



**UNIVERSITÉ
DE GENÈVE**

Dynamical Love Numbers: a Shell Effective Field Theory

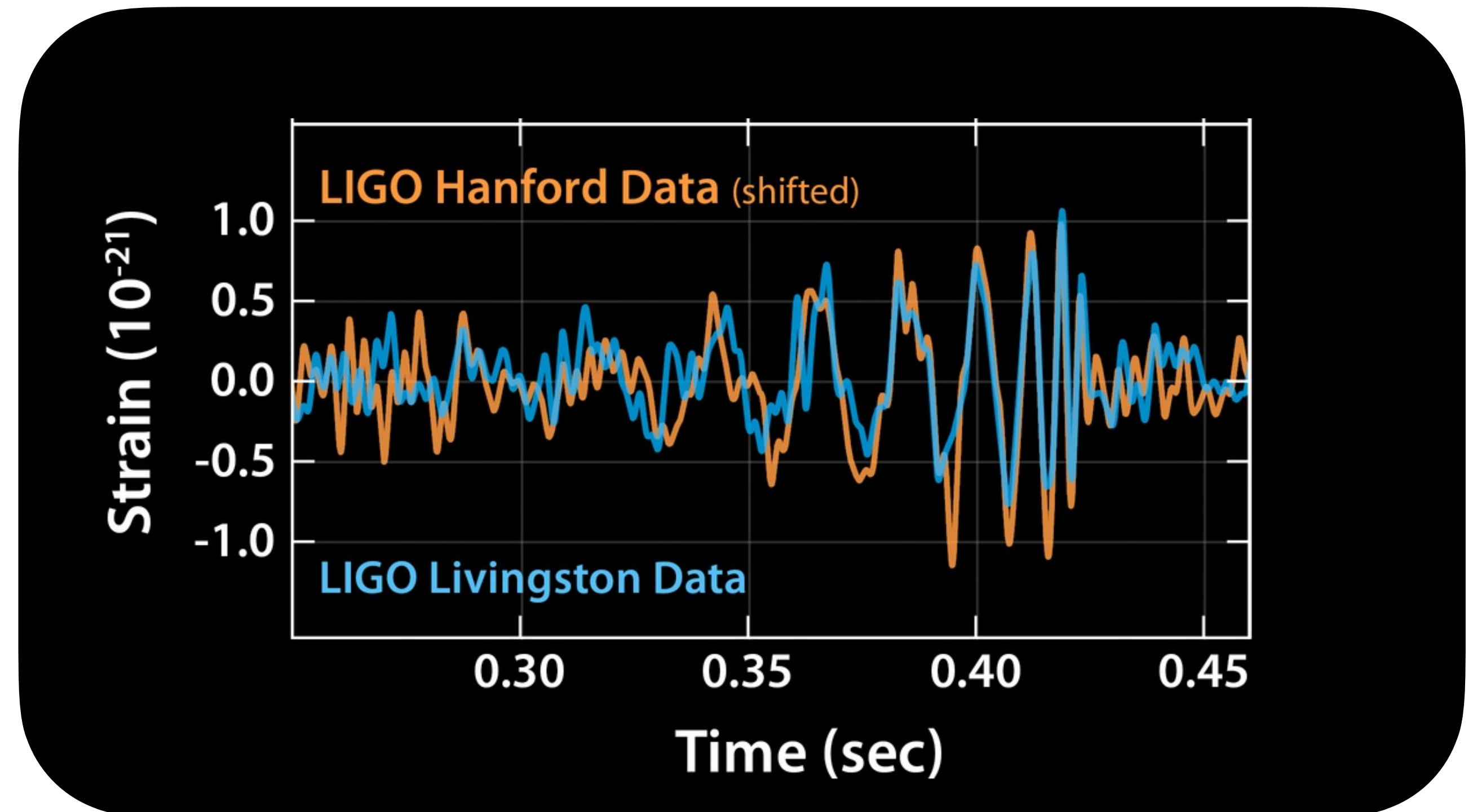
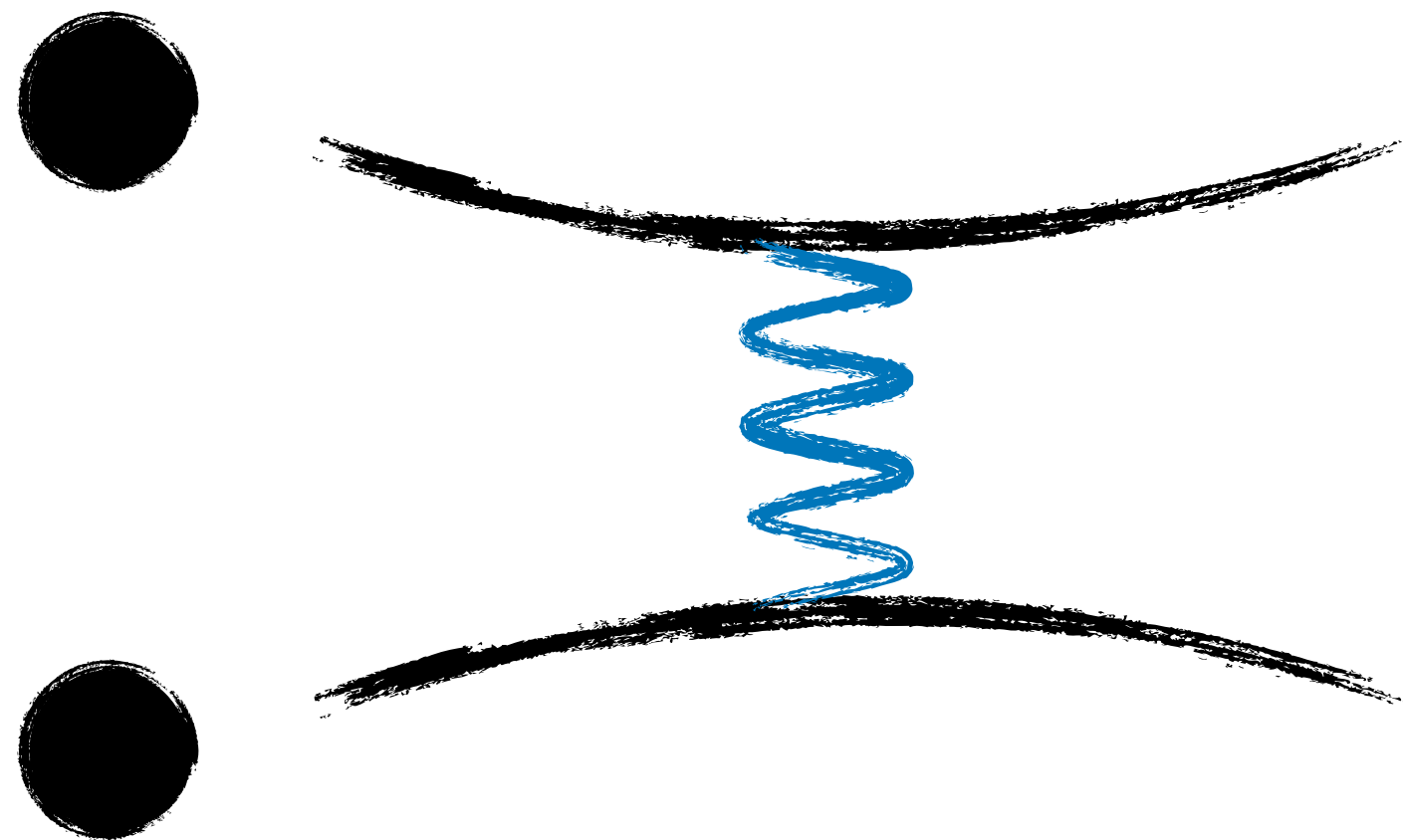
**Based on [2512.04002](#) with
D. Kosmopoulos (CERN) and M. Solon (UCLA)**

**Davide Perrone,
Swiss Cosmology Days,
Geneva, 4 June 2026.**

Particle physics tools for gravity

Using amplitudes to predict the waveforms

Effective field theory
+
Amplitude techniques



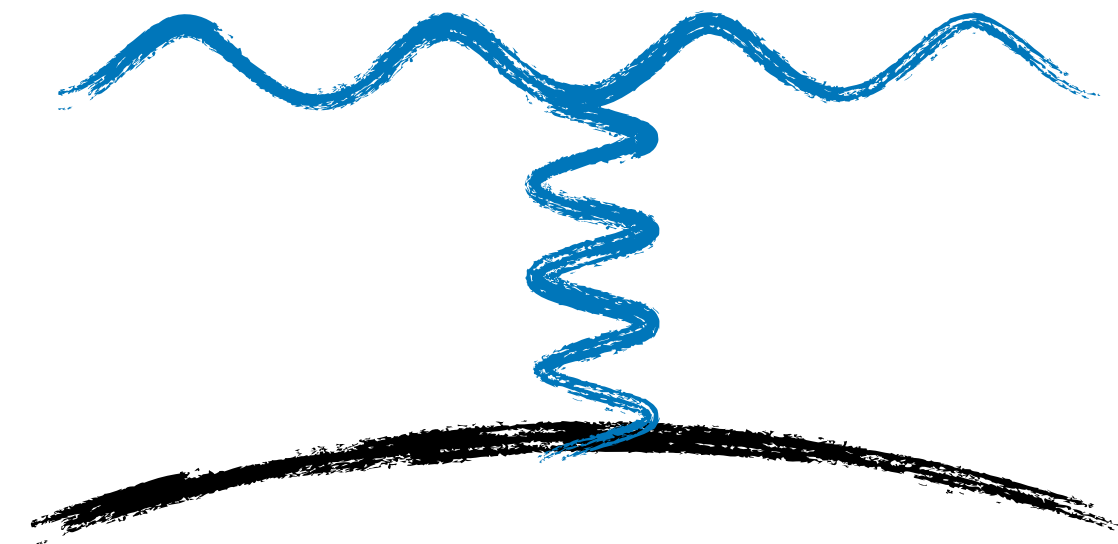
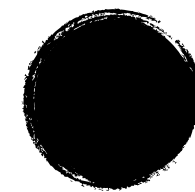
Particle physics tools for gravity

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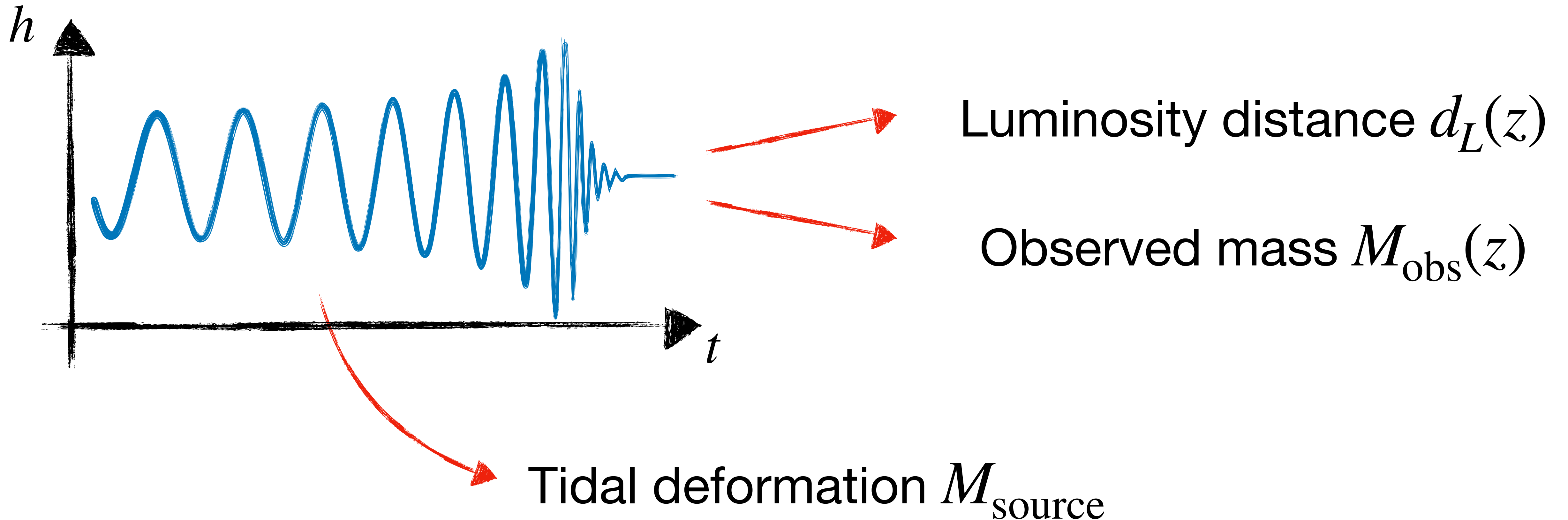
Probe



Describes tides which appear in the waveform!

Tides and Cosmology

Find the redshift from the tidal deformation



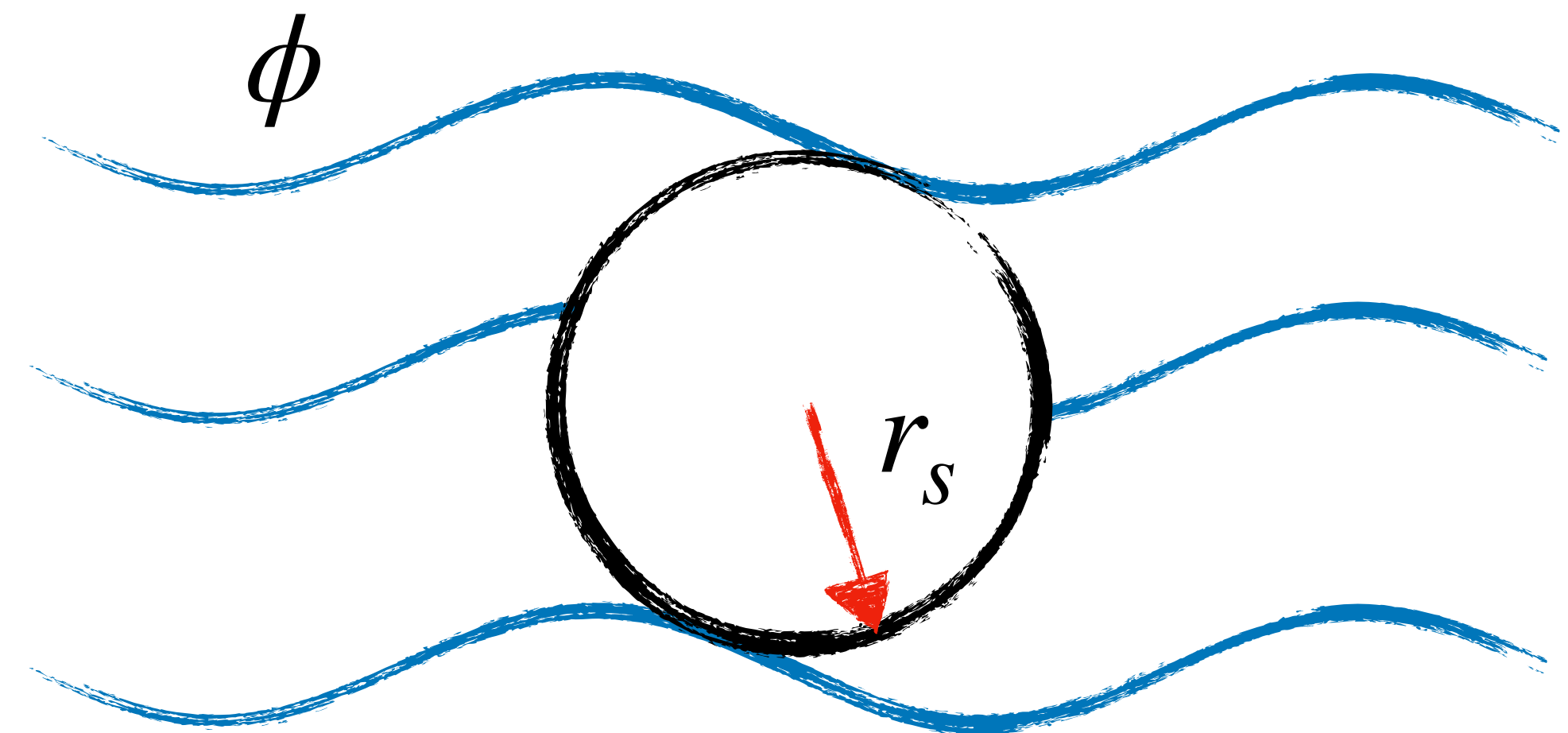
Schwarzschild black hole and a probe

Gravitational interaction with a scalar field

$$ds^2 = - \left(1 - \frac{r_s}{r} \right) dt^2 + \left(1 - \frac{r_s}{r} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad r_s = 2GM$$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \, g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\omega \ll r_s^{-1}$$



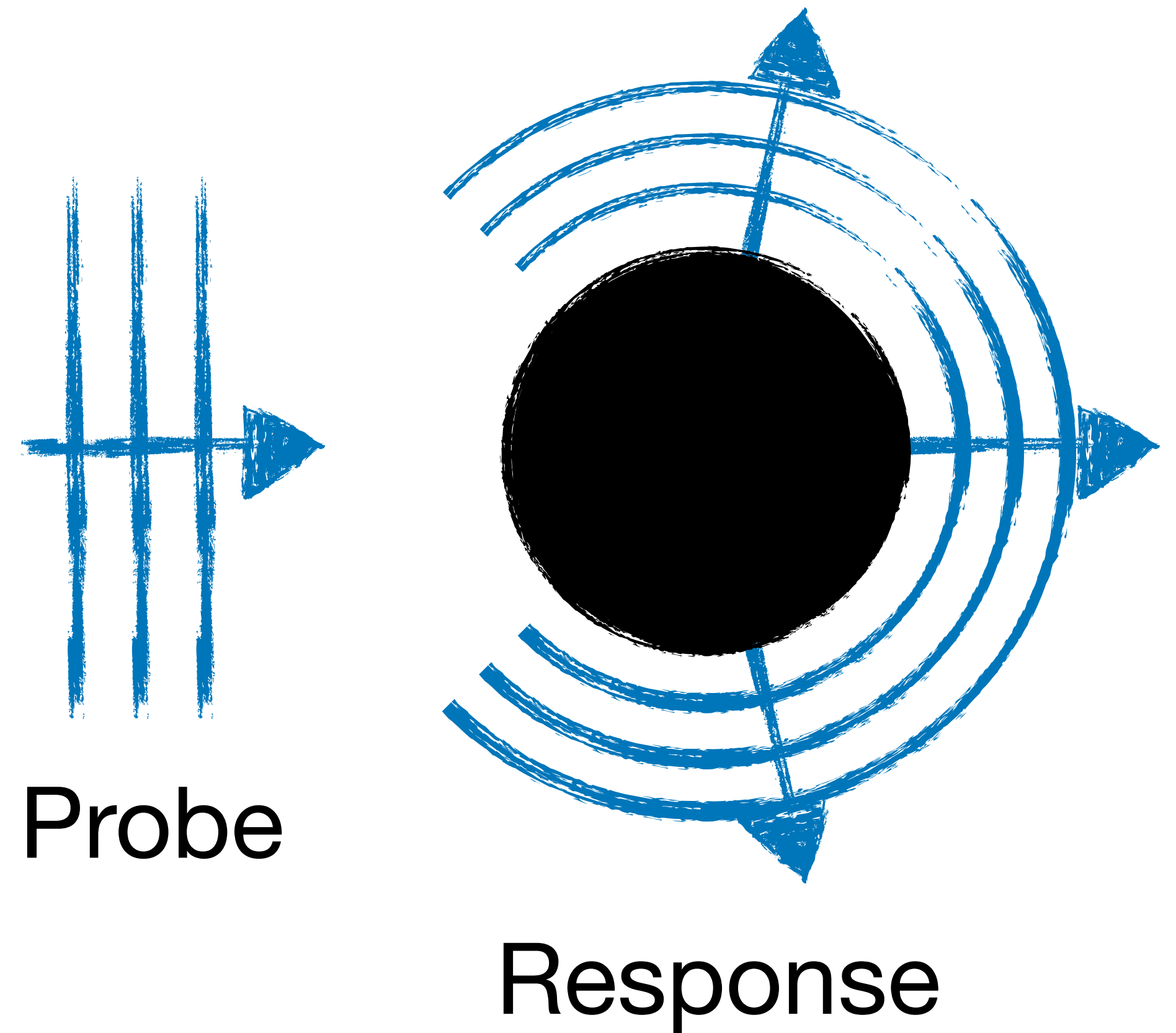
Love numbers, static

Capture the tidal response of an object to an external probe field

In the static limit, $\nabla_{\text{BH}}^2 \phi = 0$

$$\phi = \sum_{\ell} \phi_{\ell}(r) P_{\ell}(\cos \theta)$$

$$\phi_{\ell} = A_{\ell} P_{\ell} \left(\frac{2r}{r_s} - 1 \right) + B_{\ell} Q_{\ell} \left(\frac{2r}{r_s} - 1 \right)$$



Asymptotic behaviours

Probe and response far away and close to the object

$$\phi_\ell = A_\ell P_\ell \left(\frac{2r}{r_s} - 1 \right) + B_\ell Q_\ell \left(\frac{2r}{r_s} - 1 \right)$$



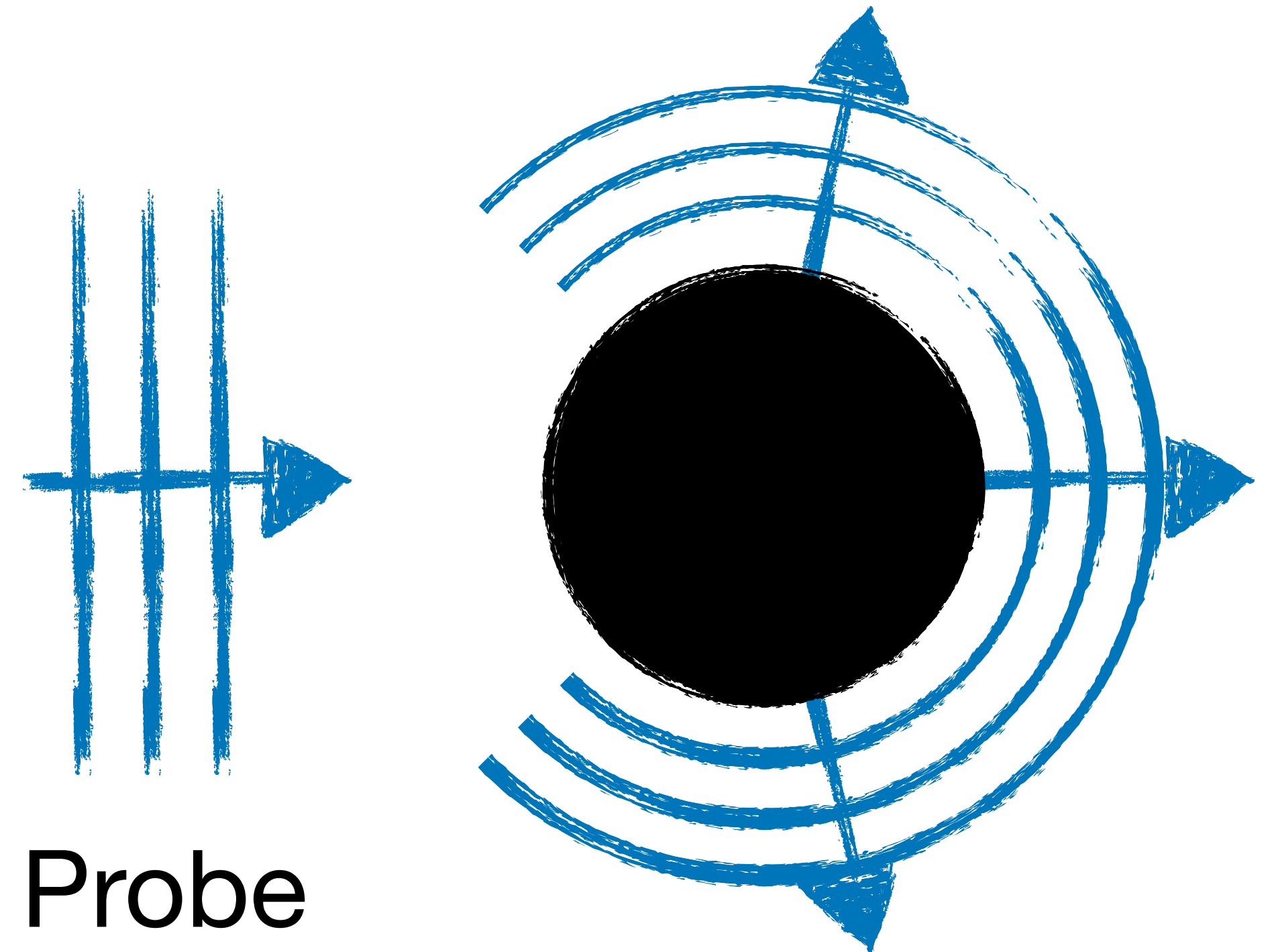
$r \rightarrow \infty$



$$A_\ell (r/r_s)^\ell$$

$$B_\ell (r/r_s)^{-\ell-1}$$

Static Love numbers $k_\ell \propto B_\ell/A_\ell$



Probe

$$\propto A_\ell$$

Response

$$\propto B_\ell$$

Asymptotic behaviours

Probe and response far away and close to the object

$$\phi_\ell = A_\ell P_\ell \left(\frac{2r}{r_s} - 1 \right) + B_\ell Q_\ell \left(\frac{2r}{r_s} - 1 \right)$$



A_ℓ

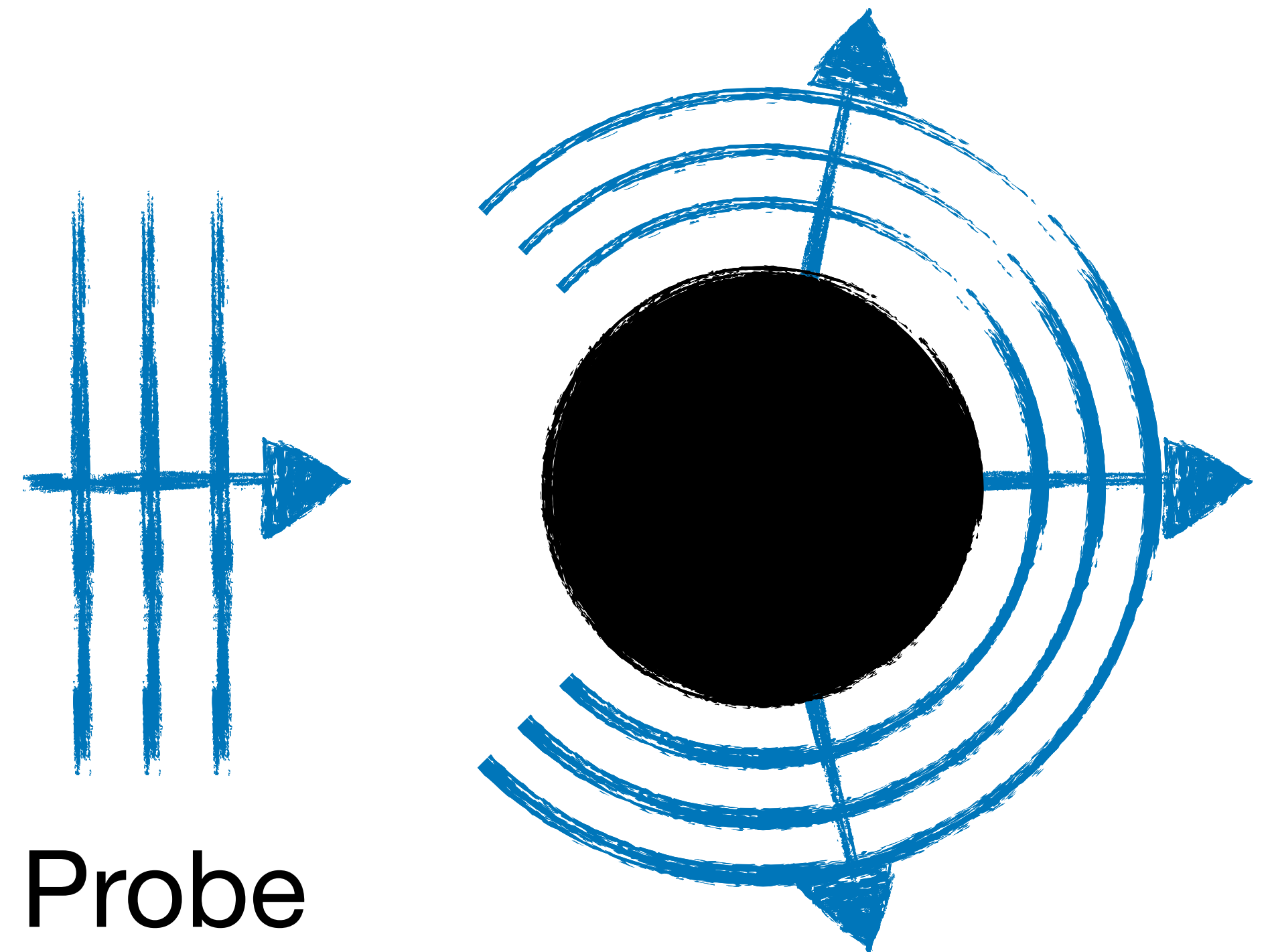
$r \rightarrow r_s$



$B_\ell \log(r/r_s - 1)$

Regularity at the horizon implies

$$B_\ell = 0 \implies k_\ell = 0$$



Probe

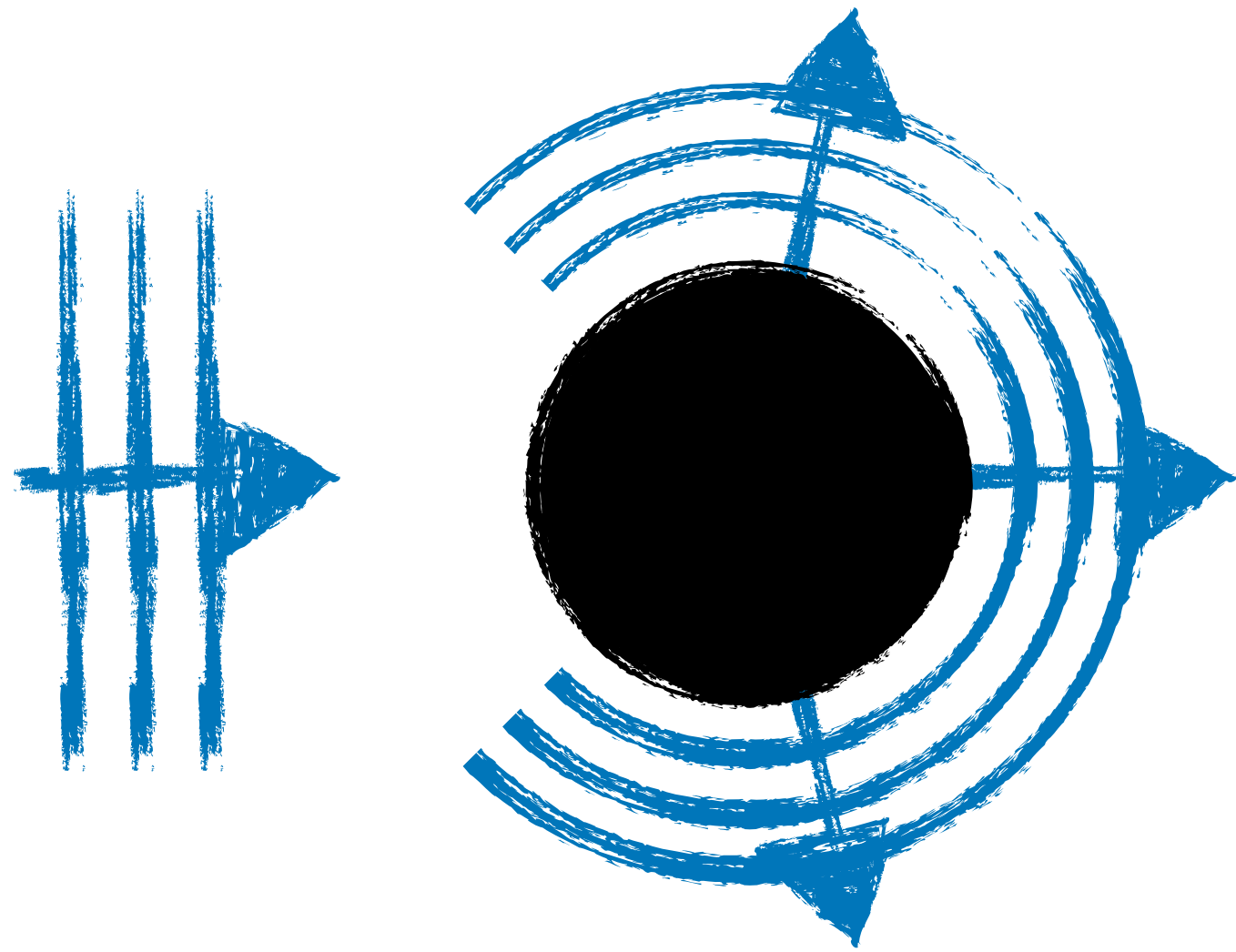
$$\propto A_\ell$$

Response

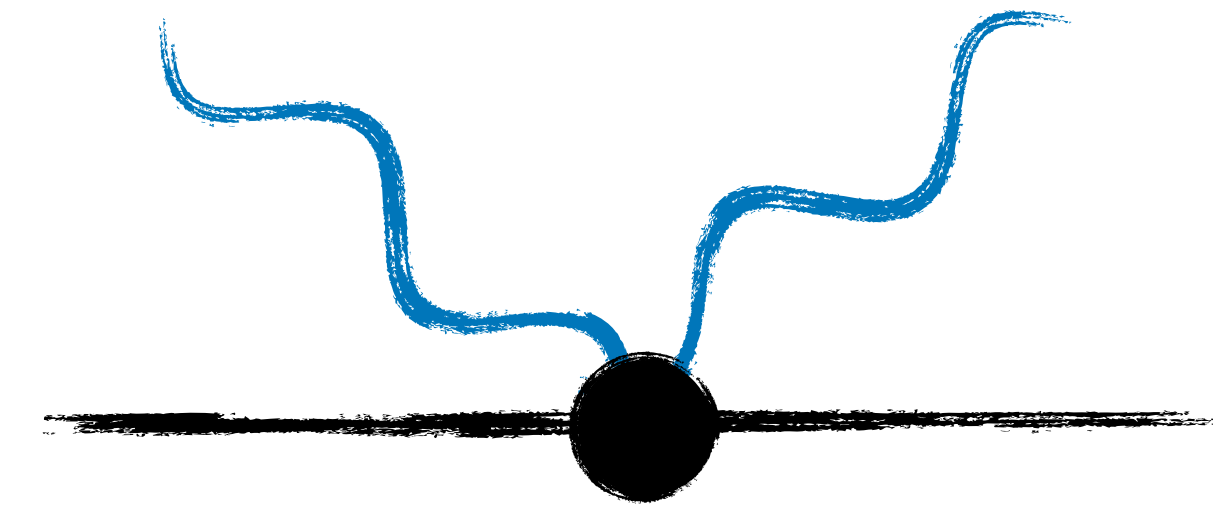
$$\propto B_\ell$$

Love numbers and amplitudes

Incorporate full solutions in the point particle description



Full solution is known



Effective point particle
description

[Leaver (1986), Mano, Suzuki, Takasugi (1996)]

Worldline effective field theory

A point particle and a toy graviton

$$S = - \int d\tau \, M \quad + \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \int d\tau \, (c_0 \phi^2 + \dots)$$

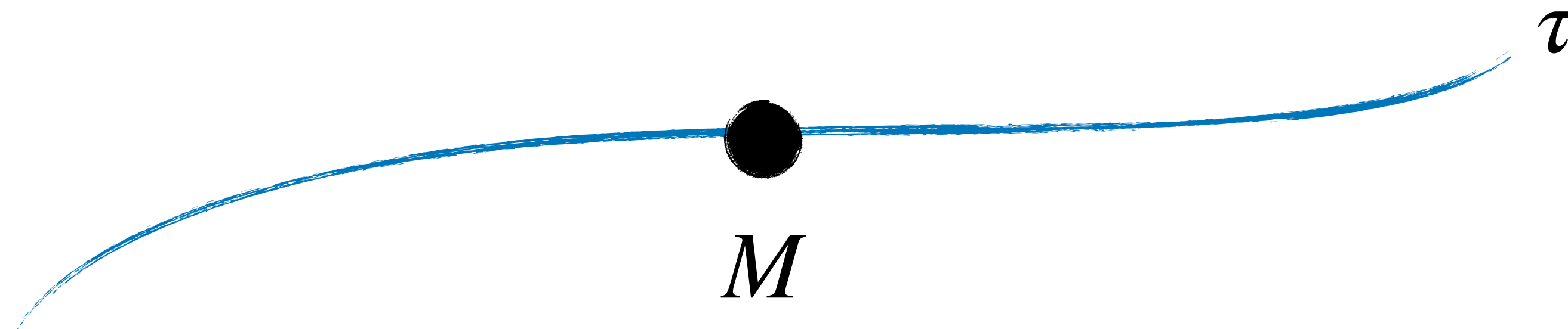
Worldline effective field theory

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$$S = - \int \boxed{d\tau M} + \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \int d\tau (c_0 \phi^2 + \dots)$$



Worldline



Worldline effective field theory

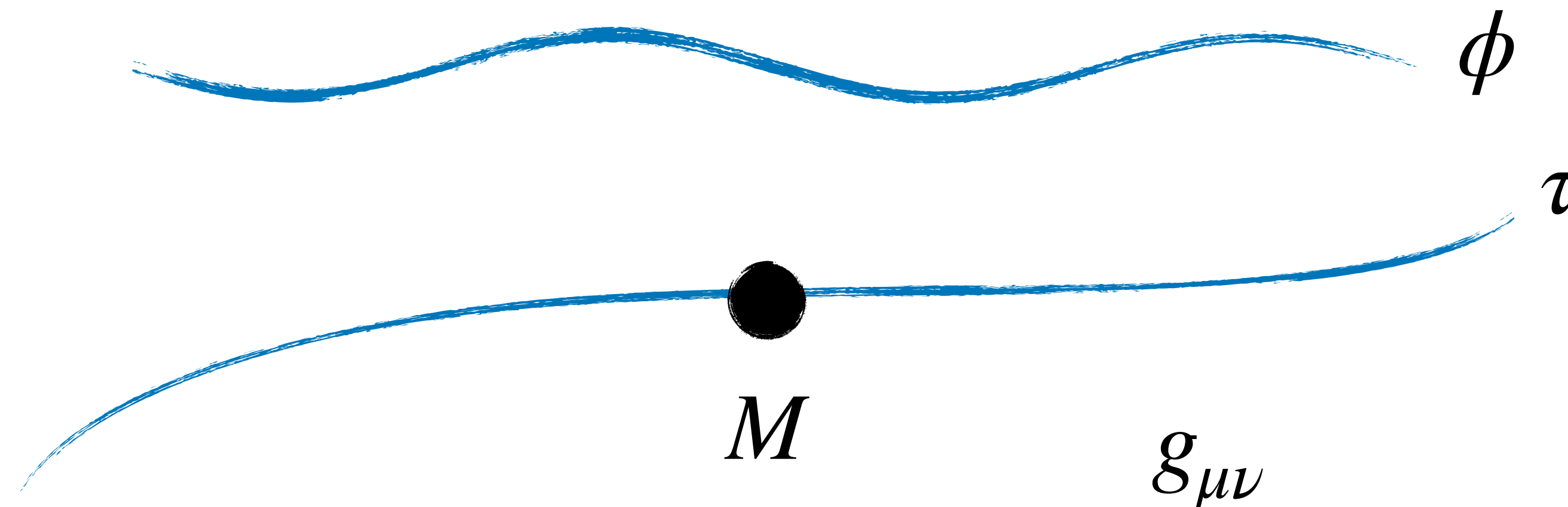
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Worldline

Kinetic term



Worldline effective field theory

A point particle and a toy graviton

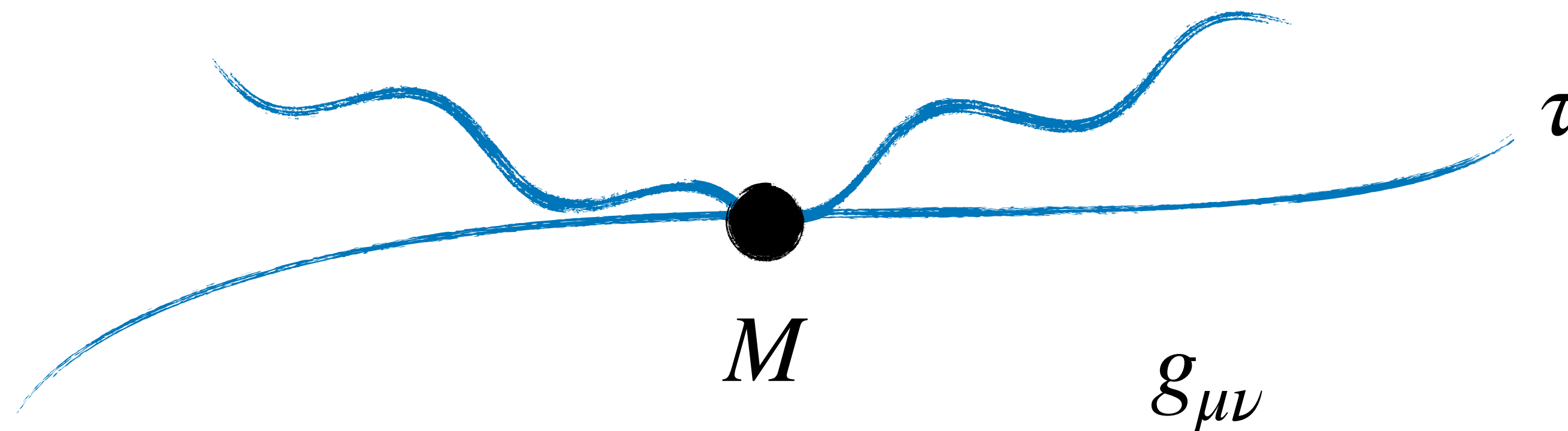
$$S = - \int d\tau M + \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \int d\tau (c_0 \phi^2 + \dots)$$



Worldline

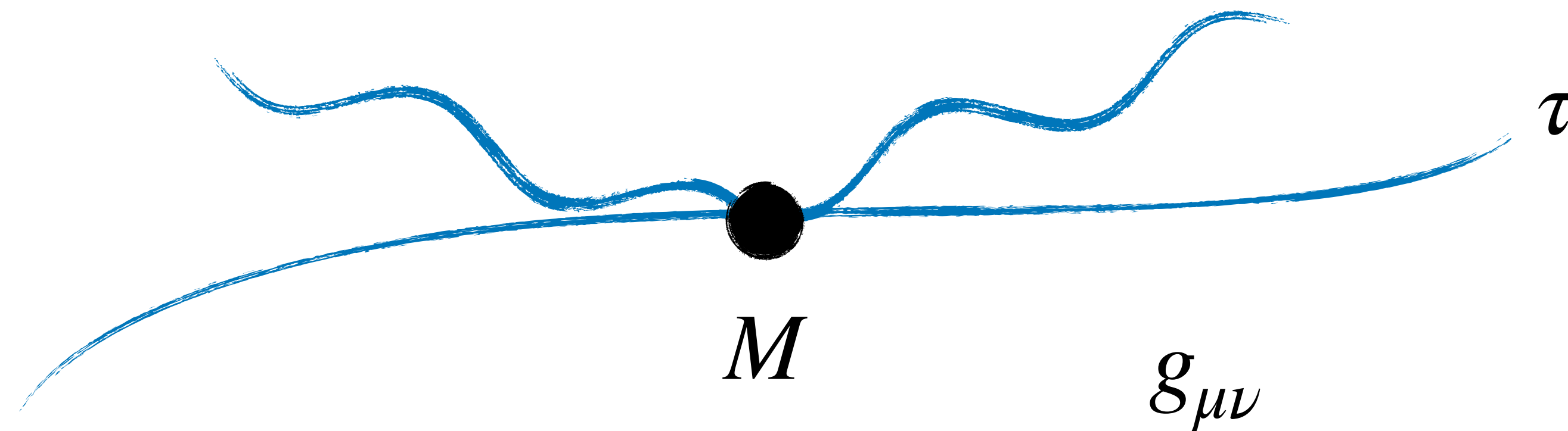
Kinetic term

Contact interactions



Worldline effective field theory

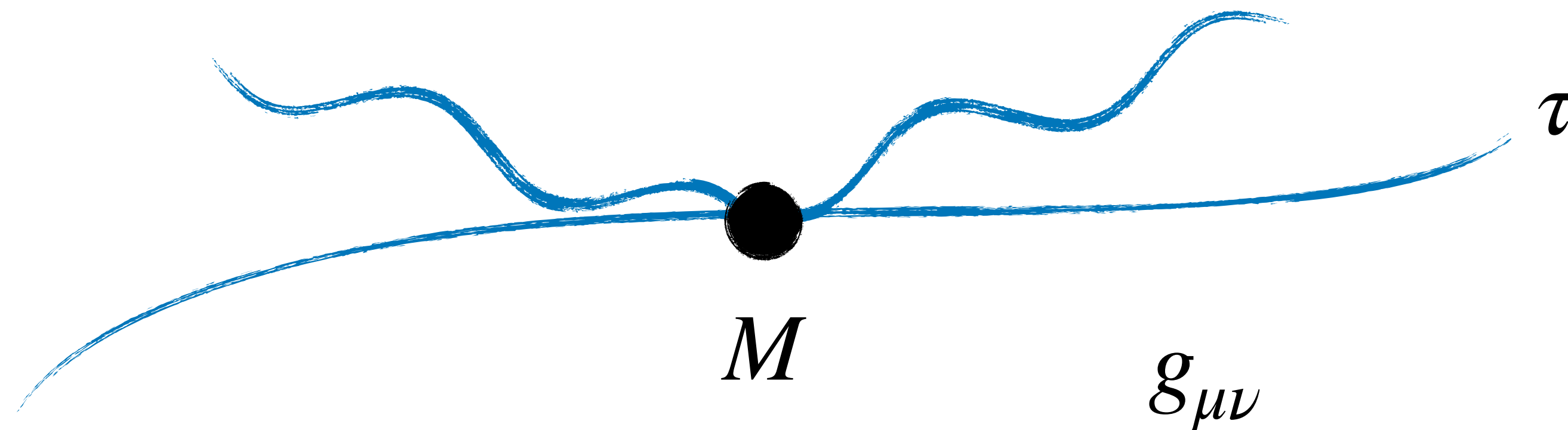
Describe a compact object with higher derivative operators



$$S = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \sum_{\ell, n} \frac{1}{\ell!} \int d\tau c_{\ell, n} \nabla_{\langle \ell \rangle} \phi \partial_\tau^n \nabla_{\langle \ell \rangle} \phi$$

Worldline effective field theory

Describe a compact object with higher derivative operators



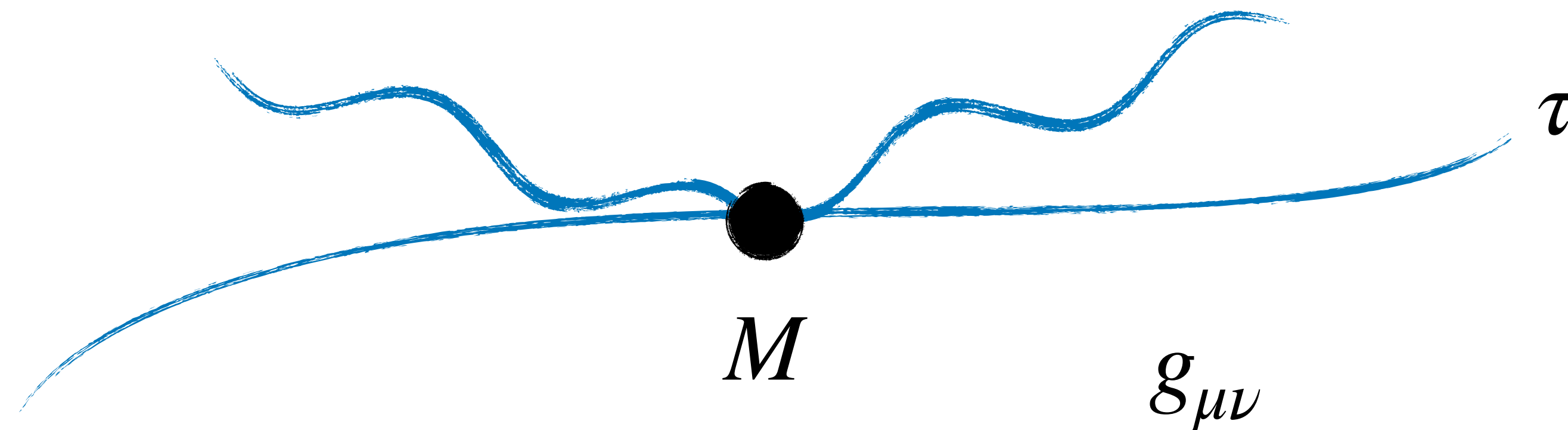
$$S = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \sum_{\ell, n} \frac{1}{\ell!} \int d\tau \, c_{\ell, n} \nabla_{\langle \ell \rangle} \phi \partial_\tau^n \nabla_{\langle \ell \rangle} \phi$$

Wilson coefficients

$$c_{\ell, n} \sim k_{\ell, n}$$

Worldline effective field theory

Describe a compact object with higher derivative operators



$$S = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \sum_{\ell, n} \frac{1}{\ell!} \int d\tau c_{\ell, n} \boxed{\nabla_{\langle \ell \rangle} \phi \partial_\tau^n \nabla_{\langle \ell \rangle} \phi}$$

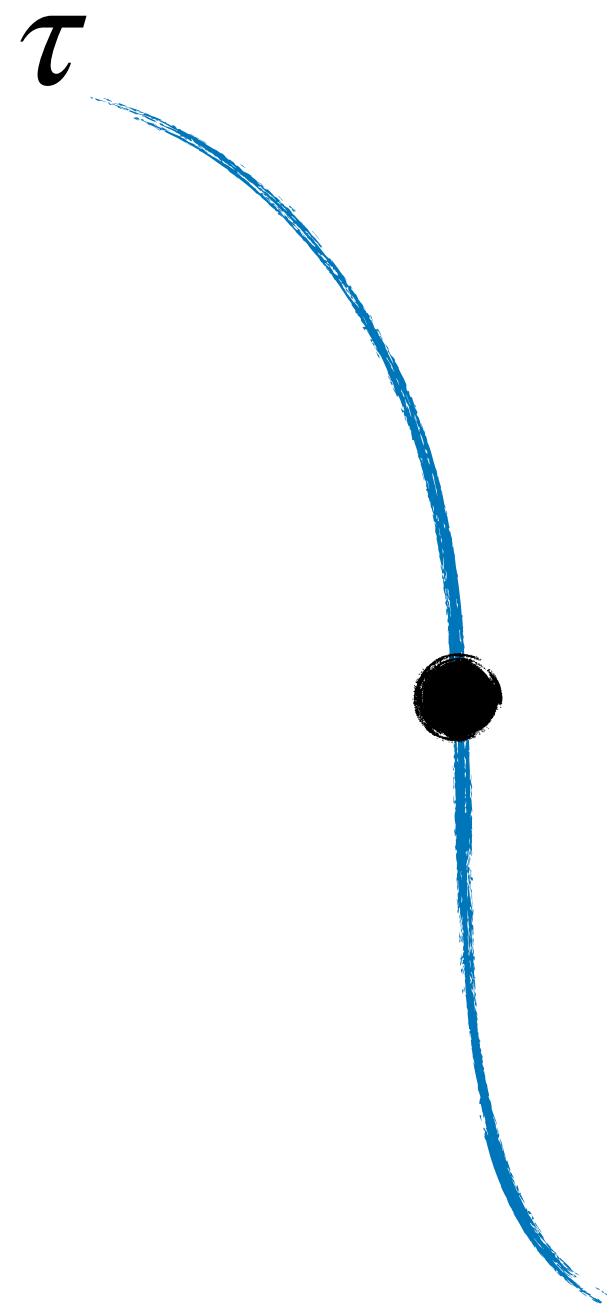
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Divergent on the
worldline

Shell regularisation

To tame the divergent behaviour on top of the worldline

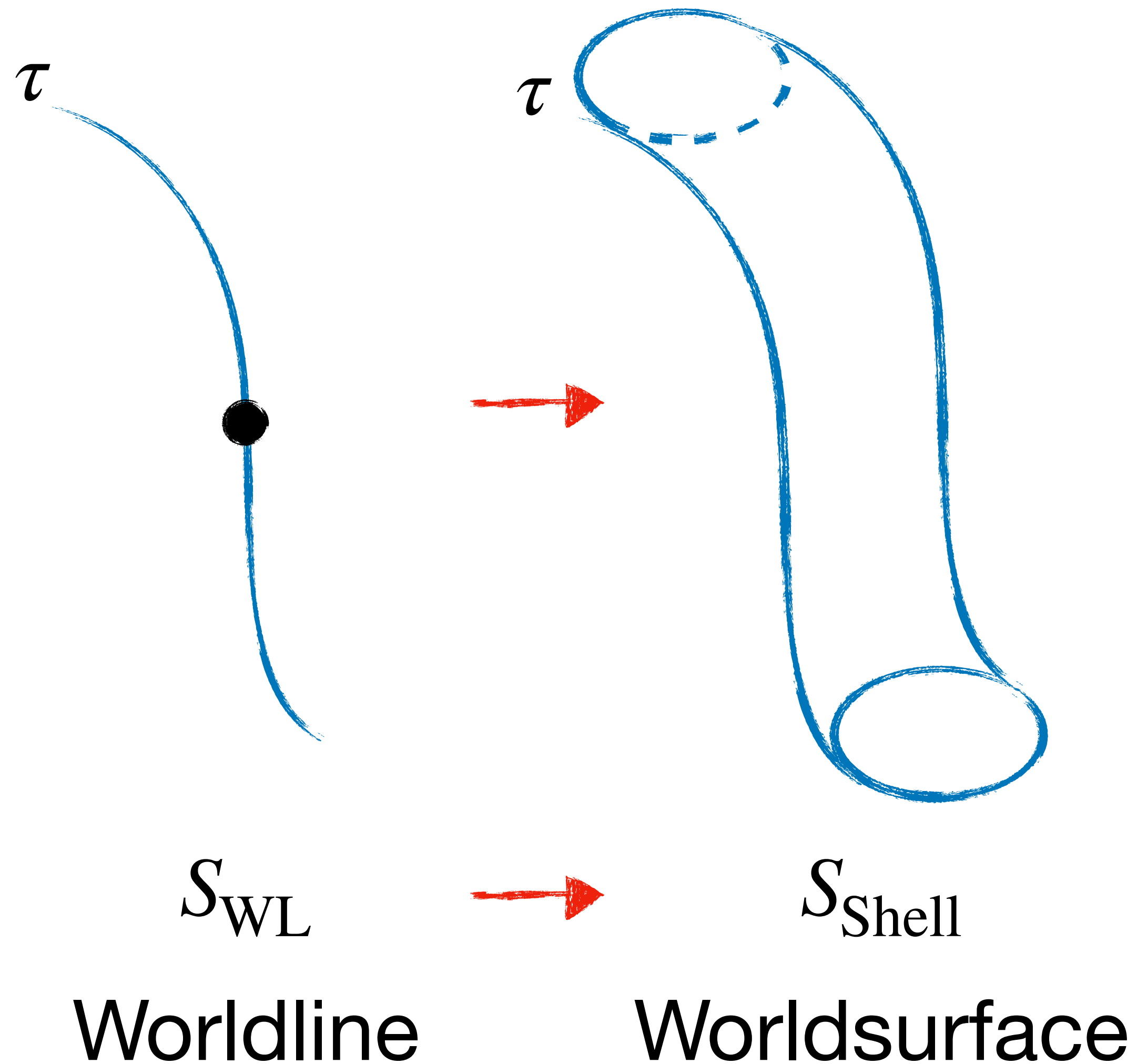


S_{WL}

Worldline

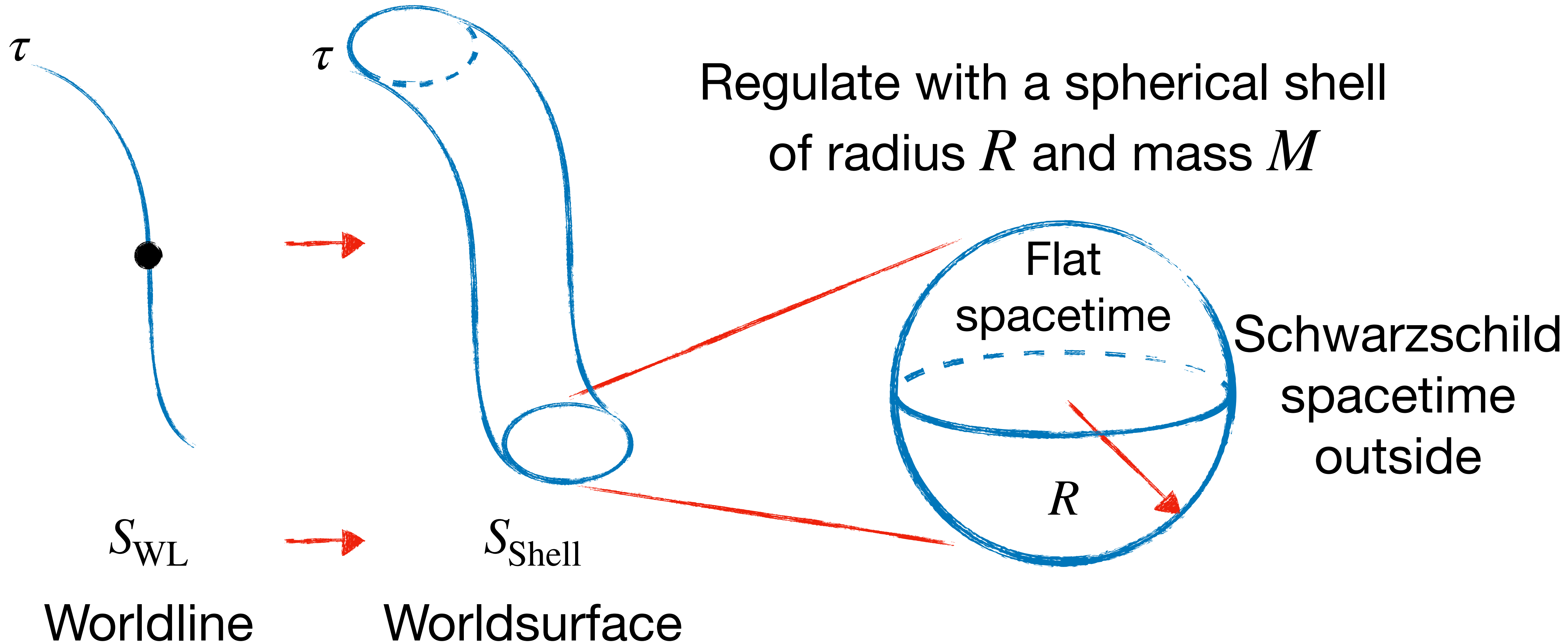
Shell regularisation

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Shell regularisation

To tame the divergent behaviour on top of the worldline



Equation of motion, with a Shell

Discontinuous solution across the Shell surface

Flat
spacetime

$$\square_{\text{flat}} \phi^- = 0$$

Schwarzschild
spacetime

$$\square_{\text{BH}} \phi^+ = 0$$

Equation of motion, with a Shell

Discontinuous solution across the Shell surface

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$$\square_{\text{BH}} \phi^+ = 0$$

$$\phi_\ell^+ \propto A_\ell \sum_{n=-\infty}^{+\infty} b_n^\ell F_{n+\nu^\ell} + B_\ell \sum_{n=-\infty}^{+\infty} b_n^\ell \bar{F}_{n+\nu^\ell}$$

Coulomb Wavefunctions,
 $\nu^\ell \sim$ renormalised angular momentum

[Leaver (1986), Mano, Suzuki, Takasugi (1996)]

Equation of motion, with a Shell

Discontinuous solution across the Shell surface

Flat
spacetime

$$\square_{\text{flat}} \phi^- = 0$$



$$\phi_\ell^- = \tilde{A}_\ell j_\ell(\tilde{r}\tilde{\omega})$$

Schwarzschild
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$$\square_{\text{BH}} \phi^+ = 0$$

$$\phi_\ell^+ \propto A_\ell \sum_{n=-\infty}^{+\infty} b_n^\ell F_{n+\nu^\ell} + B_\ell \sum_{n=-\infty}^{+\infty} b_n^\ell \bar{F}_{n+\nu^\ell}$$

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Discontinuous solution across the Shell surface

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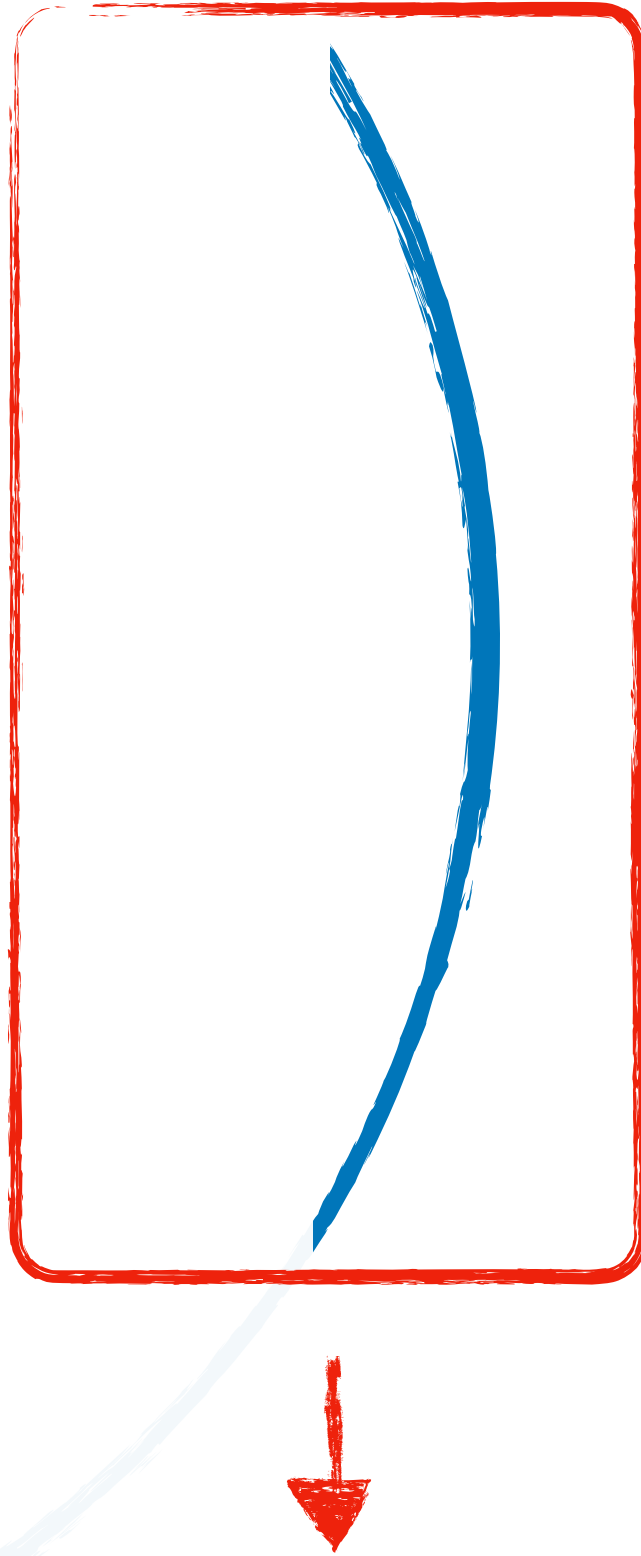
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$$\phi_\ell^+ \propto A_\ell \sum_{n=-\infty}^{+\infty} b_n^\ell F_{n+\nu^\ell} + B_\ell \sum_{n=-\infty}^{+\infty} b_n^\ell \bar{F}_{n+\nu^\ell}$$


$$\frac{\delta S_{\text{Shell}}}{\delta \phi} \propto \delta(r - R) \sum_{\ell, n} \frac{\phi_\ell}{R^{2\ell+2}} (-i\omega)^n c_{\ell, n}$$

Dynamical Love numbers

For any compact object, to all orders in G

Matching the discontinuity from EoMs with the Shell contribution

$$\mathcal{F}_\ell(\omega) = \sum_n (-i\omega)^n c_{\ell,n} = \frac{\pi 2^{\ell+2} \ell!}{(2\ell+1)!} \left(1 - \frac{R_s}{R}\right)^{\ell+1/2} R^{2\ell+2} \left(\frac{\phi_\ell'^- - \phi_\ell'^+}{\phi_\ell} \right) \Big|_R$$

1) Expand in $R_s = 2GM$

2) Take the limit $R \rightarrow 0$

Dynamical Love numbers

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Matching the discontinuity from EoMs with the Shell contribution

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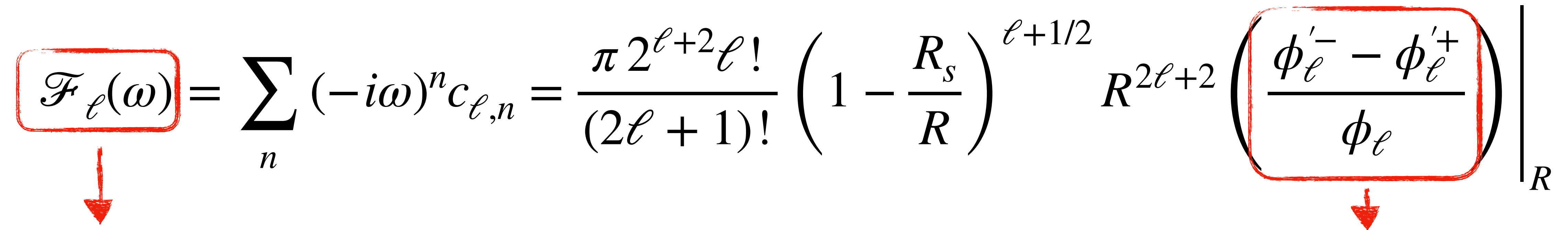

Dynamical
Love Numbers

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Dynamical
Love Numbers

Discontinuity
contribution

- 1) Expand in $R_s = 2GM$
- 2) Take the limit $R \rightarrow 0$

Dynamical Love numbers for black holes

Comparison with previous results

$$c_{0,0} = \boxed{0}$$

 Scheme independent,
match dim. reg.

$$c_{0,1} = 4\pi R_s^2 \left(\boxed{1} + \boxed{\frac{R_s}{2R}} + \mathcal{O}(R_s^2) \right)$$

 Power law divergencies

$$c_{0,2} = 4\pi R_s^3 \left(\left(\frac{7}{8} - \boxed{\frac{5R_s}{128R}} + \mathcal{O}(R_s^2) \right) + \log \frac{R_s}{R} \left(\boxed{1} + \frac{R_s}{2R} + \mathcal{O}(R_s^2) \right) \right)$$

•

•

•

$$c_{0,4} = 4\pi R_s^5 \left(\frac{13343}{34560} - \boxed{\frac{11\pi^2}{9}} + \frac{461}{72} \log \frac{R_s}{R} + \boxed{\frac{11}{6}} \left(\log \frac{R_s}{R} \right)^2 - \boxed{2\zeta_3} + \mathcal{O}(R_s^2) \right)$$

$\mathcal{O}(G^3)$ -[Ivanov, Li, Parra-Martinez, Zhou (2024)]

$\mathcal{O}(G^7)$ -[Caron-Hout, Correia, Isabella, Solon (2025)]

Conclusions

- New formalism which incorporates exact solutions from Black Hole Perturbation Theory in an Effective Field Theory,
- Generalisable to graviton case in Schwarzschild black hole, and can be compared with recent similar results,
- Easy to calculate Love Numbers at high order in G without any integration.

Stability of the shell

An equation of state to avoid collapse

$$S_{ab} = \sigma u_a u_b + p (\gamma_{ab} + u_a u_b)$$

$$\sigma = \frac{1}{4\pi R} \left(1 - \sqrt{1 - \frac{2GM}{R}} \right)$$

$$p = \frac{\left(1 - \frac{2GM}{R} \right)^{-1/2}}{8\pi R} \left(1 - \sqrt{1 - \frac{2GM}{R}} - \frac{GM}{R} \right)$$

Hidden structure of Love numbers

How to group Love numbers in transcendentality classes

$$(-1)^n \frac{\lambda_{\ell,n}}{4\pi r_s^{2\ell+n+1}} = \mathbf{L}_{\ell} \zeta_{n-1} + \sum_a \left(\mathbf{L}_{\ell,n}^{(2)} \right)_{a_1,a_2} \zeta_{a_1} \zeta_{a_2} + \sum_a \left(\mathbf{L}_{\ell,n}^{(3)} \right)_{a_1,a_2,a_3} \zeta_{a_1} \zeta_{a_2} \zeta_{a_3}$$



$$\lambda_{\ell,n} = c_{\ell,n} - \mathcal{O}(r_s/R)$$



$$a_1 + a_2 = n - 2$$



$$a_1 + a_2 + a_3 = n - 3$$

$$\zeta_1 \equiv -\frac{1}{2} \log(r_s/R)$$

Why all of this?
The scheme-independent terms
do not depend on n and a_i !

Scheme-independent coefficients

Universal values to all orders in G

For the scheme-independent ones

$$\left(\mathbf{L}_{\ell,n}^{(N)}\right)_{a_i} = \mathbf{L}_{\ell}^{(N)}$$

But n is linked to the
order in G



The result is the same for all orders!

	$\mathbf{L}_{\ell}$	$\mathbf{L}_{\ell}^{(2)}$	$\mathbf{L}_{\ell}^{(3)}$	$\mathbf{L}_{\ell}^{(4)}$
$l = 0$	-2	$22/3$	$-628/27$	$5830/81$
$l = 1$	$-1/6$	$19/90$	$-361/2025$	$-$
$l = 2$	$-1/270$	$79/28350$	$-$	$-$
$l = 3$	$-1/21000$	$-$	$-$	$-$

Resummation of black hole Love numbers

Hints towards quasi-normal modes

Our conjecture enables us to resum the Love numbers

$$\mathcal{F}_\ell = \sum_n (-i\omega)^n \lambda_{\ell,n}$$

$$\frac{\mathcal{F}_\ell}{4\pi r_s^{2\ell+1}} = -\frac{1}{2}\eta \mathbf{L}_\ell \left(1 + 2\eta \boxed{H(-\eta)} + \eta \log(r_s/R) \right) + \dots, \quad \eta = i\omega r_s$$



Has poles for frequencies $\omega = -in/r_s$