UNIVERSITÉ DE GENÈVE

Gravitational lensing of GWs: new insights from the wave optics regime

COSMOFondue

Martin Pijnenburg – PhD student,
University of Geneva

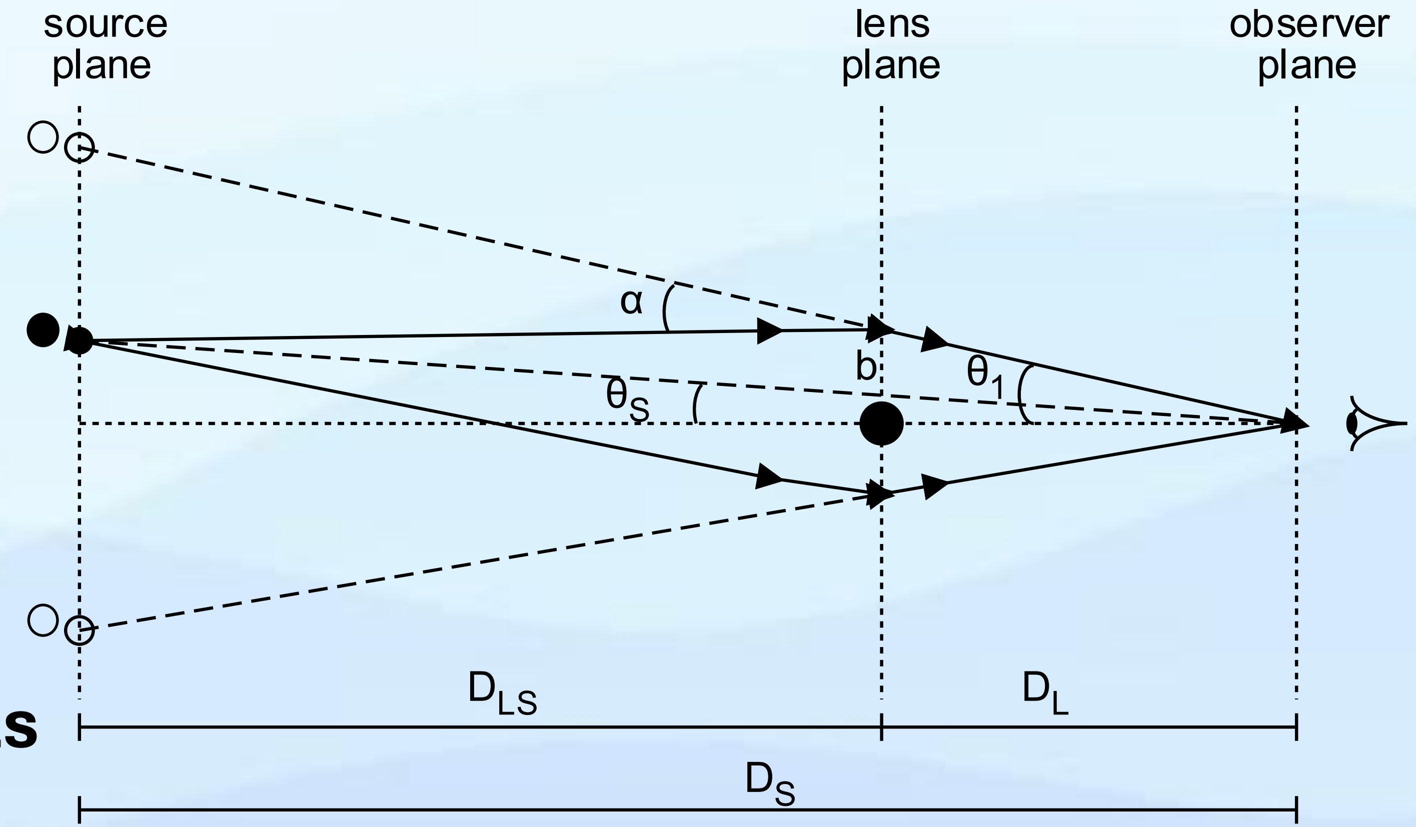
Gravitational lensing: EM waves vs gravitational waves

Null geodesics are bent by masses:

→ Usual lensing picture (deflection angle, etc.)

→ Key prediction of General Relativity

→ **Both EM and GW signals**



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Is lensing really the same for EM and GW signals ? *Some ideas...*

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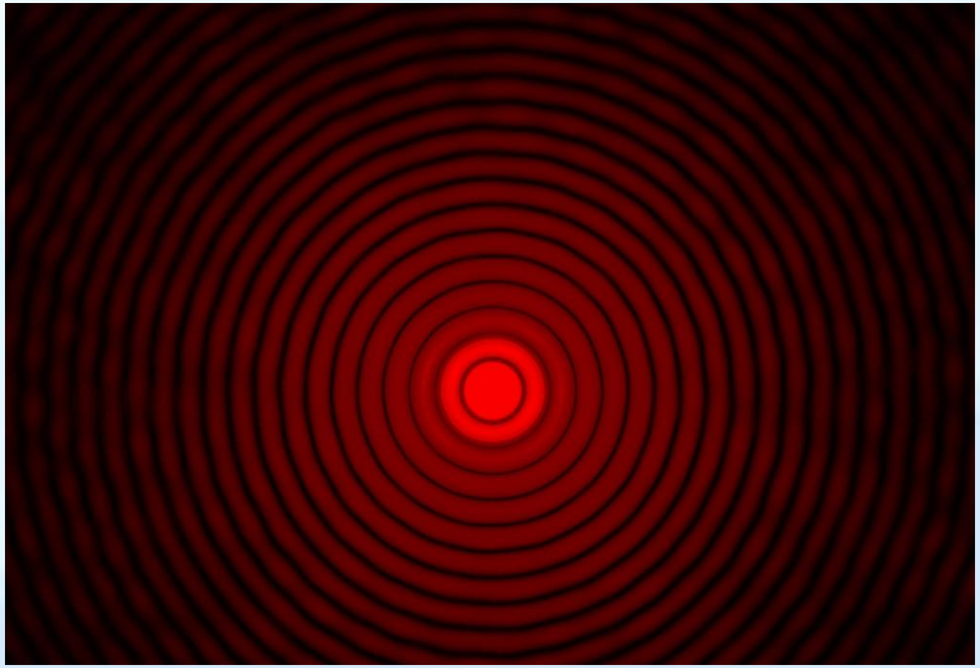
EM waves	GW
Extended sources	~ Point-like sources $\longrightarrow$ mainly magnification
Observed λ_{wave} : $\leq \mathcal{O}(1\text{m})$	Observed λ_{wave} : $\leq \mathcal{O}(5 \cdot 10^7\text{m})$ (LIGO, ET) $\leq \mathcal{O}(5 \cdot 10^{10}\text{m})$ (LISA) $\leq \mathcal{O}(5 \cdot 10^{16}\text{m})$ (PTA)
Spin 1 (photon)	Spin 2, tensorial signal

Gravitational lensing: EM waves vs gravitational waves

Point particle null-geodesics, known limitation in the EM case:

PSF (Airy disk)



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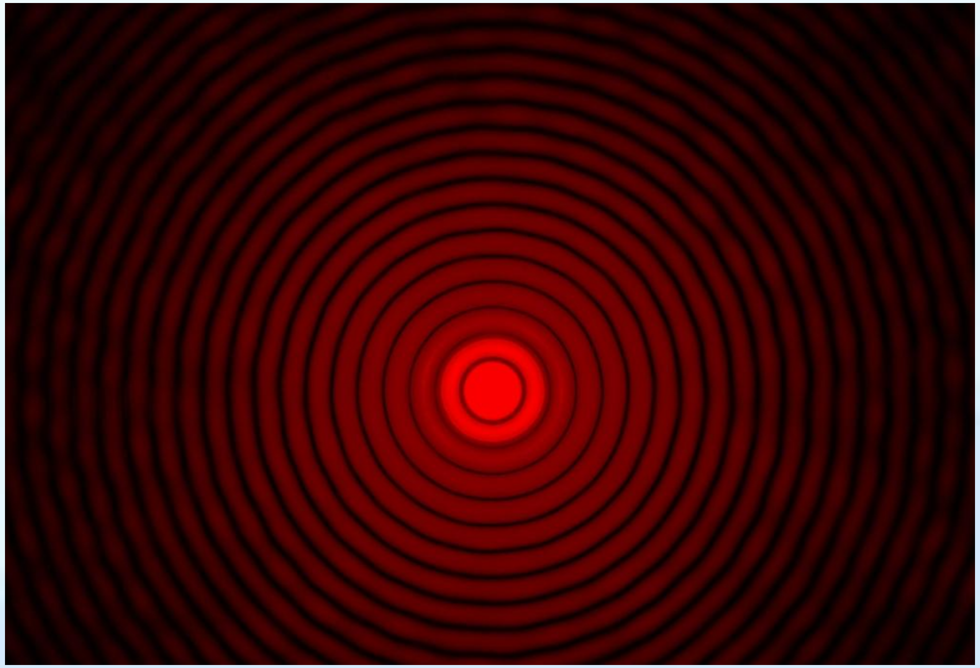

More generally : diffraction

$$\lambda_{\mathrm{wave}} \sim \text{obstacle size}$$

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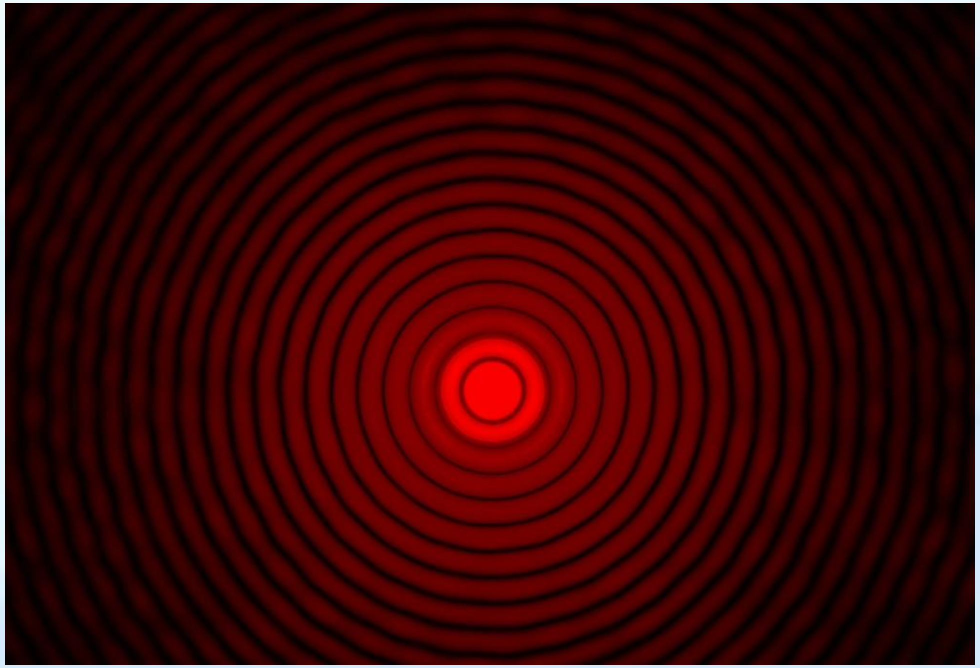

More generally : diffraction

$$\lambda_{\text{wave}} \sim \text{obstacle size}$$

e.g. aperture, or ...
gravitational lens ?

Gravitational lensing: EM waves vs gravitational waves

Diffraction: $\lambda_{\rm wave} \sim$ obstacle size (e.g. gravitational radius)

EM waves	GW
Observed $\lambda_{\rm wave}$: $\leq 10^{-4} M_\odot$	Observed $\lambda_{\rm wave}$: $\leq 10^{4} M_\odot$ (LIGO, ET) $\leq 10^{7} M_\odot$ (LISA) $\leq 10^{13} M_\odot$ (PTA)

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→ Relevant astrophysical and cosmological lenses!

Wave effects in gravitational lensing:

Assuming a **weak field** lensing

$$ds^2 = -(1 + 2U)\, dt^2 + (1 - 2U)\, d\mathbf{r}^2$$

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and a **scalar signal** ϕ, the propagation follows (in Fourier space) :

$$(\nabla^2 + \omega^2)\phi = 4\omega^2 U \phi, \qquad \omega = 2\pi f$$

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Define the signal amplification $\sqrt{\mu} = \frac{\phi}{\phi(\text{no lens})}$, (magnification)$^{1/2}$

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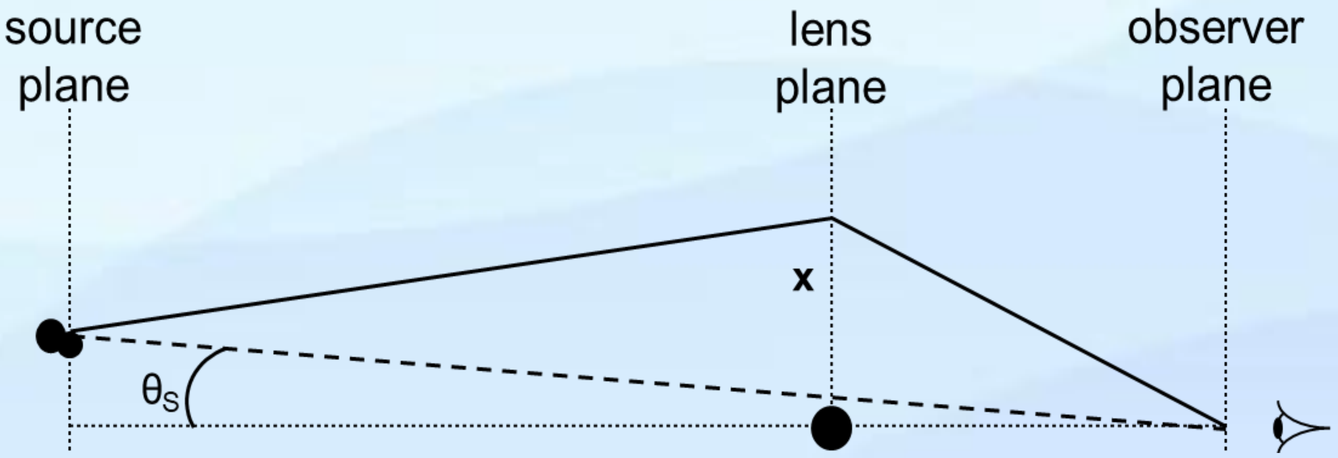
$$\sqrt{\mu} \propto f \int_{\text{lens plane}} d^2\mathbf{x} \exp\left(if\, \mathrm{T}_{\text{travel}}(\mathbf{x}, \theta_S)\right)$$



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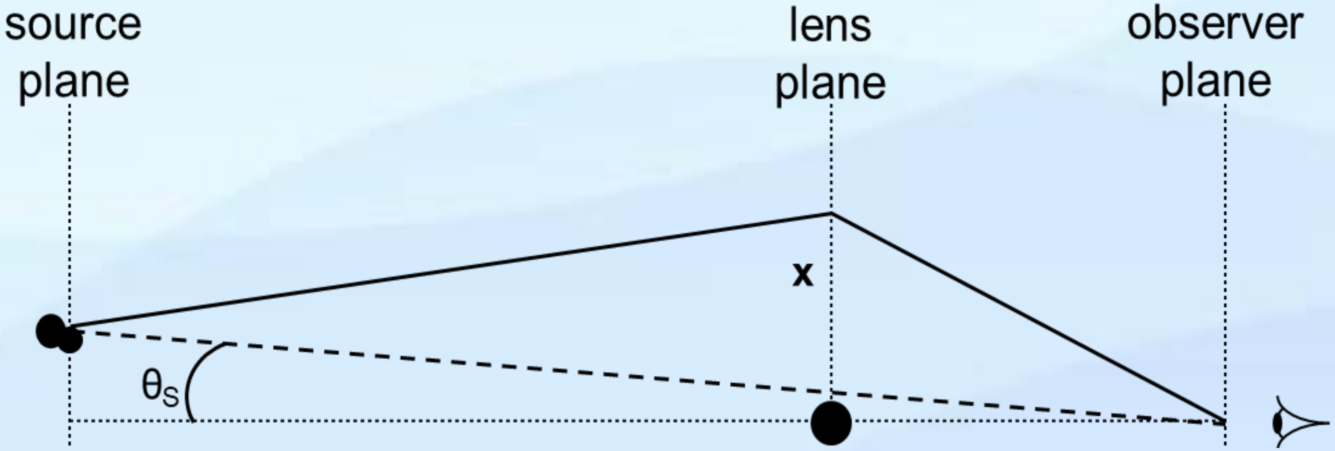
Geometrical + Shapiro travel time for the path through **x**



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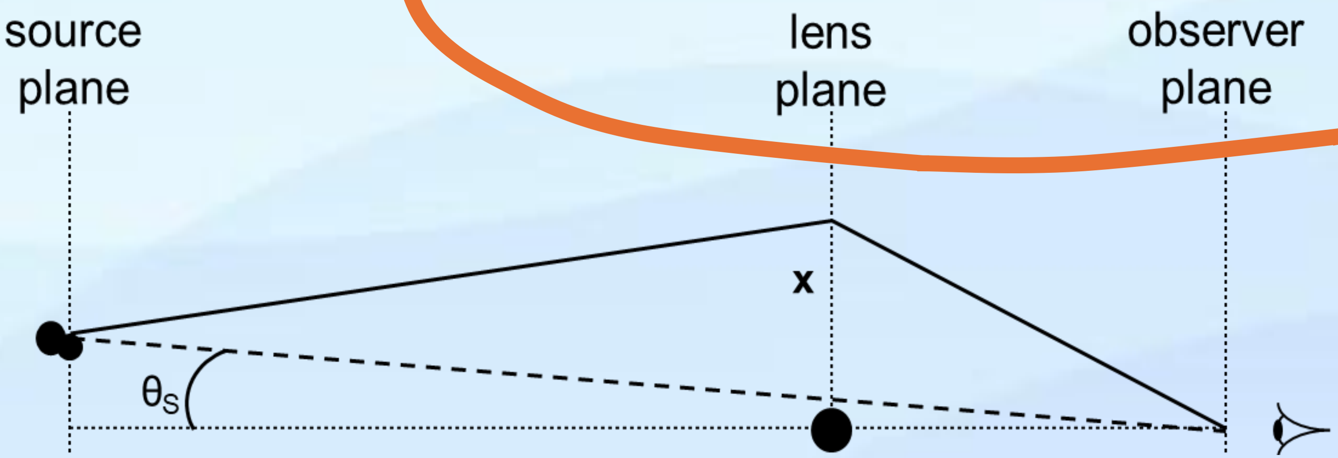
Geometrical + Shapiro travel time for the path through **x**

Integrated over all hypothetical paths through the lens plane



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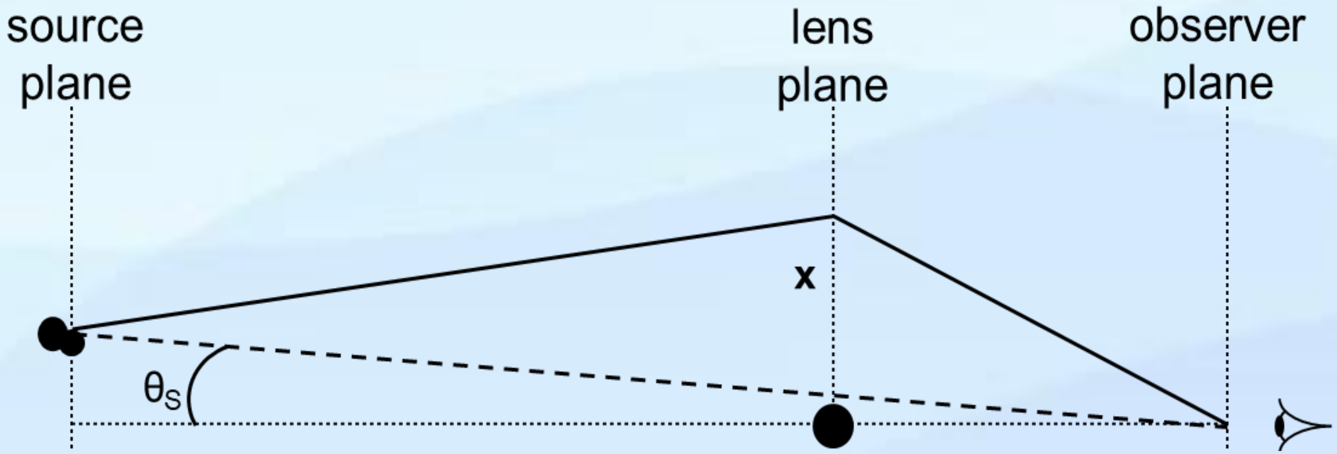

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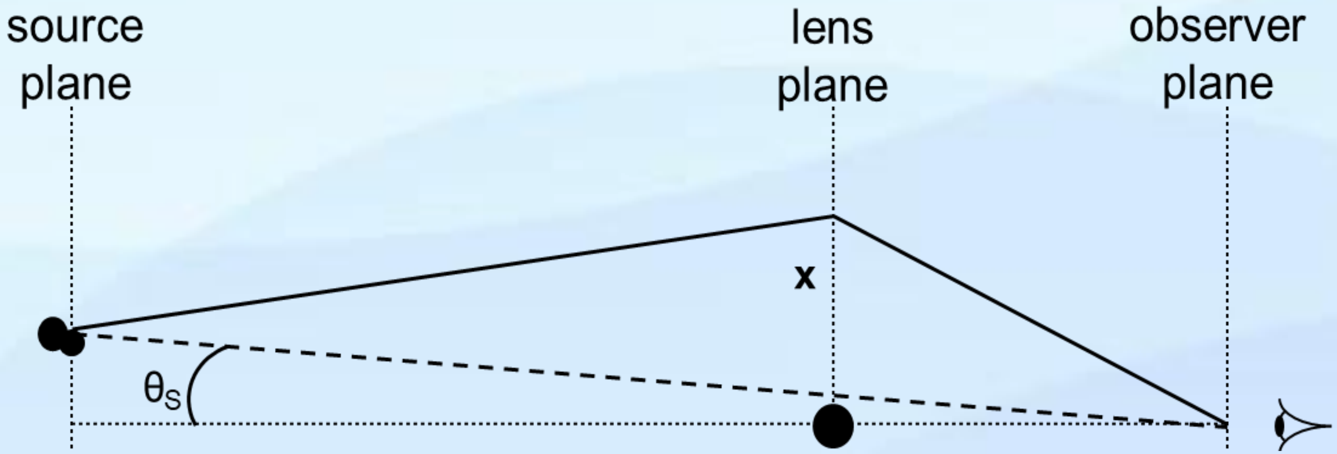

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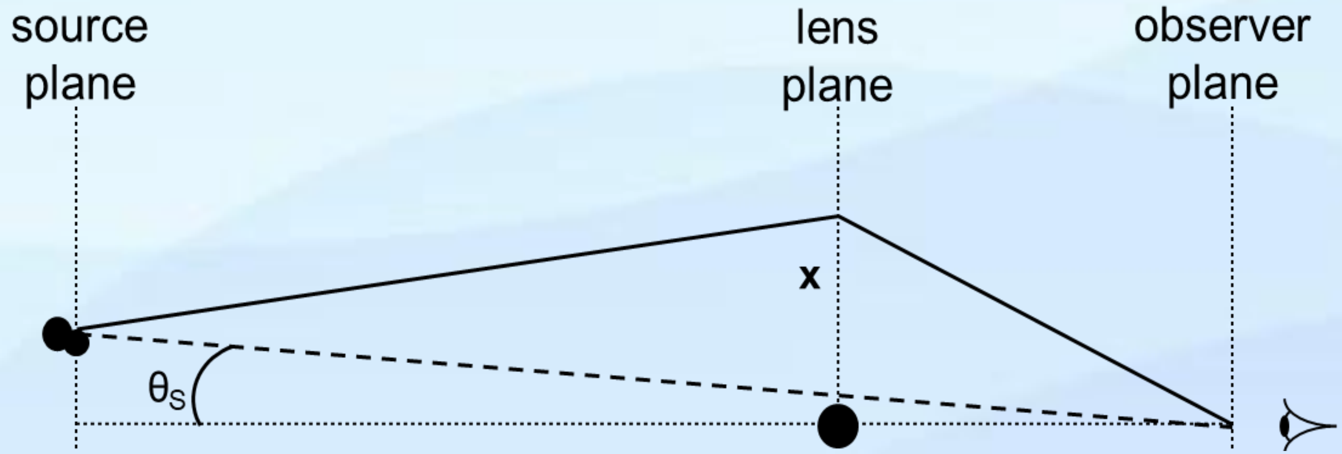

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only the stationary phase points contribute
i.e. $\nabla \mathrm{T}_{\text{travel}} = 0$

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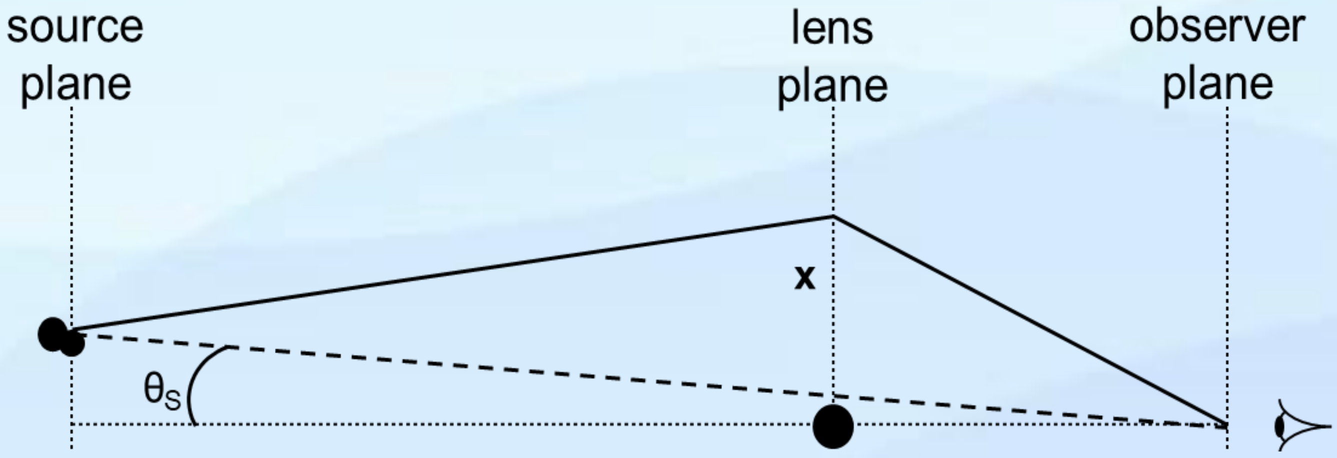

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→ **recover the geodesic limit & associated observables naturally** (Fermat principle)

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Framework established since the 1970's

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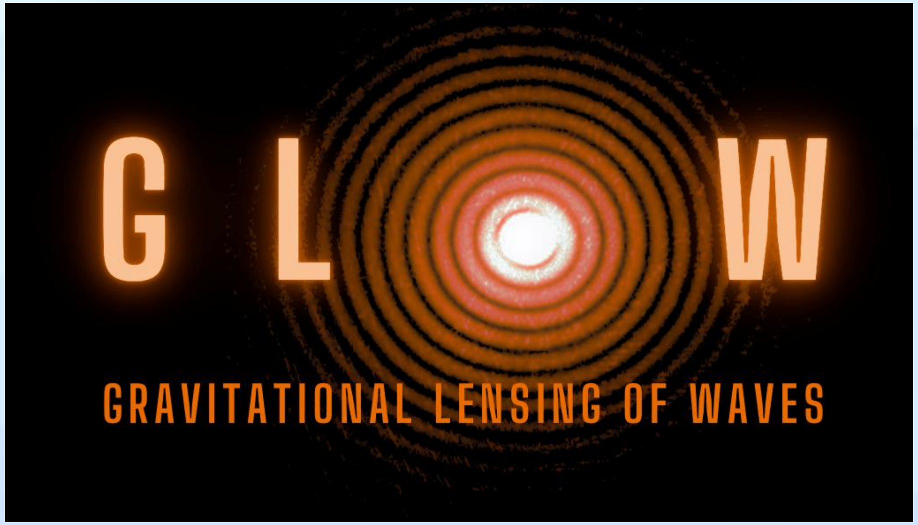
State of the art numerical methods have been developed **this year**

e.g. GLoW code for diffraction integral

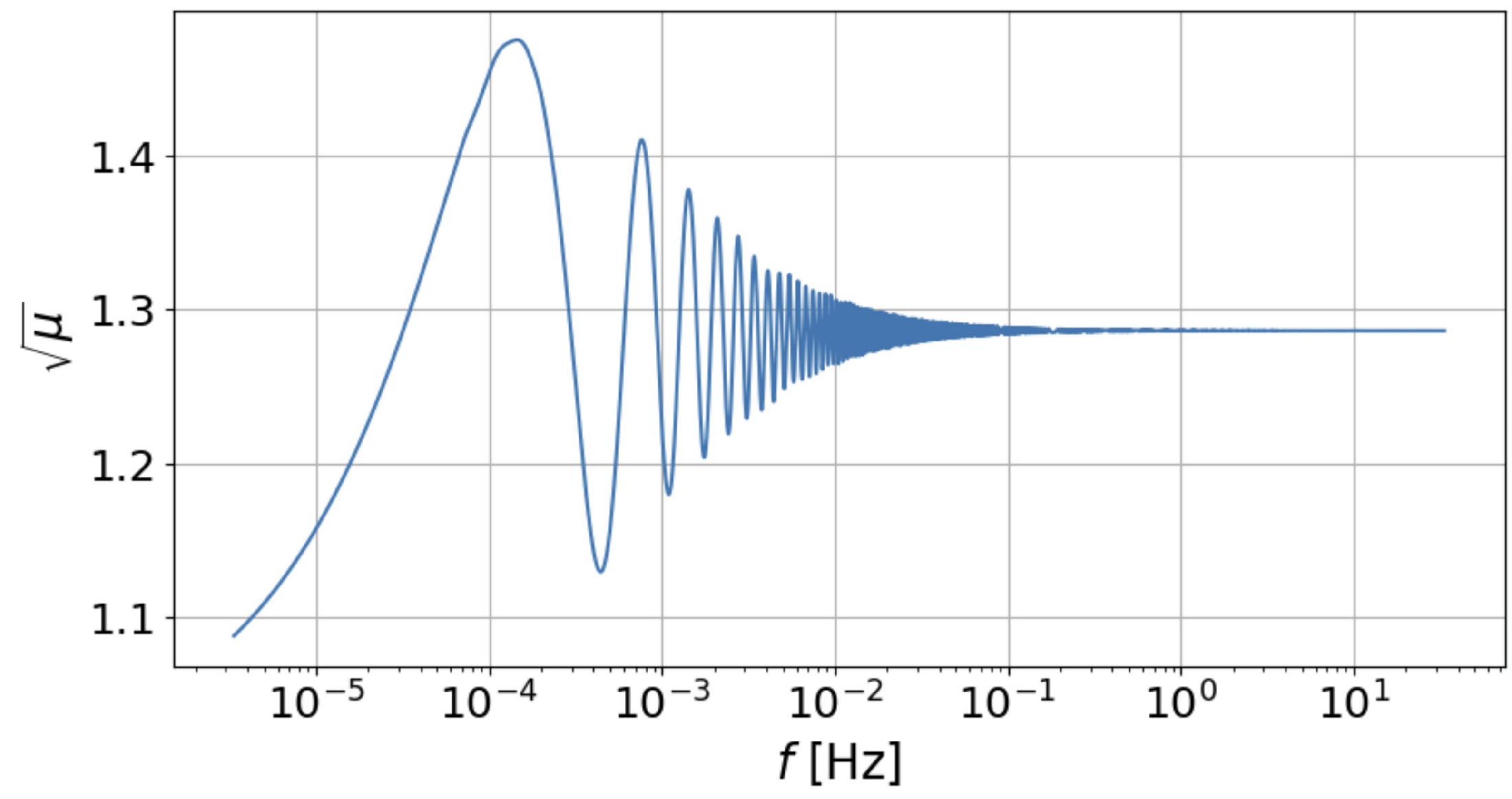
Villarrubia-Rojo+2025



Wave effects in gravitational lensing:

Illustration example:

$10^9 M_\odot$ DM (sub)halo at $z = 1$ with *truncated SIS* profile, source at $z = 5$, **weak lensing** regime, “good” alignment



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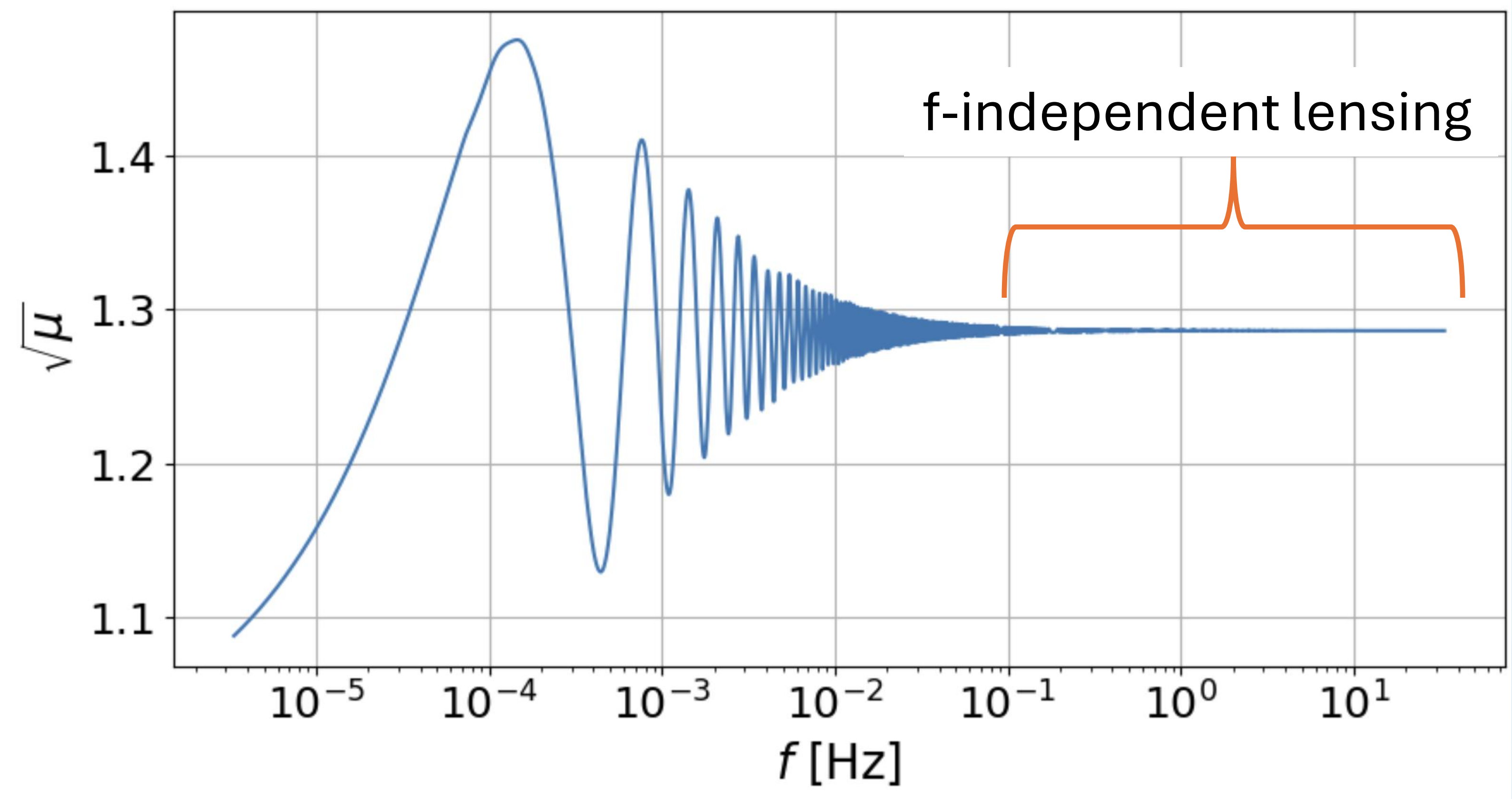

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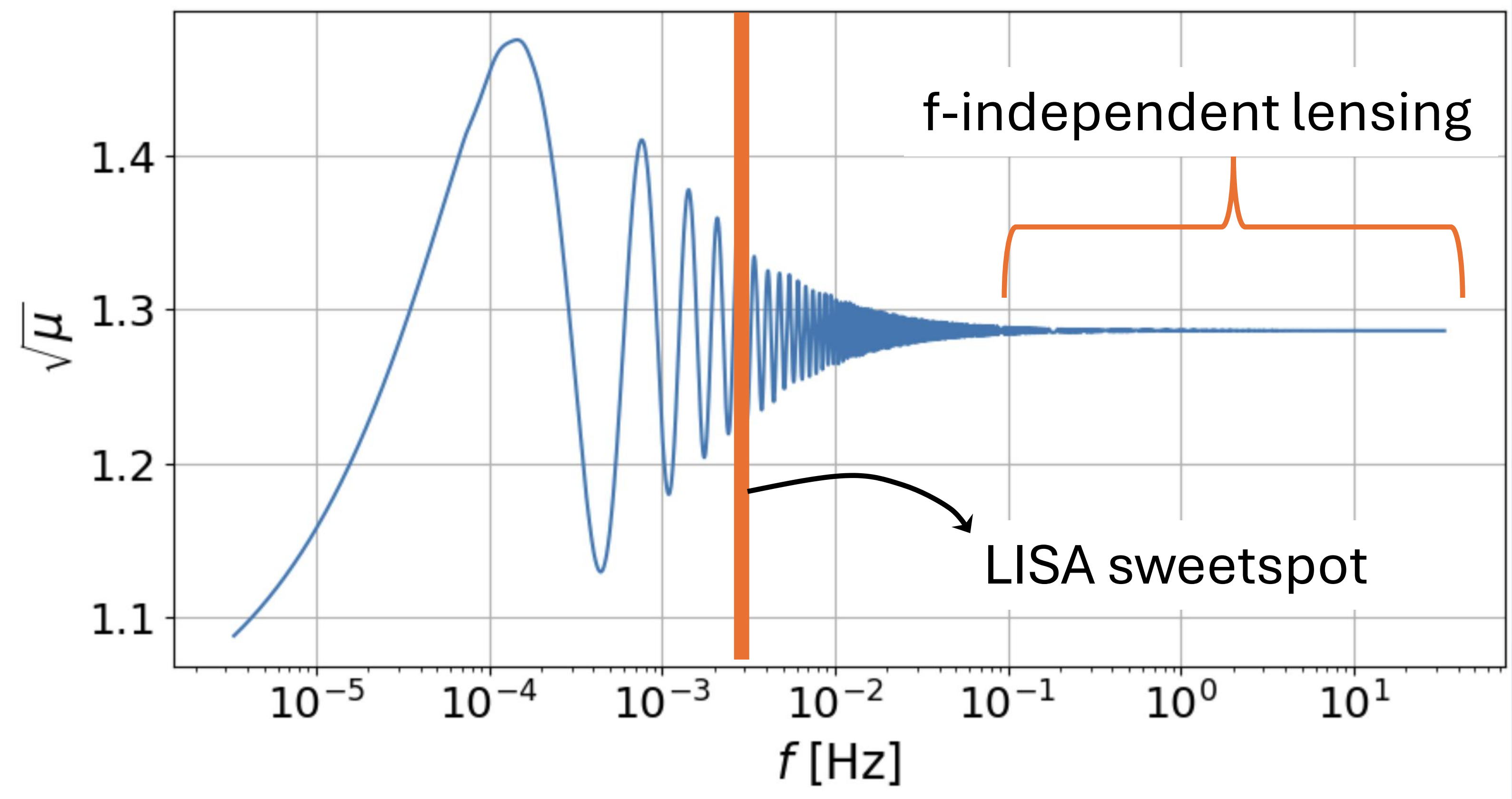

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A few results (*not mine !*) from the literature:

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- LISA observation prospects of **diffraction** : ***expected detection*** to ***low chances of detection*** depending on study
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- Very sensitive to the **abundance** and **profile** of the low-mass end (e.g. $< 10^{11} M_\odot$) DM halos : potential to distinguish DM models
  Savastano+ 2023, Brando+2024, Singh+2025

Beyond initial assumptions:

This framework relied on **weak fields potentials, scalar signals…**

… But **GW** are not scalars!

$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0$$

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Expect even **richer phenomenology !**

… but a practical and generic framework for diffractive gravitational lensing is missing

Tensorial lensing by a strong field:

If the lens has high symmetry e.g. Schwarzschild BH:

Black hole perturbation theory provides approximate analytical results for long $R_{\text{Schw}}/\lambda_{\text{wave}} < 1$

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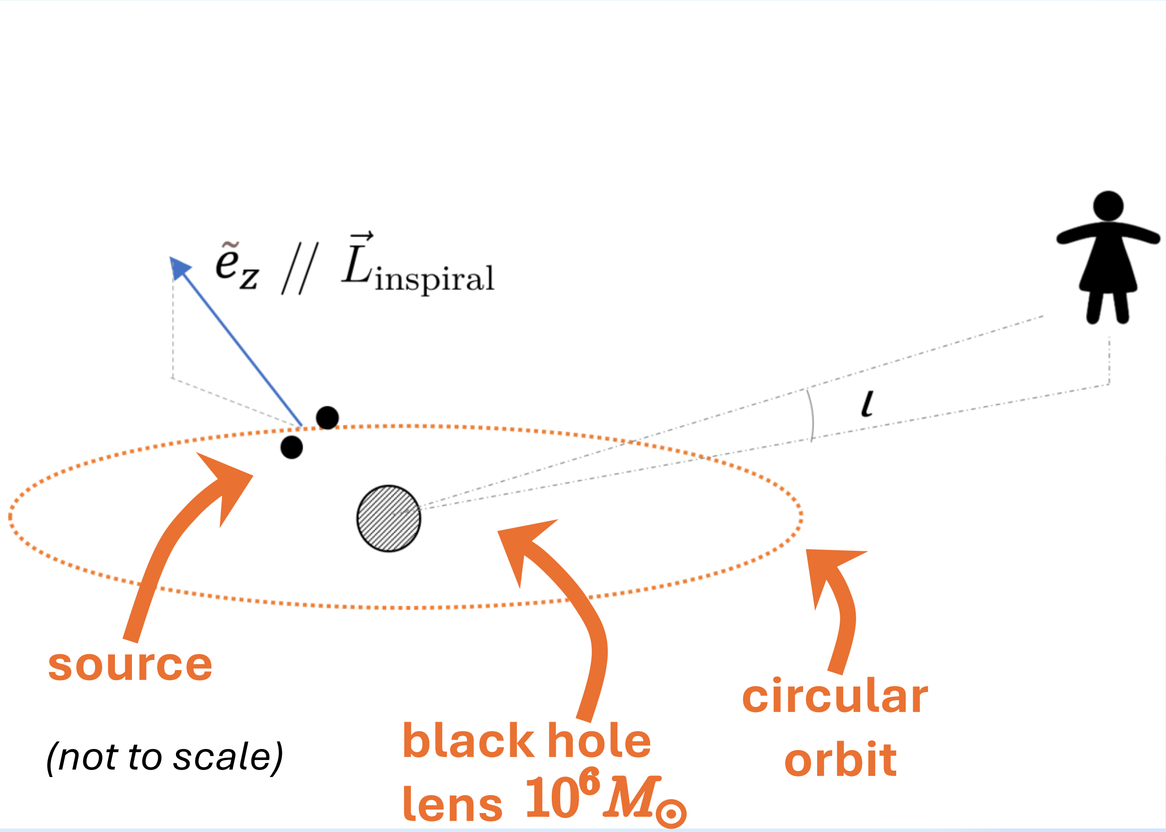
$\rightarrow$ **lensing by a BH depends on the helicity of the signal**

$\rightarrow$ **non-trivial effects on polarisations,** the polarization content is **not** preserved by lensing   Pijnenburg+, 2024

Wave optics lensing in triple systems: towards a phenomenology

Example: consider a microlensing-like **dynamical** setup



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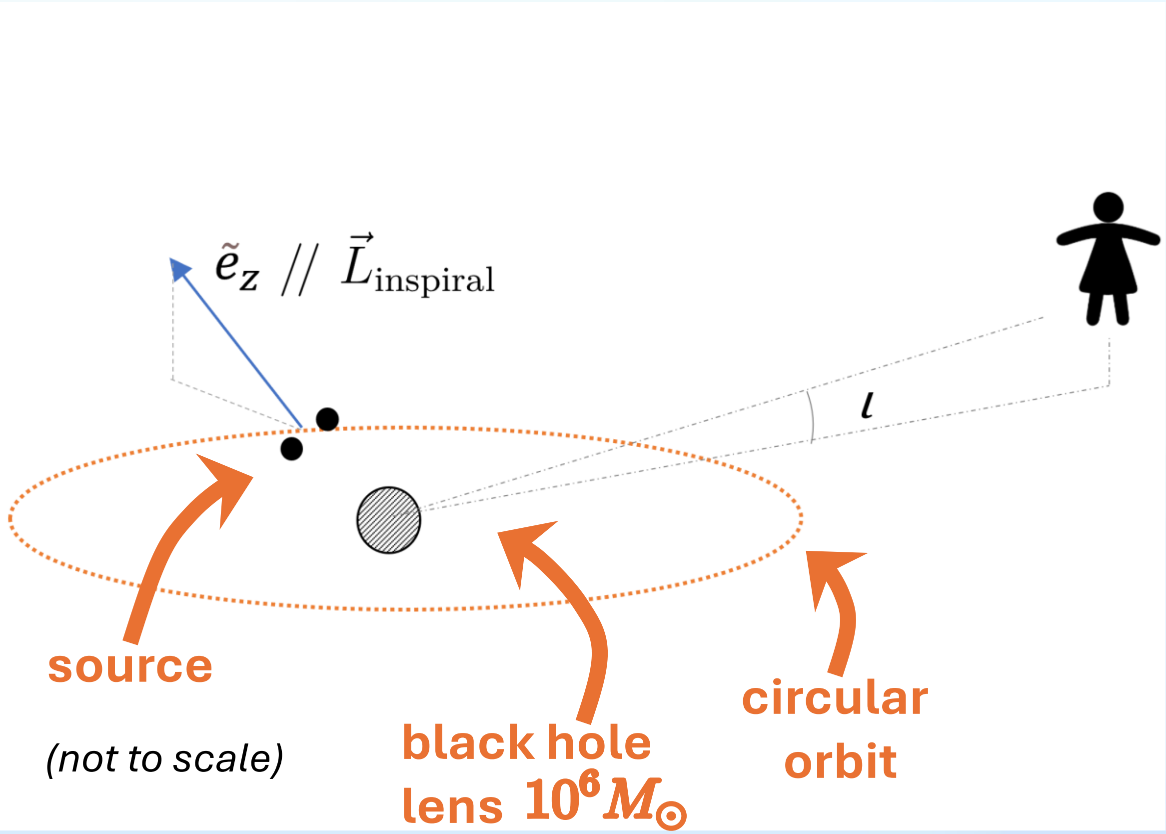

Stokes parameters of GWs:

I : total intensity

V : circularly-polarized intensity

$\mathcal{V} = V/I$: circularly polarized fraction of the intensity

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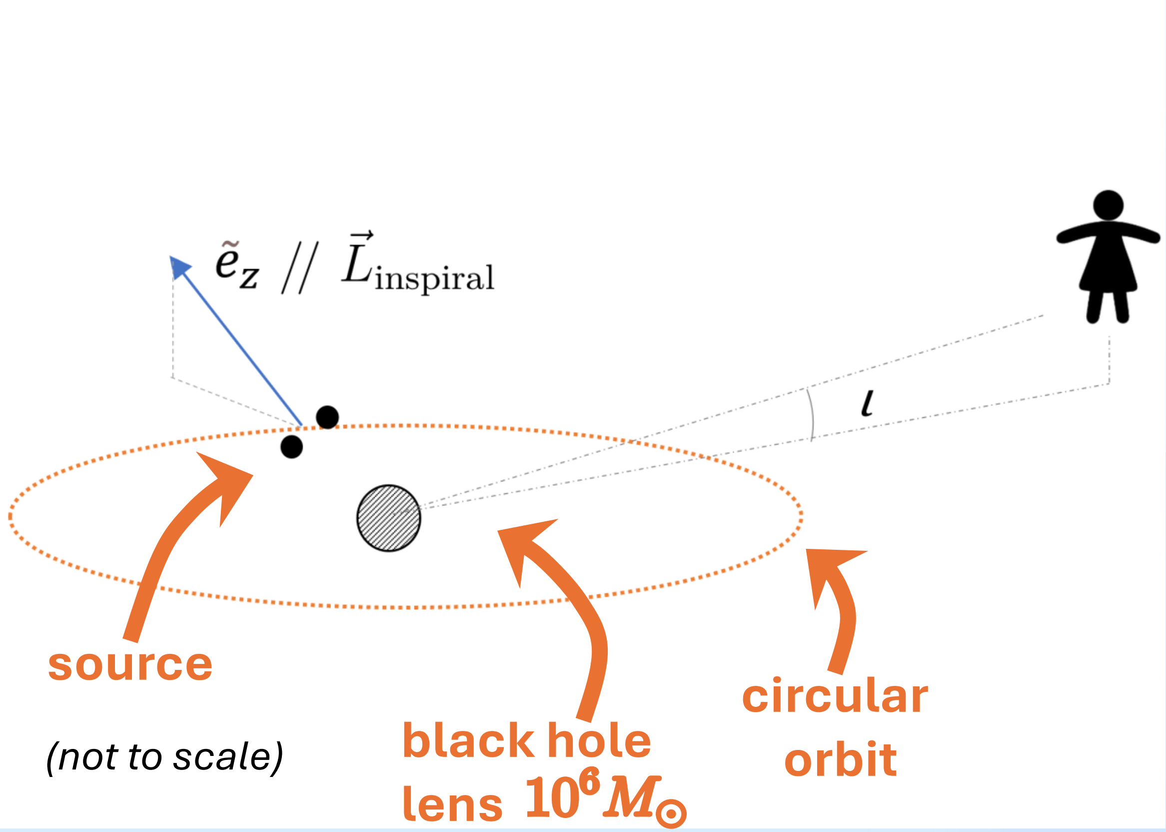

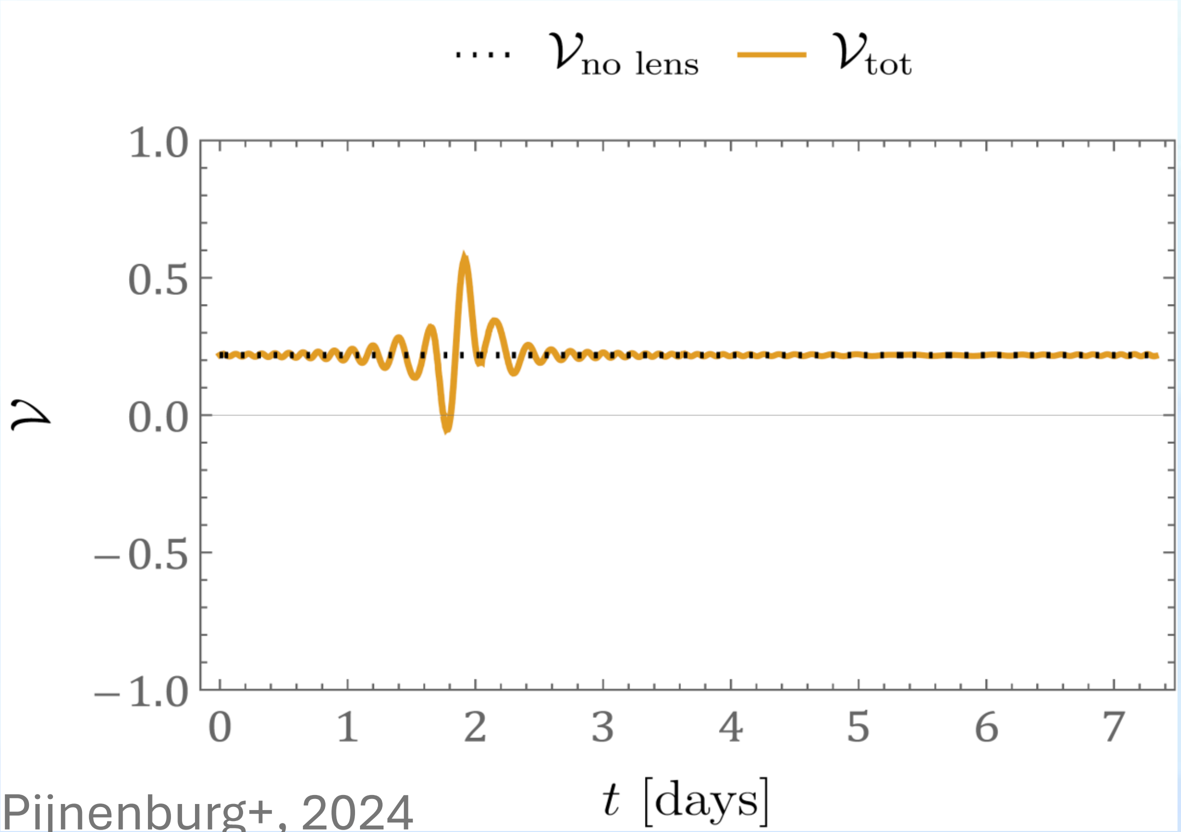

Pijnenburg+, 2024

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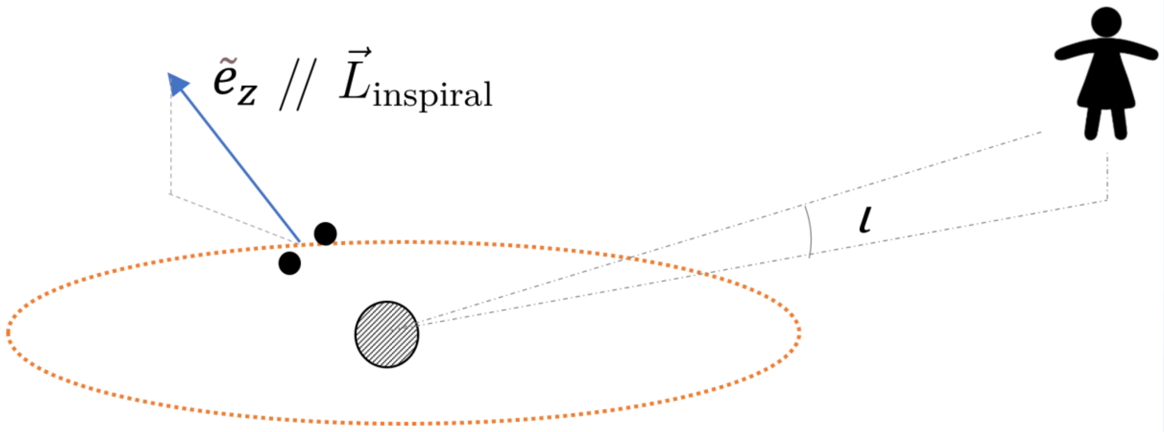
- **Wave optics** is a regime in the range of future GW observations
- Relevant lenses include **DM subhalos, supermassive black holes**, ...
- **Without new physics, new lensing phenomenology** wrt EM case
  - Frequency-dependent magnification
  - Changes in the polarisation/helicity content (BH lens)

Wave optics lensing in triple systems: towards a phenomenology



(not to scale)

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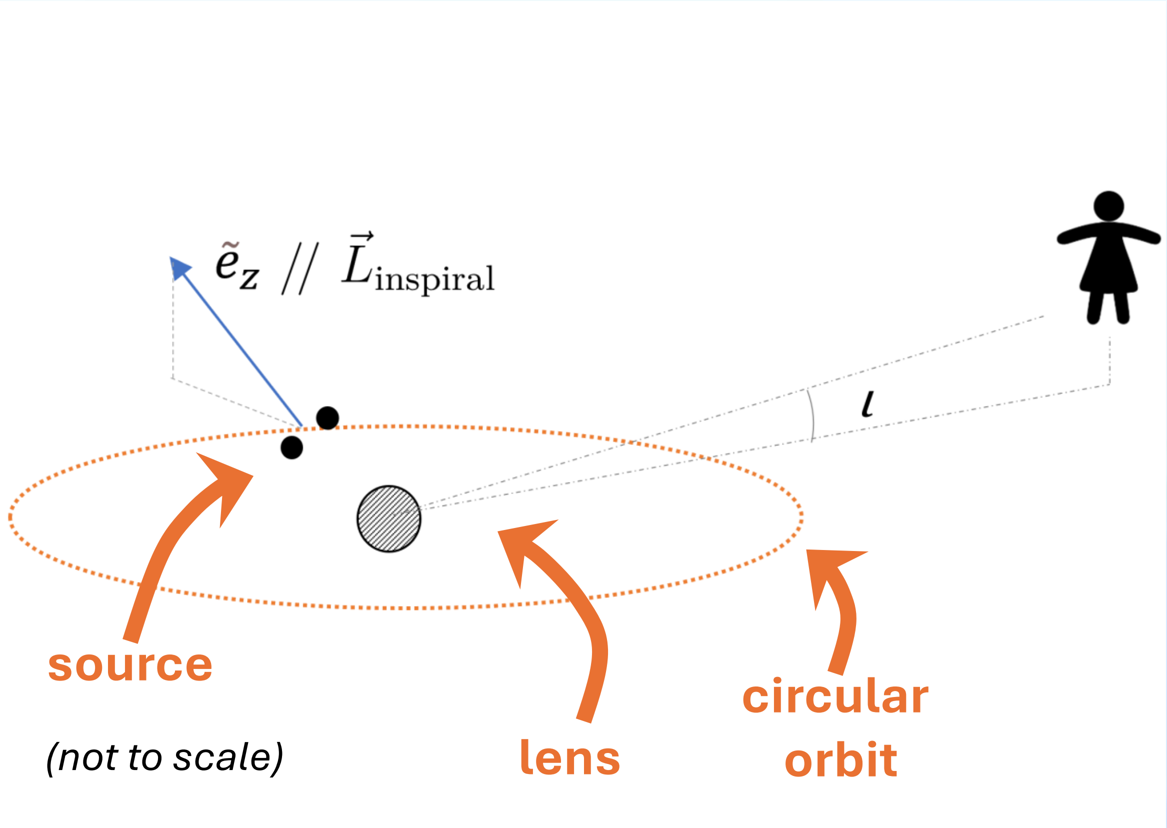

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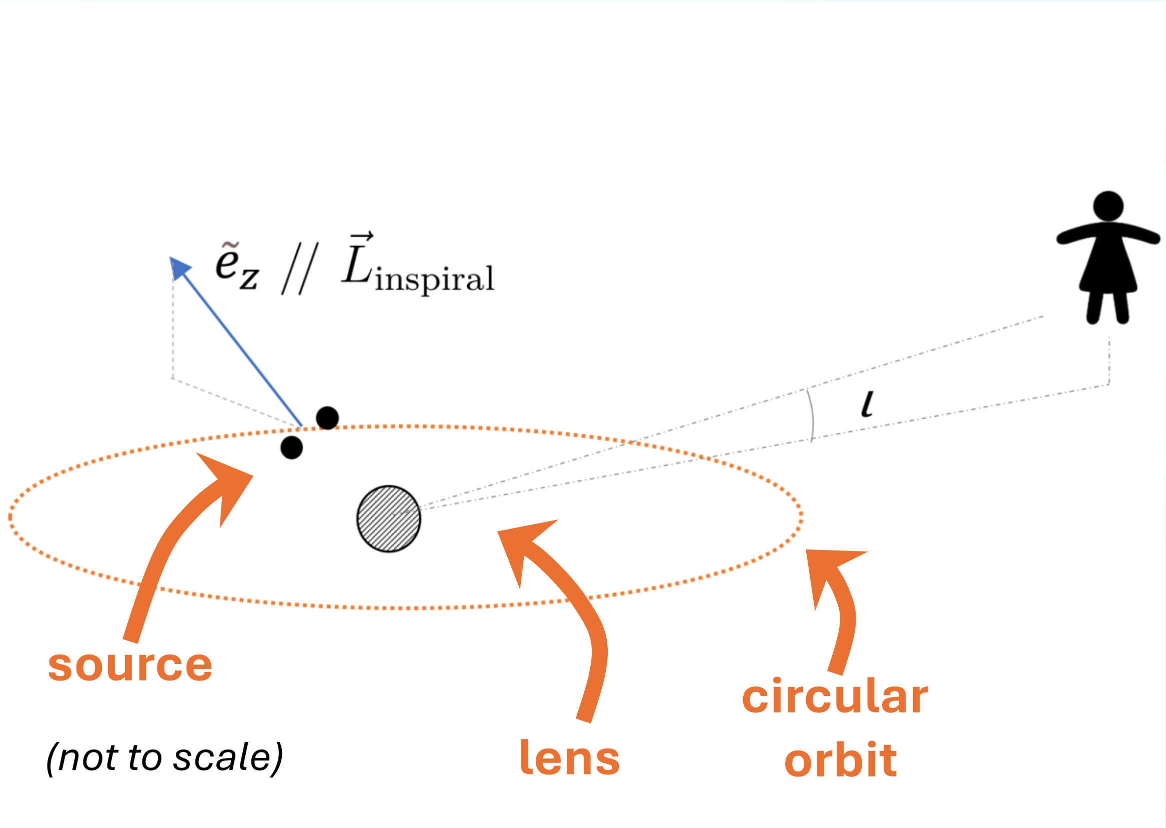
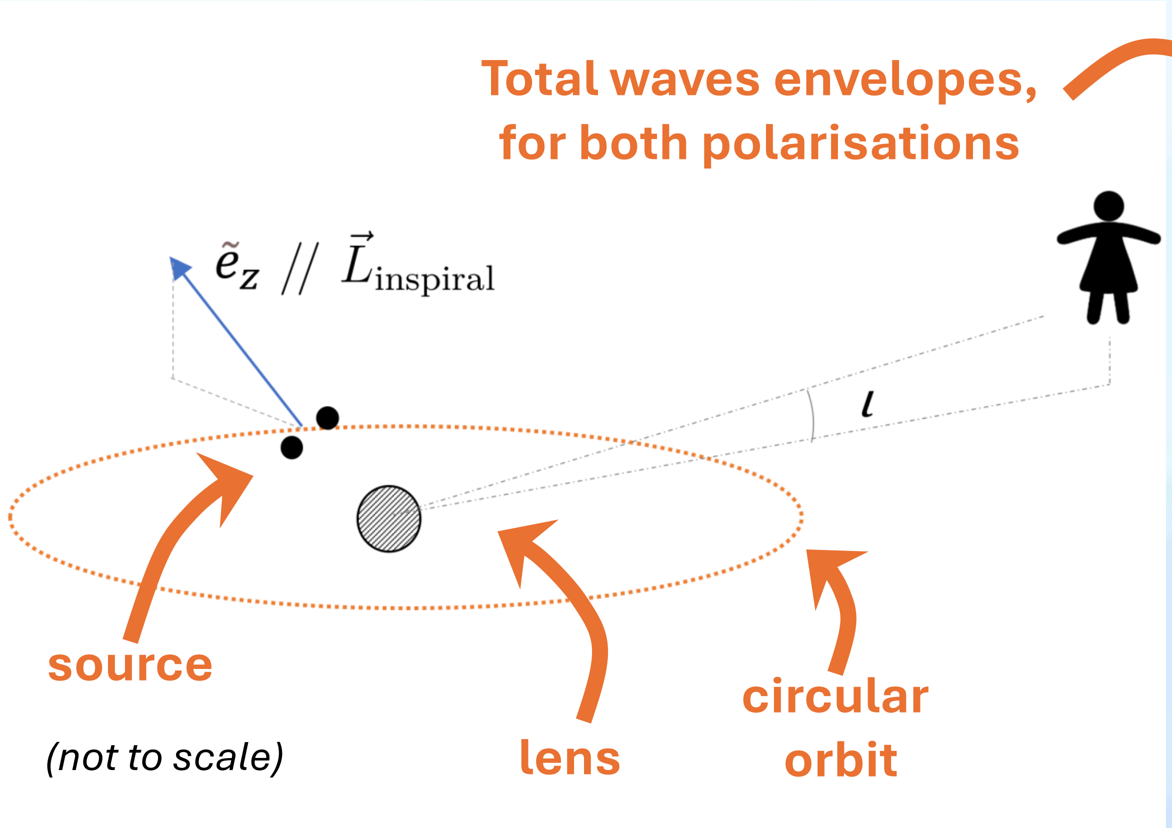


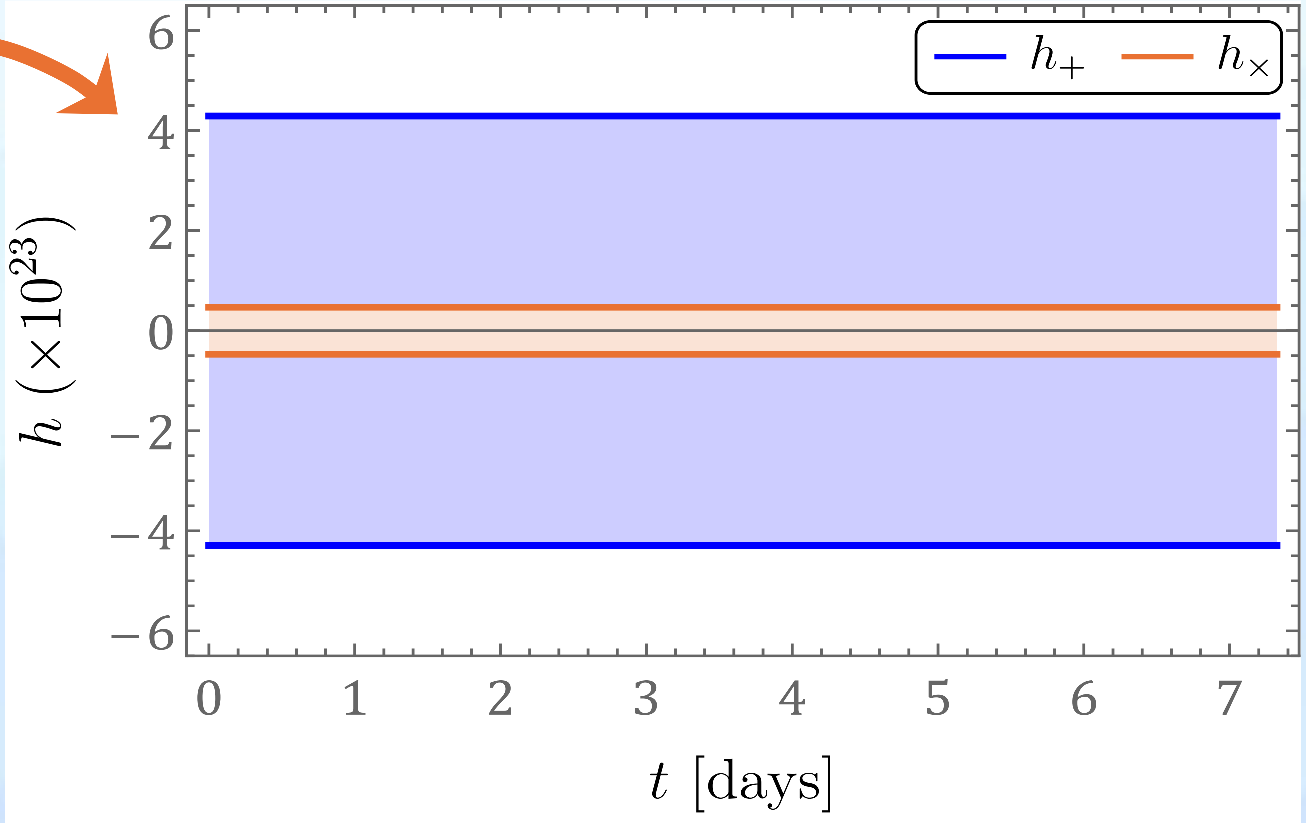
Such a **triple system** is suspected in the GW event

GW190521

Wave optics lensing in triple systems: towards a phenomenology

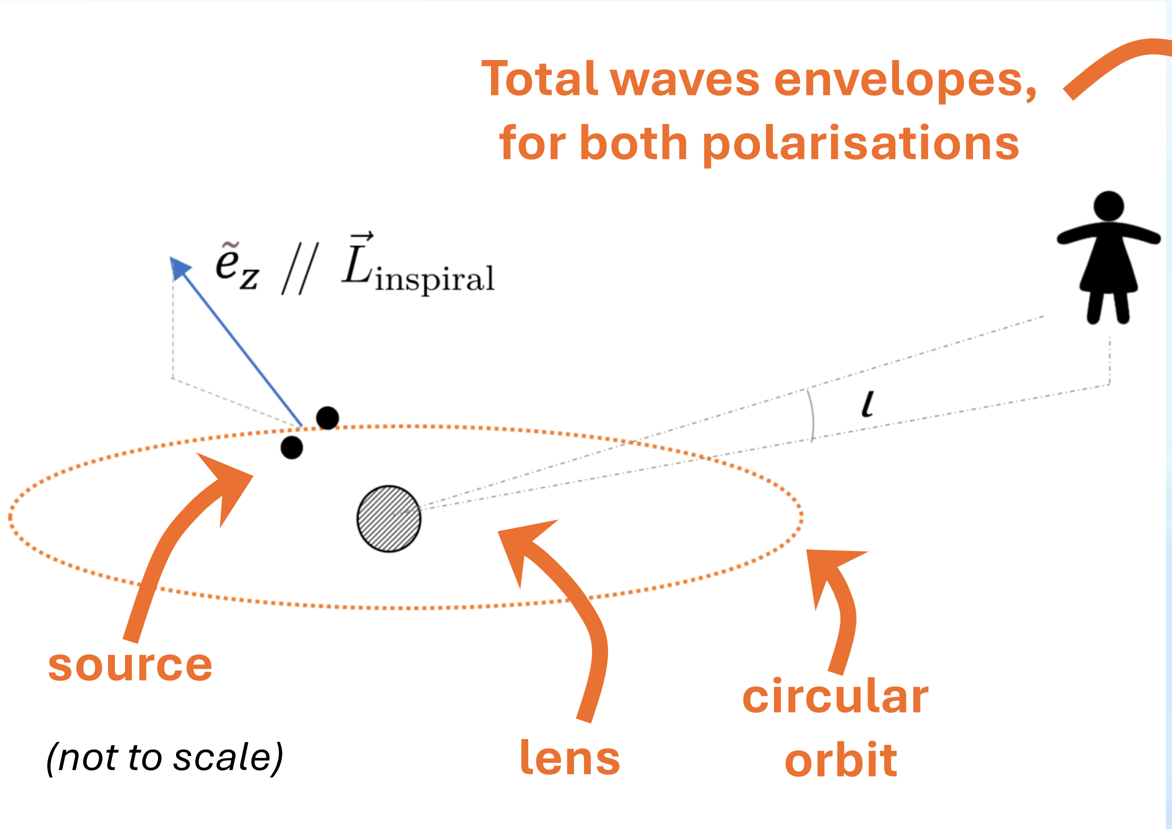


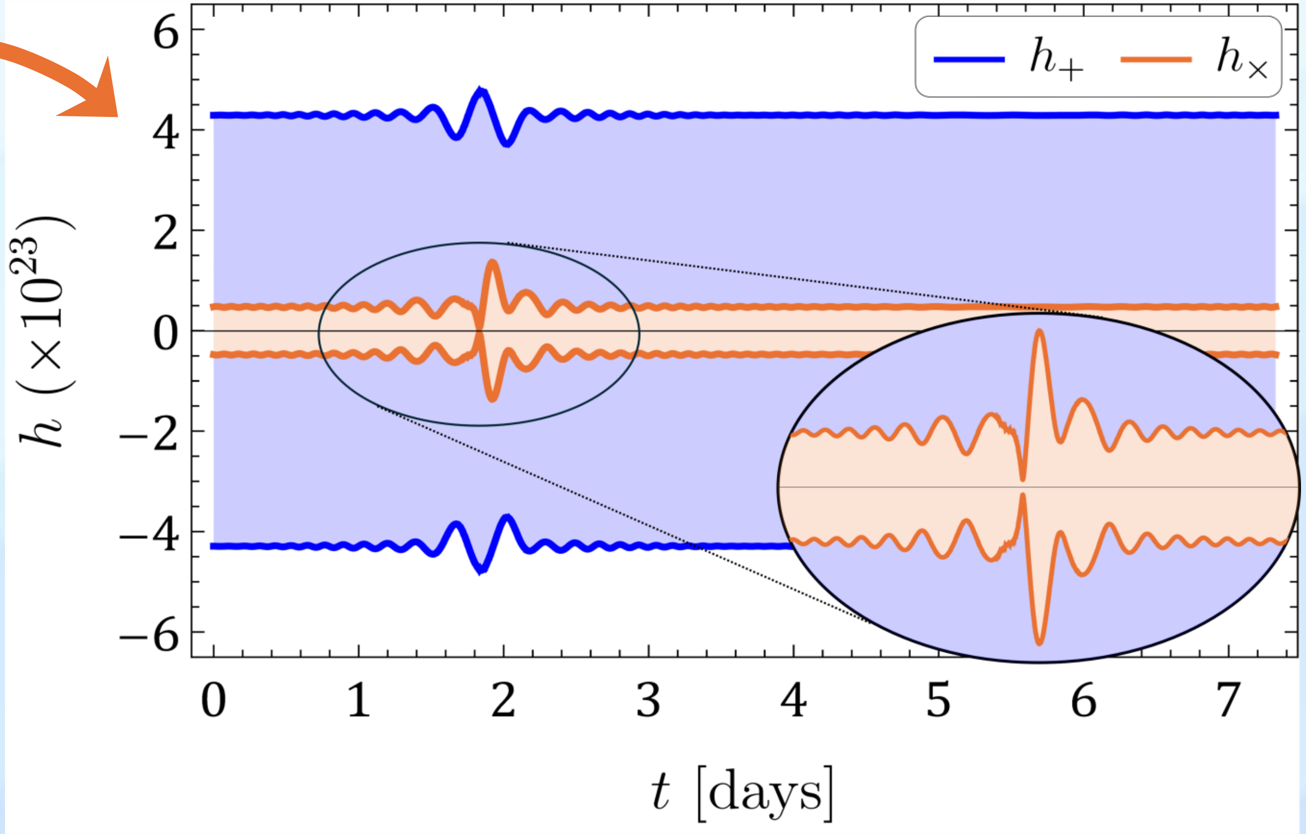
Inspiral phase *without lens*



Wave optics lensing in triple systems: towards a phenomenology

Toy GW190521-inspired source, in LISA- « optimal » wave optics





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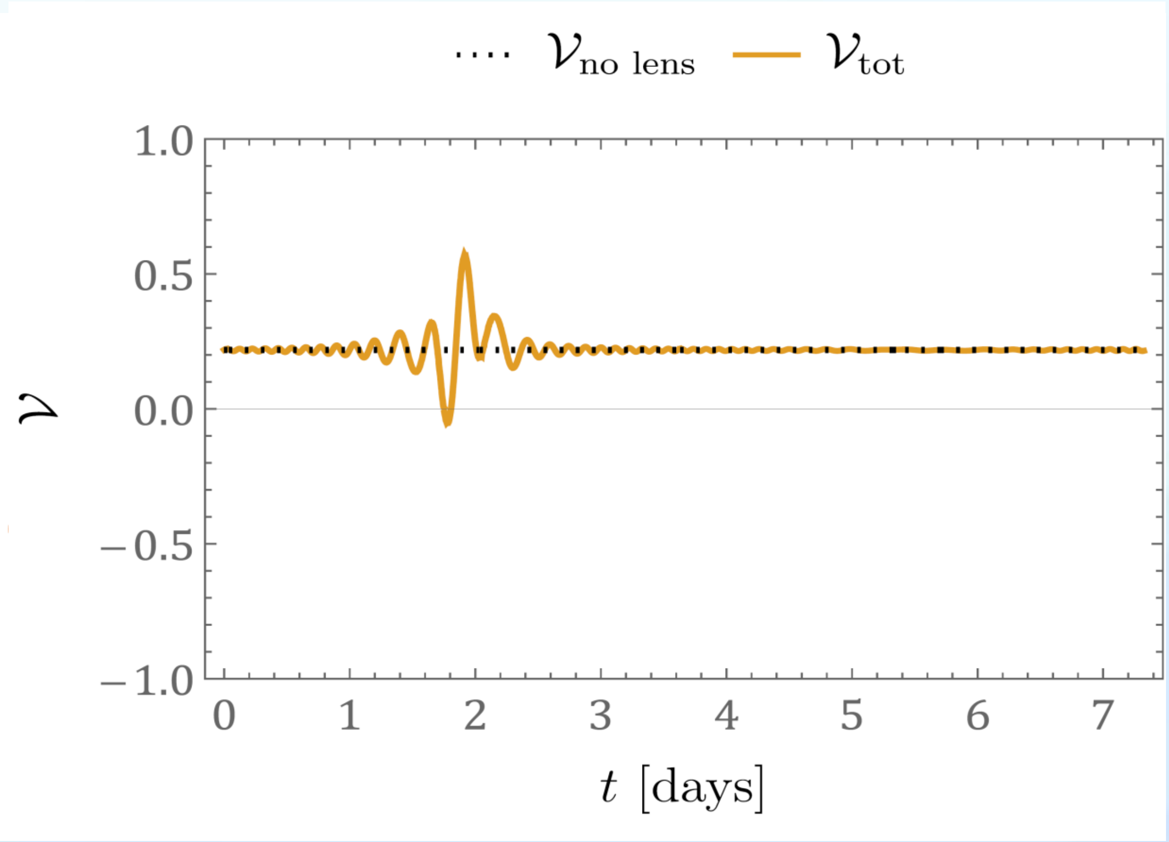
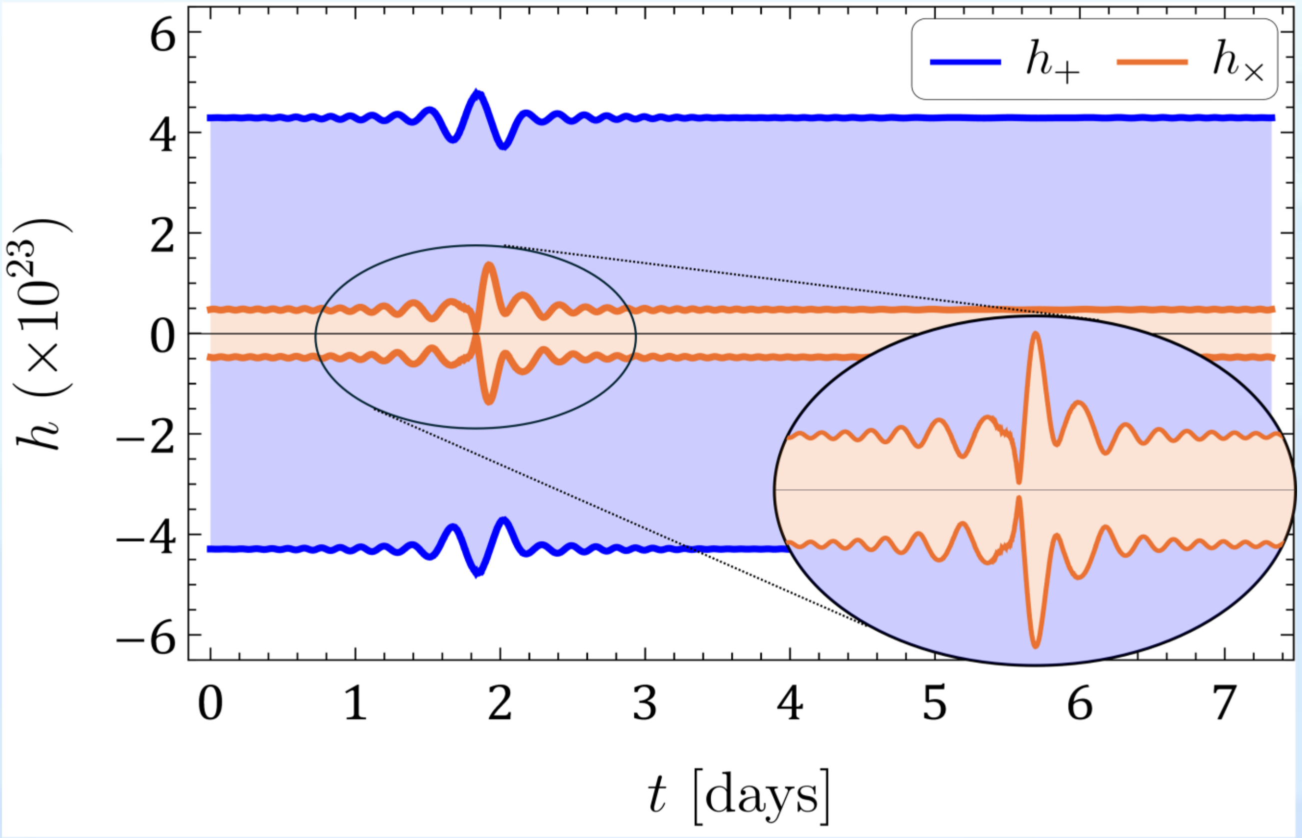
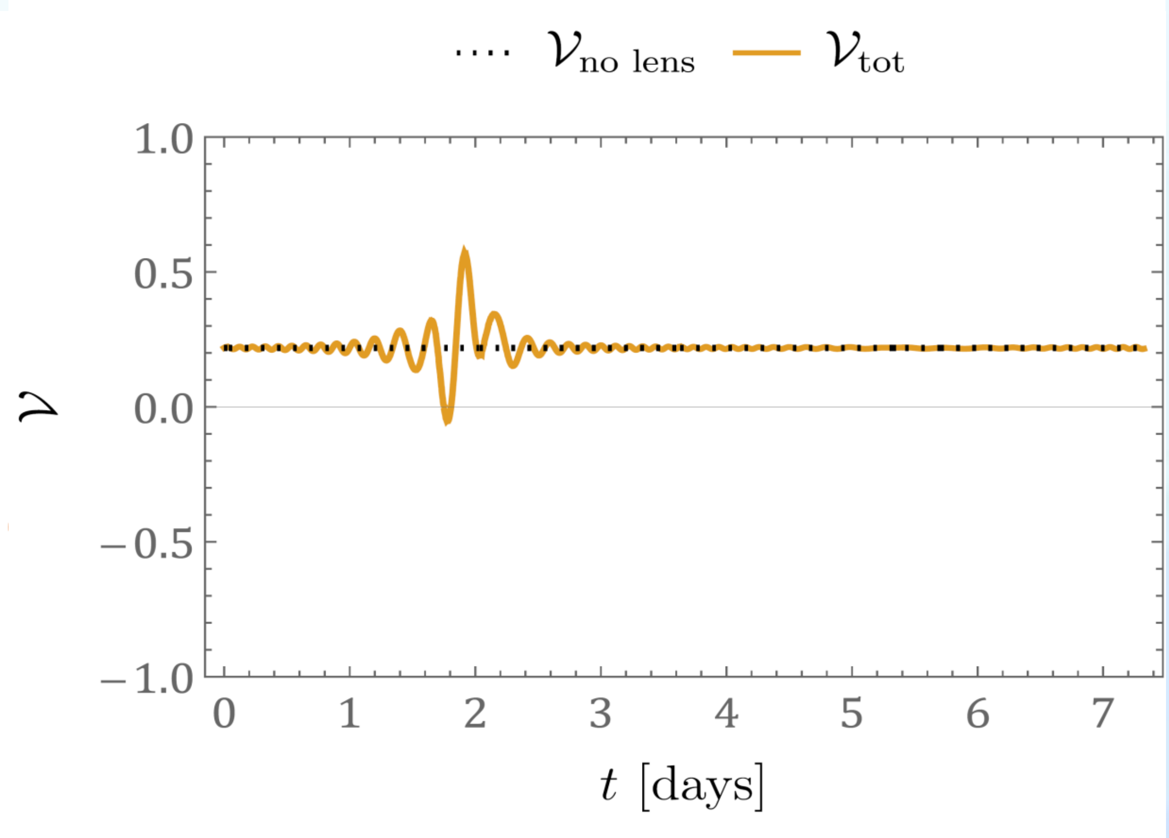
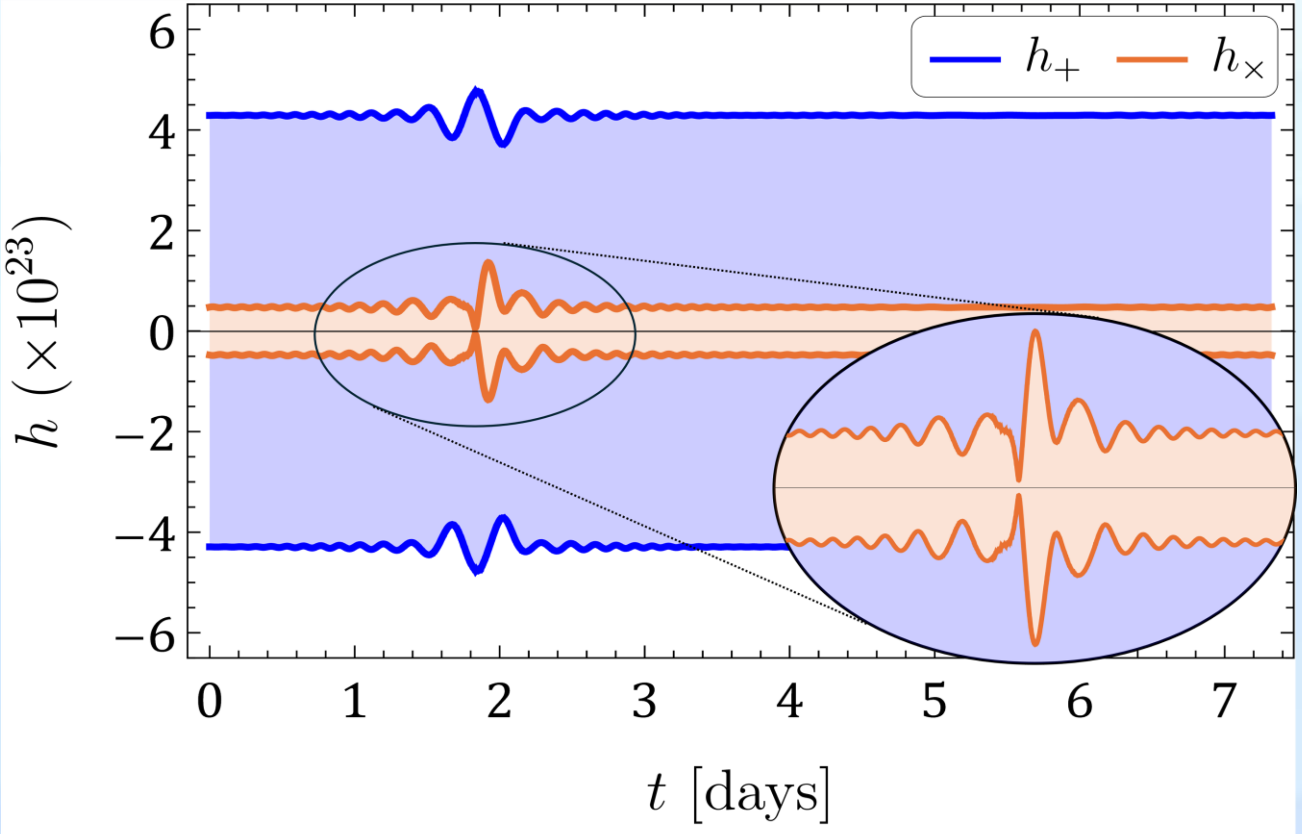




Wave optics lensing in triple systems: towards a phenomenology

LISA **detectable** with SNR > 100 if at $z \sim 0.01$





GW lensing: wave optics

Start with : $\mathrm{d}s^2 = (\bar{g}_{\mu\nu} + h_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu$

E.g. gauge fixing : $h^{\nu}_{\mu;\nu} = 0, \quad h^{\mu}{}_{\mu} = 0$

$\longrightarrow$ Wave equation :

$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0, \quad \text{with} \quad h_{\mu\nu;\alpha}{}^{;\alpha} \equiv \Box h_{\mu\nu}.$$

GW lensing: wave optics

BH lenses, historical works, at the formal level :

- Matzner (1968)
- Peters (1976)
- Chrzanowski *et al.* (1976)
- De Logi, Kovacs (1977)
- Futterman *et al.* (1988)

…

More recently: Dolan (2018)

GW lensing: wave optics

Reference work for phenomenology :

Wave effects in gravitational lensing of gravitational waves from chirping binaries

Ryuichi Takahashi (Kyoto U.), Takashi Nakamura (Kyoto U.)

May, 2003

28 pages

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$$h_{\mu\nu;\alpha}{}^{;\alpha} + 2\bar{R}_{\alpha\mu\beta\nu}\, h^{\alpha\beta} = 0$$

Assume $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Solve for ϕ

Tensorial wave optics

In *Pijnenburg, et al.*, *2024*, we treat lensing by a Schwarzschild BH

avoiding the assumption $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Tensorial wave optics

In *Pijnenburg, et al., 2024*, we treat lensing by a Schwarzschild BH

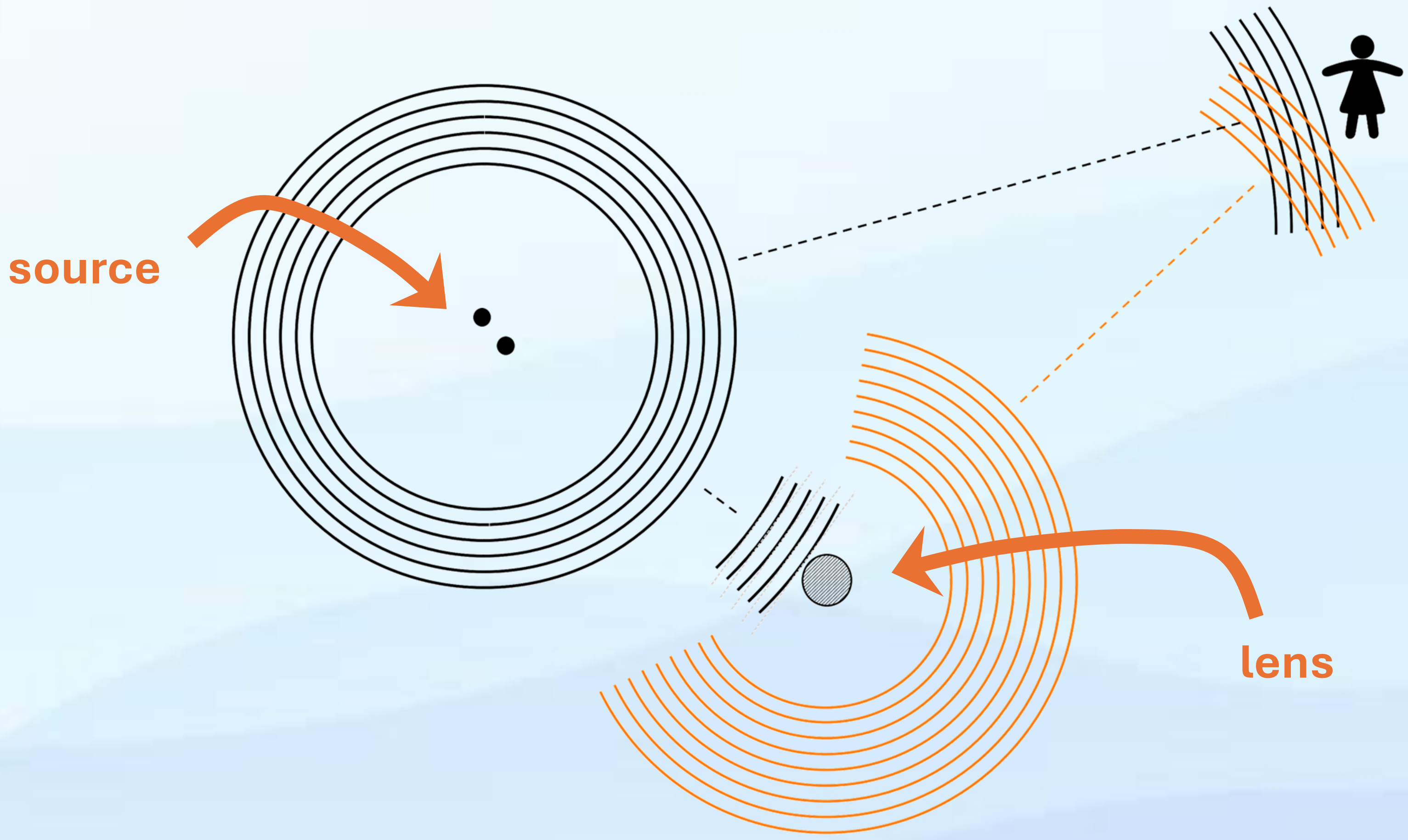
avoiding the assumption $h_{\mu\nu} = \phi \cdot e_{\mu\nu}$

Rather use : - black hole perturbation theory (BHPT)
- (quantum) waves scattering (e.g. phase shifts)

since the equations are quantum like (RW, Zerilli)

to keep track of the full polarisation structure analytically

Tensorial wave optics



Polarisation

Quantifying the signal polarisation content $\mathcal{V} \in [-1, 1]$:

$$\mathcal{V} \equiv \frac{2\mathrm{Im}[\tilde{h}_+ \tilde{h}_\times^*]}{|\tilde{h}_+|^2 + |\tilde{h}_\times|^2} = V/I \qquad \text{in terms of the Stokes parameters } V, I\,.$$

$$= \frac{|\tilde{h}^{(2)}|^2 - |\tilde{h}^{(-2)}|^2}{|\tilde{h}^{(2)}|^2 + |\tilde{h}^{(-2)}|^2}$$

constant in geometric optics and scalar wave optics

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constant in geometric optics and scalar wave optics

in general **not constant** in tensorial wave optics for $\lambda_{GW} \gg \frac{2GM_{\mathrm{lens}}}{c^2}$

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- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\mathrm{Hz}$

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- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\mathrm{Hz}$
- AGN lens mass : $M = 1.2 \times 10^6 M_\odot \implies \boxed{\omega M = 0.11}$

Wave optics lensing in triple systems: towards a phenomenology

- LISA-band system : $\omega = 2\pi f = 2\pi \times 3 \times 10^{-3}\text{Hz}$
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LISA **detectable** with SNR > 100 if at $z \sim 0.01$

Tensorial wave optics : BHPT

Project $h_{\mu\nu}$ on basis functions on the sphere with even (Y) and odd (X) parity :

$$h_{rr} = \sum_{\ell m} h_{rr}^{\ell m} \, Y^{\ell m}, \qquad \text{(radial)}$$

$$h_{rA} = \sum_{\ell m} h_r^{\ell m} \, X_A^{\ell m} + j_r^{\ell m} \, Y_A^{\ell m}, \quad A = \theta, \phi, \qquad \text{(radial/angular)}$$

$$h_{AB} = \sum_{\ell m} h_2^{\ell m} X_{AB}^{\ell m} + r^2 G^{\ell m} Y_{AB}^{\ell m} + r^2 K^{\ell m} \Omega_{AB} Y^{\ell m}, \quad A, B = \theta, \phi, \qquad \text{(angular)}$$

Tensorial wave optics : BHPT

From metric multipoles, define two **gauge invariant** master functions:

$$\Psi^{\ell m}_{\text{odd}} = \frac{2r}{(\ell-1)(\ell+2)}\left(\frac{\partial}{\partial r}\hat{h}^{\ell m}_t - \frac{\partial}{\partial t}\hat{h}^{\ell m}_r - \frac{2}{r}\hat{h}^{\ell m}_t\right)$$

$$r^{-1}\Psi^{\ell m}_{\text{even}} \propto \hat{K}^{\ell m} + \frac{2(1-2M/r)}{(\ell-1)(\ell+2)+6M/r}\left((1-2M/r)\hat{h}^{\ell m}_{rr} - r\frac{\partial}{\partial r}\hat{K}^{\ell m}\right)$$

Martel, Poisson. *Physical Review. D* 71.10 (2005)

Tensorial wave optics : BHPT

$\Psi_{\bullet}^{\ell m}$ obey Zerilli & Regge-Wheeler equations, $\bullet = \text{even, odd}$

$$\frac{\mathrm{d}^2\Psi_\bullet}{\mathrm{d}r_*^2} + \left(\omega^2 - V_\bullet\right)\Psi_\bullet = 0, \qquad \text{with } r_*(r) = r - 2M\ln\left(\frac{r}{2M} - 1\right)$$

Schrödinger-like, for given potentials $V_\bullet(\ell, r, M)$

Poisson, Sasaki. *Physical Review D* 51.10 (1995)

Tensorial wave optics : BHPT

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Schrödinger-like, for given potentials $V_\bullet(\ell, r, M)$

For the scattering problem :

Asymptotic solutions **for $\boldsymbol{\omega M \ll 1}$** are known, expect $\Psi_\bullet^{\ell m} \sim \Psi_\bullet^{\text{plane}} + \Psi_\bullet^{\text{sph}}$

Poisson, Sasaki. *Physical Review D* 51.10 (1995)

Tensorial wave optics : BHPT

Assume initial

$$h_{\mu\nu}^{\text{source}}$$

(absence of lens)

Derive corresponding

$$\Psi_{\bullet}^{\text{source}}$$

Solve differential equ. such that

$$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$$

Derive final

$$h_{\mu\nu}$$

(incl. lensing)

Tensorial wave optics : BHPT

« Observables »

Assume initial

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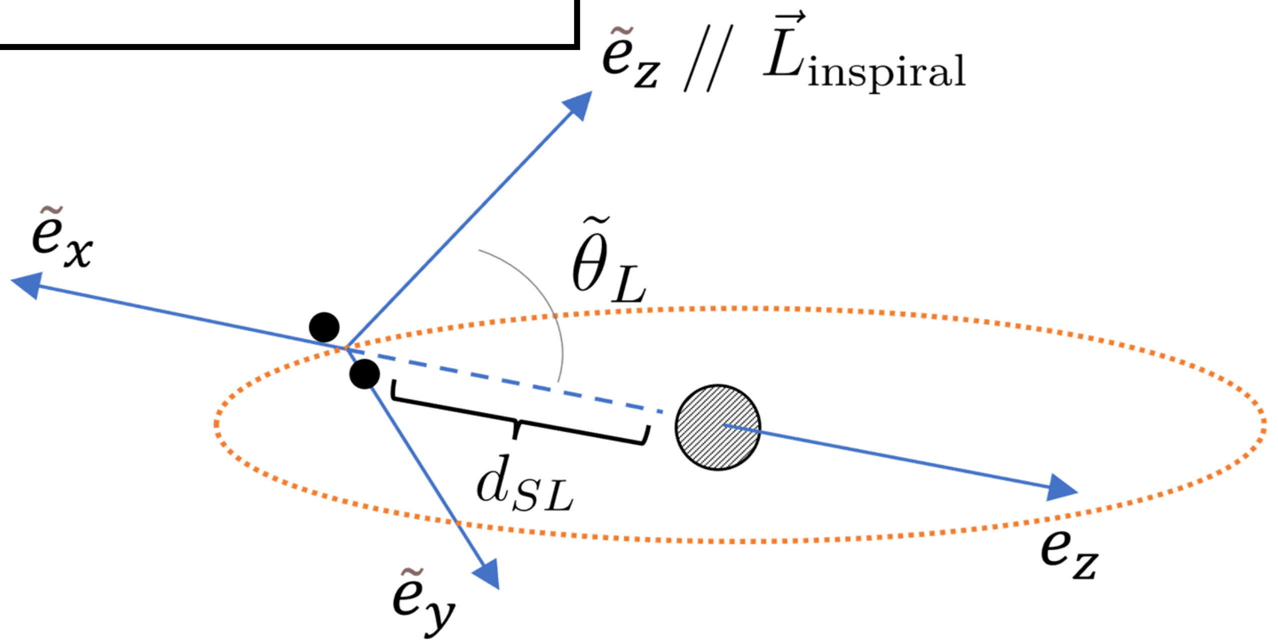

TT gauge, propagation along e_z :

$$h_{ij}^{\text{source}} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}$$

$$h_+ = \frac{A_{\text{in}}}{\tilde{r}} \frac{1+\cos^2\tilde{\theta}_L}{2} \cos[\omega(t-\tilde{r}) - 2\tilde{\phi}_L]$$

$$h_\times = \frac{A_{\text{in}}}{\tilde{r}} \cos\tilde{\theta}_L \sin[\omega(t-\tilde{r}) - 2\tilde{\phi}_L]$$

(locally plane wave)

Tensorial wave optics : BHPT

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(absence of lens)

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(incl. lensing)

Tensorial wave optics : BHPT

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(absence of lens)

Derive corresponding

$\Psi_{\bullet}^{\text{source}}$

Using scattering phase shifts

Solve differential equ. such that

$\Psi_{\bullet}^{\ell m} \sim \Psi_{\bullet}^{\text{source}} + \Psi_{\bullet}^{\text{lensed}}$

Derive final

$h_{\mu\nu}$

(incl. lensing)

Tensorial wave optics : BHPT

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(absence of lens)

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Tensorial wave optics : BHPT

Technicality : in principle, should sum $\lim_{r\to\infty} \sum_{\ell m} \Psi_\bullet^{\ell m}$

In practice : $\sum_{\ell m} \lim_{r\to\infty} \Psi_\bullet^{\ell m}$

Derive final

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Solve differential equ. such that

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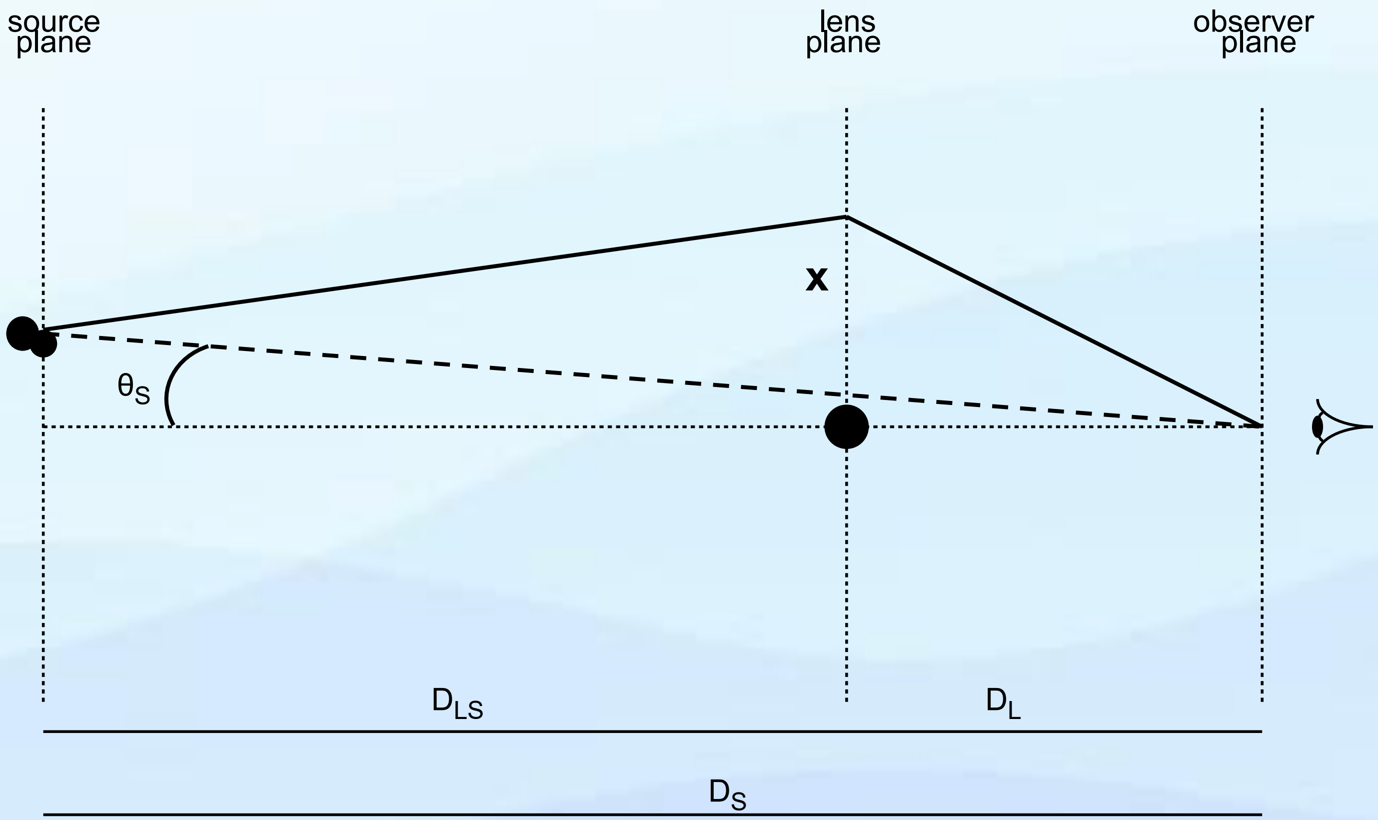
Technicality : in principle, should sum $\lim_{r\to\infty} \sum_{\ell m} \Psi_\bullet^{\ell m}$

In practice : $\sum_{\ell m} \lim_{r\to\infty} \Psi_\bullet^{\ell m}$ ... **diverges** analytically & numerically

Derive final $h_{\mu\nu}$ (incl. lensing) ← **non standard summation methods** ← Solve differential equ. such that $\Psi_\bullet^{\ell m} \sim \Psi_\bullet^{\text{source}} + \Psi_\bullet^{\text{lensed}}$

Gravitational lensing: EM waves vs gravitational waves



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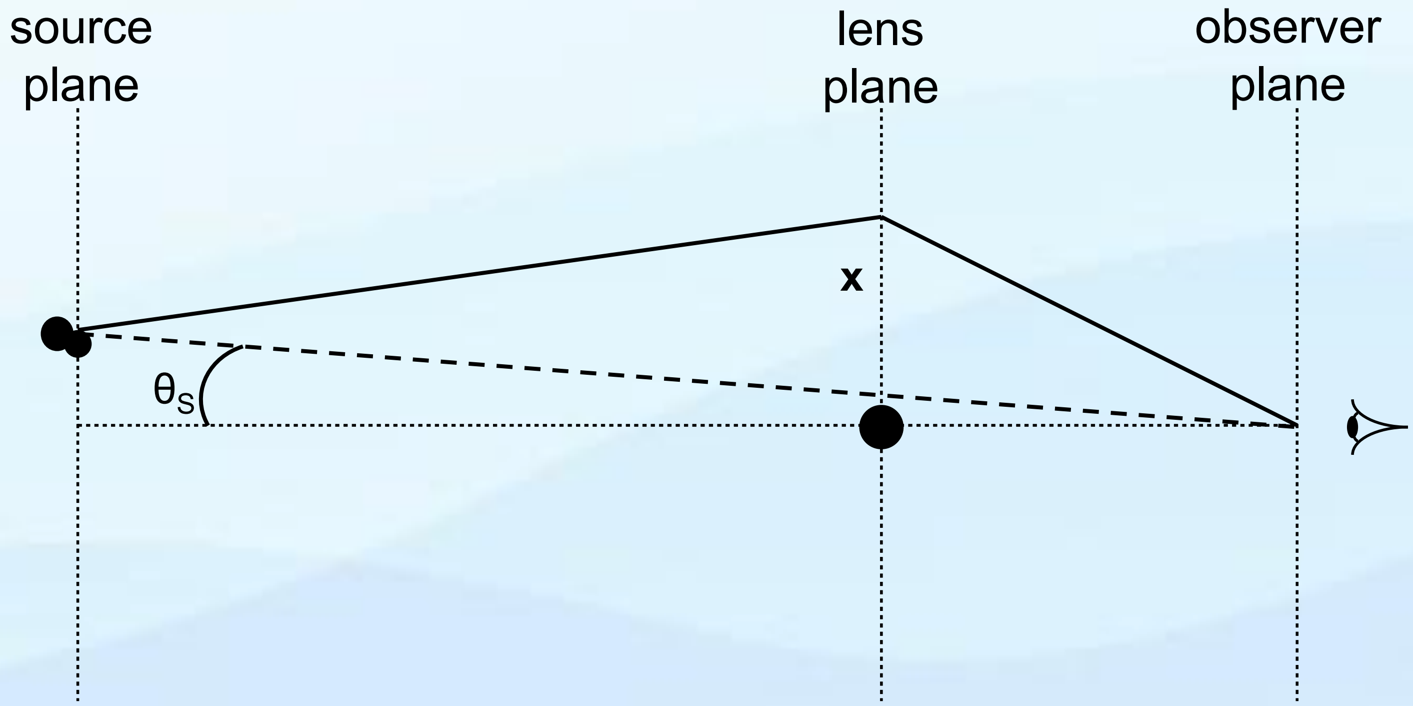

Gravitational lensing: EM waves vs gravitational waves