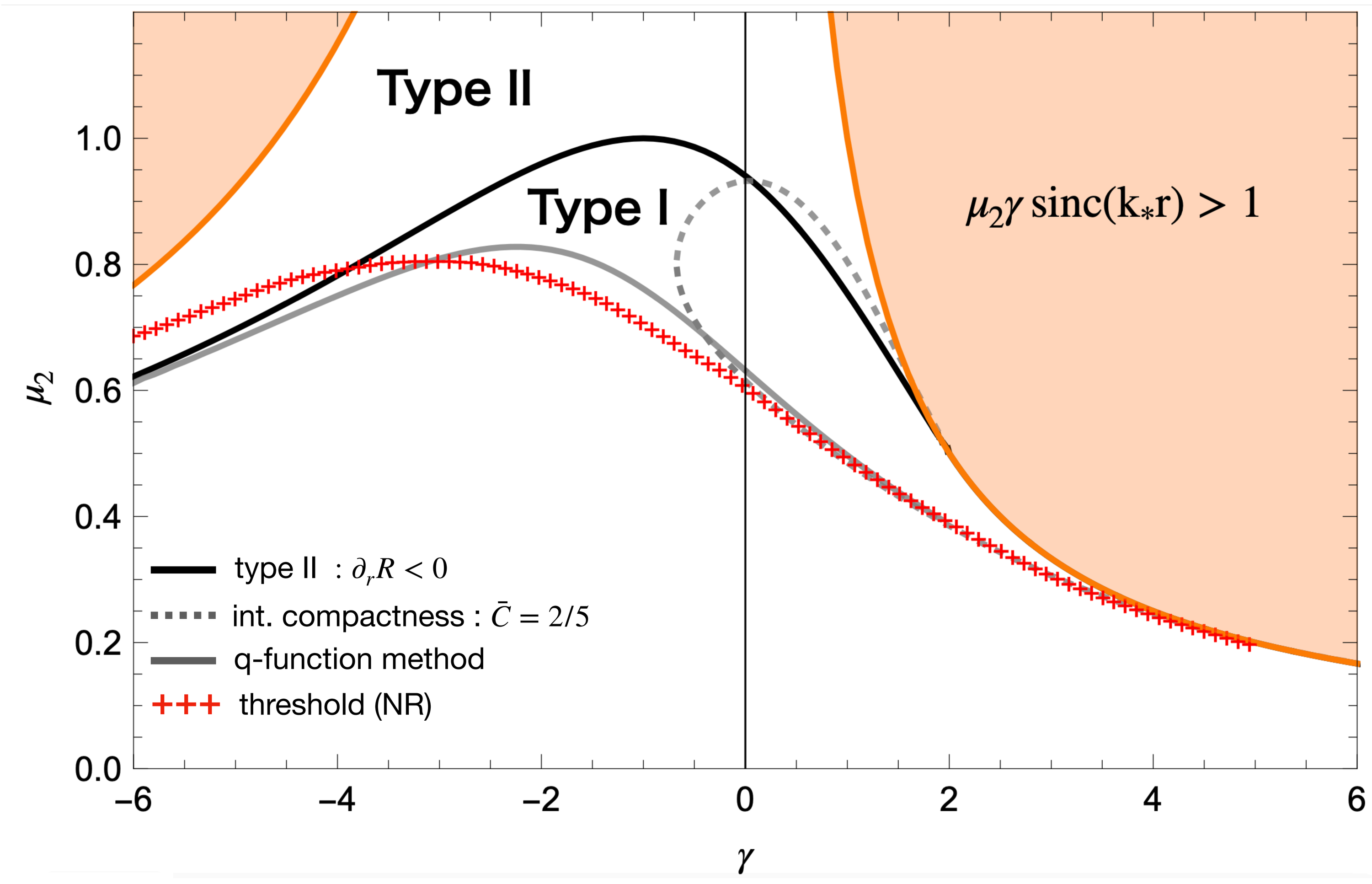
$$\mathscr{P}_G(k) = A_G\delta(\ln k - \ln k_\star) \qquad \zeta_G(r) = \mu_2\frac{\sin(kr)}{kr} \qquad \omega = \frac{1}{3}$$

PBH Formation Thresholds (Num. Relativity)

Knowing the thresholds precisely, it is crucial to compute the expected PBH abundances and the induced gravitational wave signal. We rely on full Numerical Relativity simulations to precisely compute the thresholds μ_c as a function of the Non-Gaussianity parameter γ.

PBH masses can be analytically computed by using the critical collapse relation $M_{\rm PBH} \propto \left(\mu_2 - \mu_{2,c}\right)^p (k_* r_{\rm m})^2 M_H$, and the abundances by using the peak-theory formalism: [Kitajima, Tada, Yokoyama, Yoo, 2109.00791]

$$f_{\rm PBH}(M) = \frac{\rho_{\rm PBH,0}}{\rho_{\rm DM,0}} = \left(\frac{\Omega_{\rm DM,0}h^2}{0.12}\right)^{-1}\left(\frac{M}{10^{20}\,{\rm g}}\right)\left(\frac{k_*}{1.56\times10^{13}\,{\rm Mpc}^{-1}}\right)^3\left(\frac{\left|\frac{d\ln M}{d\mu_2}\right|^{-1} f\left(\frac{\mu_2(M)}{\sqrt{A_G}}\right)\mathcal{N}(\mu_2(M),\sqrt{A_G})}{1.4\times10^{-14}}\right)$$



Note: Type I and Type II fluctuations distinguishes between space-time geometries with monotonic/non-monotonic areal radius $R = a(t)re^{\zeta(\vec{x})}$. Contrary to expectations, Type II fluctuations do not necessarily lead to PBH formation, i.e. when strong negative non-Gaussianity is present ($\gamma < -4$).

PBH Scalar Induced GWs: Observables

From the GW tensors equations of motion, [Abe, Inui, Tada, Yokoyama, 2209.13891]

$$\ddot{h}_\lambda + 2H\dot{h}_\lambda + k^2 h_\lambda = 4S_\lambda \qquad S_\lambda \propto \zeta(\vec{q})\zeta(|\vec{k}-\vec{q}|)$$

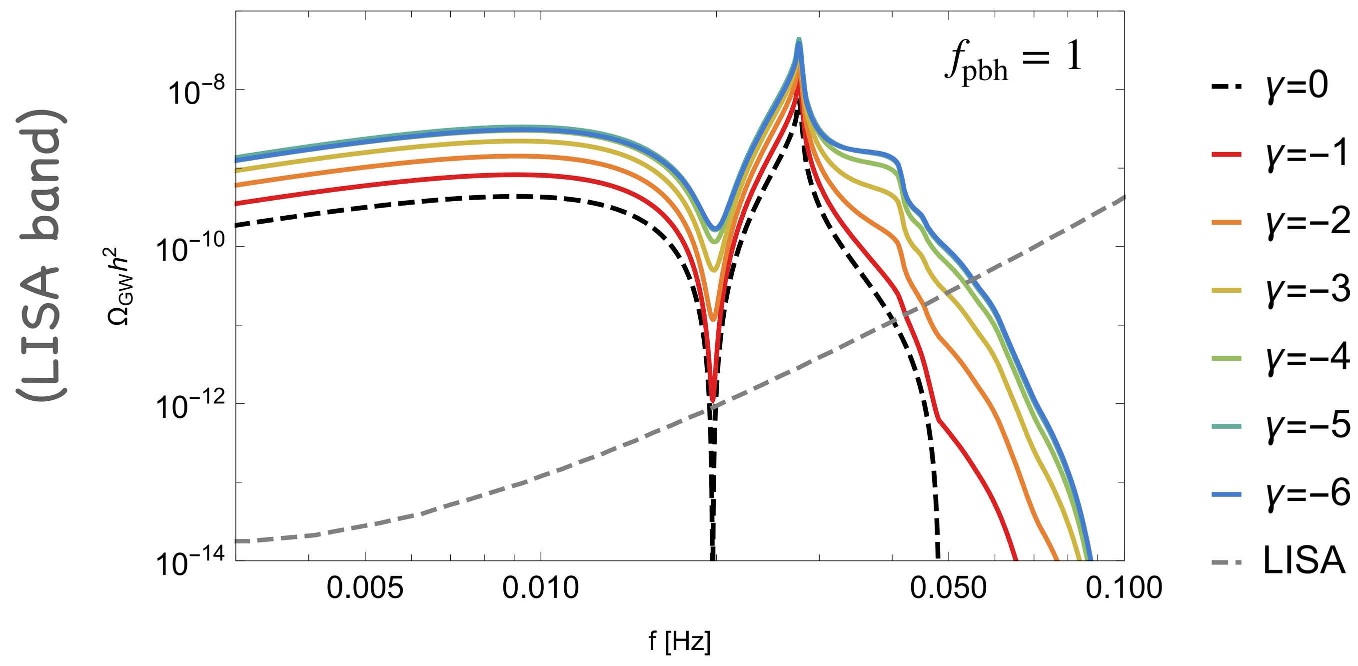
$$h_\lambda = \int\frac{d^3q}{(2\pi)^3}Q_\lambda(k,\vec{q})I_k(|\vec{k}-\vec{q}|,\vec{q},t)\zeta(\vec{q})\zeta(|\vec{k}-\vec{q}|)$$

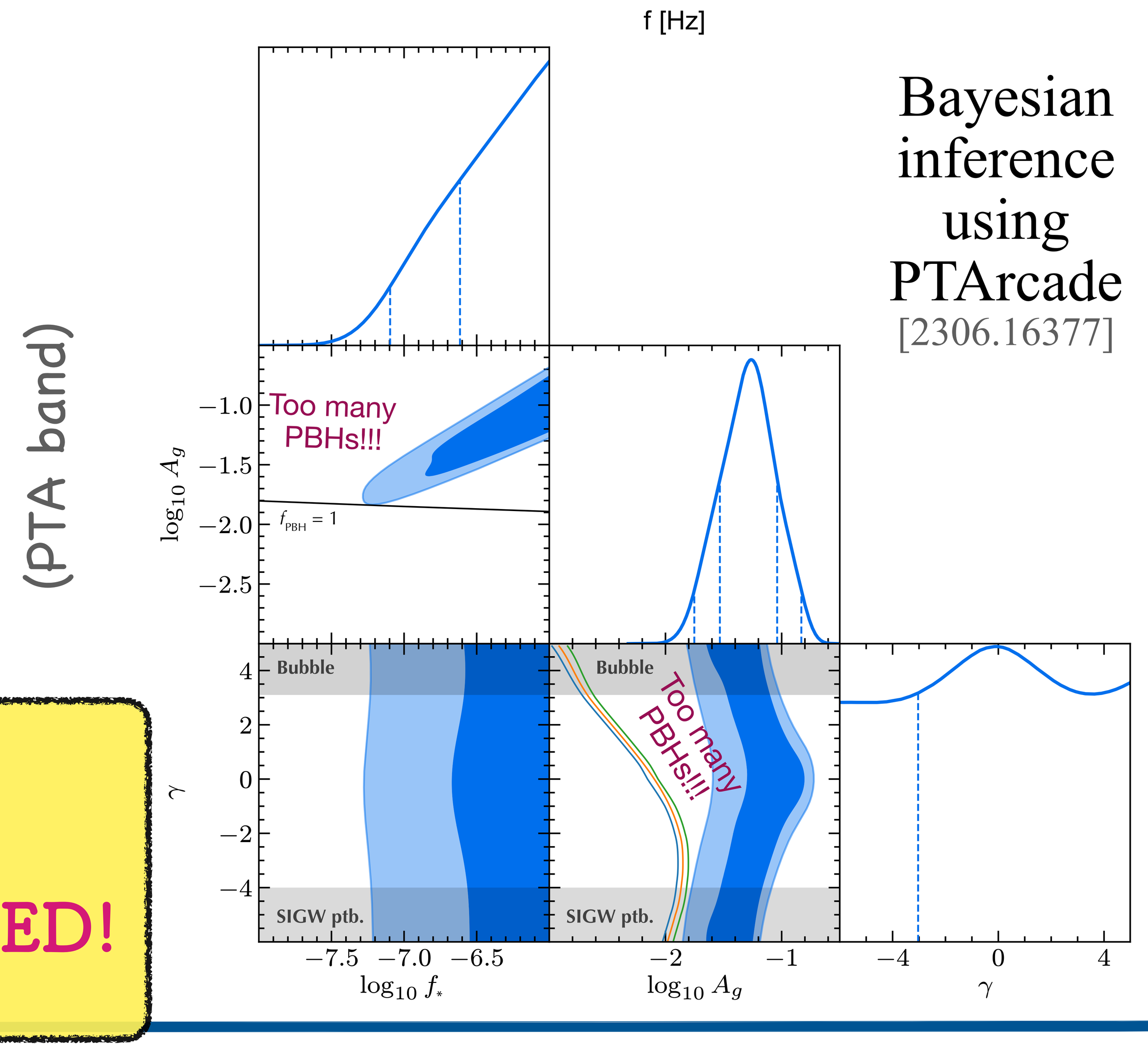
TT mode — gravitation FF — curvature perturb.,

the SIGWs perturbative are computed

$$\Omega_{\rm GW} = \frac{1}{48}\sum_{\lambda,\lambda'}\overline{\mathscr{P}_{\lambda\lambda'}} \propto \langle hh\rangle$$

$$\langle hh\rangle \propto \langle\zeta\zeta\zeta\zeta\rangle \propto \quad \langle\zeta_G\zeta_G\zeta_G\zeta_G\rangle + \dots\mathcal{O}(A_g^2)$$
$$+\quad f_{\rm NL}\langle\zeta_G\zeta_G\zeta_G^2\zeta_G^2\rangle + g_{\rm NL}\langle\zeta_G\zeta_G\zeta_G\zeta_G^3\rangle + \dots\mathcal{O}(A_G^3)$$
$$+\quad f_{\rm NL}^4\langle\zeta_g^2\zeta_G^2\zeta_G^2\zeta_G^2\rangle + f_{\rm NL}^2 g_{\rm NL}^2\langle\zeta_G\zeta_G^2\zeta_G^2\zeta_G^3\rangle + \dots\mathcal{O}(A_G^4)$$





Bayesian inference using PTArcade [2306.16377]

To explain the PTA signal, **PBH overproduction is the main problem.**

can it be avoided? Probably! for Non-monochromatic $\mathscr{P}_\zeta(k)$. STAY TUNED!